Soft contributions to the shear and bulk viscosities

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Contents

Experimental indications

RHIC data exhibit strong collective phenomena in the asymmetric azimuthal distribution around the beam axis:

$$p_0 \frac{d^3N}{dp^3}\bigg|_{p_z=0} = v_0(p_\perp)\left[1 + 2v_2(p_\perp)\cos(2\phi) + 2v_4(p_\perp)\cos(4\phi) + \cdots\right],$$

where $(p_x, p_y, p_z) = (p_\perp \cos\phi, p_\perp \sin\phi, p_z)$.

The large elliptic flow $v_2 \simeq 0.06$ cannot be described just by two-body interactions between partons.

Particles of different mass are emitted from the fireball with a common fluid velocity.

Relativistic hydrodynamics reproduces v_2 very well, up to $p_\perp \sim 1.5\,\mathrm{GeV}$ (P. Huovinen, U.W. Heinz, '01).

Experimental indications

$L_{\rm mfp}$ of a parton, which traverses an (ideal quantum) liquid is much smaller than the thermal wavelength $\sim \beta \equiv \frac{1}{T}$, i.e.

$$\frac{L_{\rm mfp}^{\rm liq.}}{\beta} \ll 1.$$

Instead, in the dilute-gas model of the QGP,

$$L_{\rm mfp}^{\rm gas} \sim (\rho\sigma_t)^{-1},$$

where $\rho \sim T^3$ is the particle-number density, $\sigma_t \sim g_T^4 \beta^2 \ln g_T^{-1}$ is the Coulomb transport cross-section, and g_T is the perturbative finite-T QCD coupling

$$\Rightarrow \qquad \frac{L_{\rm mfp}^{\rm gas}}{\beta} \sim \frac{1}{g_T^4 \ln g_T^{-1}} \gg 1$$

$\Rightarrow$ the experimental results could have only been reproduced by the dilute-gas model if σ_t were larger by an order of magnitude (D. Molnar, M. Gyulassy, '02).

General properties of the viscosities

Shear viscosity η represents the ability to transport momentum:

$$\frac{\eta}{s} \sim \frac{L_{\rm mfp}}{\beta},$$

where s is the entropy density $\Rightarrow \frac{\eta}{s}$ is large in the dilute-gas model of the QGP, and gets smaller for a strongly interacting QGP.

E.g., for $T \sim 200\,{\rm MeV}$ and $L_{\rm mfp} \sim 0.1\,{\rm fm}$, $\frac{\eta}{s} \sim 0.1$.

When a parton propagates through the QGP over the distance $L_{\rm mfp}$, its mean momentum change Δp is $\sim T$

$$\Rightarrow \qquad \frac{\eta}{s} \sim \frac{L_{\rm mfp}}{\beta} \sim L_{\rm mfp} \cdot \Delta p$$

is nonvanishing due to the Heisenberg uncertainty principle
$\Rightarrow \eta$ cannot vanish completely.

General properties of the viscosities

- How small can $\frac{\eta}{s}$ be ?
- What is the temperature behavior of $\frac{\eta}{s}$?

The minimal possible value of $\frac{\eta}{s}$ is conjectured to be that in $\mathcal{N}=4$ SYM (G. Policastro, D.T. Son, A.O. Starinets, '01):

$$\left.\frac{\eta}{s}\right|_{\mathcal{N}=4\,\mathrm{SYM}} = \frac{1}{4\pi} \simeq 0.08.$$

It is a temperature-independent constant (because $\mathcal{N}=4$ SYM is a CFT).

Rather, in perturbative QCD (P. Arnold et al., '01, '03),

$$\left.\frac{\eta}{s}\right|_{\mathrm{pQCD}} \sim \frac{1}{g_T^4 \ln g_T^{-1}} \gg 1.$$

Note that plasma instabilities can generate an anomalous viscosity η_A (M. Asakawa, S.A. Bass, B. Müller, '06):

$$\frac{\eta_A}{s} \sim \frac{1}{g_T^{3/2}} < \left.\frac{\eta}{s}\right|_{\mathrm{pQCD}}, \quad \text{but still} \quad \gg 1.$$

General properties of the viscosities

For known liquids, $\frac{\eta}{s} \gg 1$ for small and large T, where η is dominated by potential- and kinetic-energy contributions, respectively.

