QFT II - Problem Set 9

(45) Mass beats loops

Consider a scalar ϕ^4 theory. We computed in (39) that the one-loop correction to $U'(\rho)$ is

$$U'(\rho)_{1l} = \left.\frac{\partial U(\rho)}{\partial \rho}\right|_{1l} = \frac{3}{2}\lambda \int_q \frac{1}{q^2 + m^2 + 3\lambda\rho}. \tag{1}$$

a) For a fixed UV-cutoff Λ, what is $U'(\rho)_{1l}$ if the mass $m^2 \to \infty$, i.e. $m^2 \gg \Lambda^2$?

b) Just qualitatively, what about diagrams with more than one loop ?

(46) Momentum depended additional mass

As we have seen in (45), we can suppress quantum corrections using large masses. So suppose we add a momentum depended mass term $R_k(p)$ to the Lagrangian

$$\mathcal{L} = \frac{1}{2}[p^2 + m^2 + R_k(p)]\Phi^2 + \frac{\lambda}{8}\Phi^4.$$

a) Suppose that

$$R_k(p) = \frac{Z_k(p)p^2}{e^{p^2/k^2} - 1},$$

where $Z_k(p)$ is supposed to vary only mildly. Sketch $R_k(p)$ as a function of p^2. What happens for $p^2 \lessapprox k^2$?

b) Sketch the behavior of another choice for $R_k(p)$

$$R_k(p) = Z_k(p)(k^2 - p^2)\theta(k^2 - p^2)$$

c) Finally, sketch the behavior of

$$R_k(p) = \frac{Z_k(p)p^2}{e^{-p^2/\Lambda^2} - e^{-p^2/k^2}} \left[1 - e^{-p^2/\Lambda^2} + e^{-p^2/k^2}\right].$$

What happens in the limits of:

(i) $k^2 \ll p^2 \ll \Lambda^2$

(ii) $p^2 \ll k^2 \ll \Lambda^2$

(iii) $k^2 \to \Lambda^2$

(47) YOUR FIRST OWN FLOW EQUATION

A flow equation for the effective action is

$$\partial_t \Gamma_k = \frac{1}{2} Tr \frac{\partial_t R_k}{\Gamma^{(2)} + R_k},$$

where $t = \ln k$ and e.g. for a scalar field theory

$$\Gamma^{(2)} = \frac{\delta^2 \Gamma}{\delta\varphi\delta\varphi}.$$

We would like to consider the running of the parameters of a φ^4 theory. For simplicity, let us consider

$$\Gamma[\rho] = \int \frac{\mathrm{d}^d q}{(2\pi)^d} \left\{ Z_k(\rho, q) q^2 \rho + \frac{\lambda_k}{2} (\rho - \rho_0(k))^2 \right\}$$

where as usual $\rho = \frac{1}{2}\varphi^2$.

a) Convince yourself that for constant φ, this leads to the running of the potential according to

$$\partial_t U_k(\rho) = \frac{1}{2} \int \frac{\mathrm{d}^d q}{(2\pi)^d} \frac{\partial_t R_k(q)}{U' + 2\rho U'' + Z_k(\rho, q^2) q^2 + R_k(q)}, \tag{2}$$

where $'$ denotes derivatives with respect to ρ.

b) Let us take the *Litim* cut-off

$$R_k(p) = Z_k(p, \rho)(k^2 - p^2)\theta(k^2 - p^2).$$

Compute $\partial_t R_k$. You can neglect the derivative of θ (why?).

c) Use

$$\eta \equiv \frac{\partial_t Z_k}{Z_k}$$

to re-write your expression of $\partial_t R_k$.

d) Plug your result for $\partial_t R_k$ and the expression for R_k into Equation (2)

e) Suppose that Z_k does not depend on p, i.e. $Z_k = Z_k(\rho)$. Perform the q-integration in the flow equation using the θ function and

$$\int \frac{\mathrm{d}^d q}{(2\pi)^d} = \frac{\Omega_d}{(2\pi)^d} \int \mathrm{d}q q^{d-1}.$$

f) To get a flow equation for λ_k, we use the fact that

$$U'' = \lambda,$$

i.e. to get a flow equation for λ_k, derive your result of (e) twice with respect to ρ. For convenience, we set $Z_k = const$, i.e. it does not depend on ρ.