Around the liquid-gas phase transition, these two contributions are nearly equal, and $\frac{\eta}{s}$ has a minimum, corresponding to the most difficult condition to transport momentum.

This behavior is exhibited by liquids of a very different nature, such as helium, nitrogen, and water.

The empirical minima of $\frac{\eta}{s}$ are at least by one order of magnitude larger than $\frac{1}{4\pi}$.

General properties of the viscosities

The energy-momentum tensor of an ideal liquid:

$$\Theta_{\mu\nu} = -p \cdot g_{\mu\nu} + Ts \cdot u_\mu u_\nu,$$

where u_μ is the velocity of energy transport.

The principal deviation from the ideality:

$$\Delta\Theta_{\mu\nu} = \eta \cdot (\Delta_\mu u_\nu + \Delta_\nu u_\mu) + \left(\frac{2}{3}\eta - \zeta\right) H_{\mu\nu} \partial_\rho u_\rho,$$

where $H_{\mu\nu} = u_\mu u_\nu - g_{\mu\nu}$, $\Delta_\mu = \partial_\mu - u_\mu u_\nu \partial_\nu$.

The bulk viscosity ζ is the other first-order transport coefficient.

While η characterizes a change in shape of a fixed volume, ζ characterizes a change in volume of the liquid of a fixed shape.

General properties of the viscosities

Cf. the Navier–Stokes' equation in hydrodynamics:

$$\rho \cdot \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v}\right] = -\mathrm{grad}\, p + \eta \cdot \Delta \mathbf{v} + \left(\zeta + \frac{\eta}{3}\right) \cdot \mathrm{grad}\,\mathrm{div}\,\mathbf{v} \quad \Rightarrow$$

– η enters foremost through $\eta \cdot \Delta \mathbf{v}$;

– ζ is relevant only when $\mathrm{div}\,\mathbf{v} \neq 0$, i.e. for compressible liquids.

For helium, nitrogen, and water, $\frac{\zeta}{s}$ has a maximum near the liquid-gas phase transition.

In $\mathcal{N} = 4$ SYM, $\zeta \equiv 0$ (again because it is a CFT), unlike QCD, where the non-conformality effects are manifest in $\varepsilon - 3p$ up to $T = (2 \div 3) T_c$.

The Kubo formulae

η and ζ are defined through the spectral densities,

$$\eta = \pi \frac{d\rho_T^{(s)}}{d\omega}\bigg|_{\omega=0} \quad \text{and} \quad \zeta = \frac{\pi}{9}\frac{d\rho_T^{(b)}}{d\omega}\bigg|_{\omega=0},$$

which can be obtained from the Euclidean Kubo formulae (A. Hosoya et al., '84; F. Karsch & H.W. Wyld, '87)

$$\int_0^\infty d\omega\, \rho_T^{(s),(b)}(\omega)\, \frac{\cosh\left[\omega\left(x_4 - \frac{\beta}{2}\right)\right]}{\sinh(\omega\beta/2)} = \int d^3x \sum_{n=-\infty}^{+\infty} U_T^{(s),(b)}(\mathbf{x}, x_4 + \beta n),$$

where

$$U_T^{(s)}(\mathbf{x}, x_4) = \left\langle \Theta_{12}(0)\Theta_{12}(\mathbf{x}, x_4)\right\rangle_T, \quad U_T^{(b)}(\mathbf{x}, x_4) = \left\langle \Theta_{\mu\mu}(0)\Theta_{\nu\nu}(\mathbf{x}, x_4)\right\rangle_T,$$

and in the Yang–Mills theory

$$\Theta_{12} = g^2 F_{1\mu}^a F_{2\mu}^a, \quad \Theta_{\mu\mu} = \frac{\beta(g)}{2g}(F_{\mu\nu}^a)^2.$$

The Kubo formulae

$\Theta_{\mu\nu}$ of the gluon plasma receives contributions from

– stochastic background fields, characterized by $\left\langle g^2 (F_{ij}^a)^2 \right\rangle_T$ and μ_T;

– valence gluons, which are confined at large *spatial* separations.

Such a two-component model of the gluon plasma is efficient to describe

– radiative energy loss of a parton traversing the plasma (H.-J. Pirner & D.A., '08);

– pressure and interaction measure $(\varepsilon - 3p)/T^4$ of the plasma (H.-J. Pirner, M.G. Schmidt & D.A., '09).

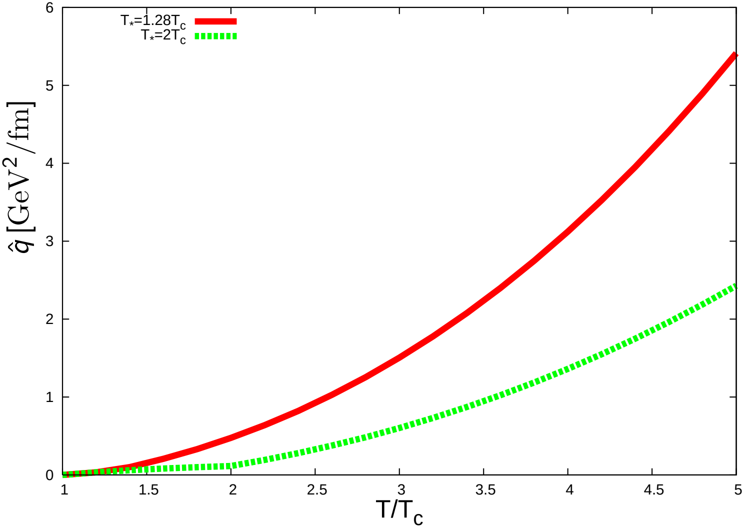

Figure: The jet quenching parameter $\hat{q}(T)$ for various values of the dimensional-reduction temperature, $T_* = 1.28\,T_c$ and $T_* = 2\,T_c$.

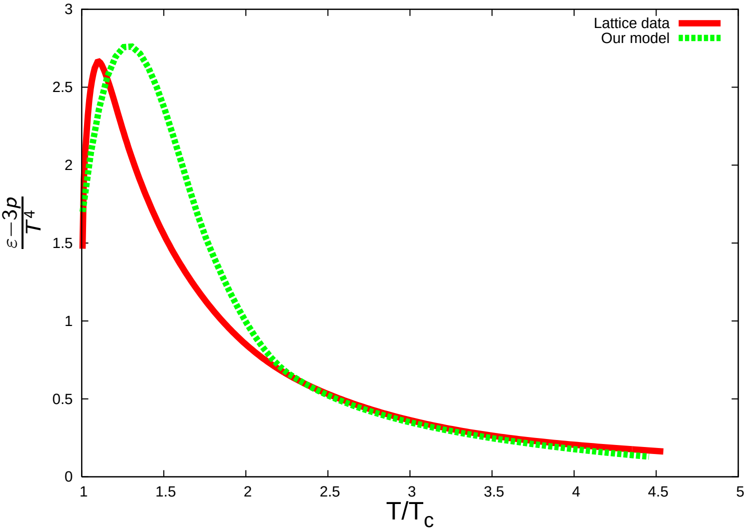

Figure: Lattice data on the interaction measure $(\varepsilon - 3p)/T^4$ (courtesy of F. Karsch) compared to the prediction of the two-component model.

The Kubo formulae

At $T \gg T_c$, the two contributions become strictly additive:

$$\rho = \rho_{\rm backgr} + \rho_{\rm pert},$$

and $\rho_{\rm pert} \propto g_T^4 \omega^4$ together with $\langle \Theta_{\mu\nu}(0)\Theta_{\lambda\rho}(x)\rangle_{\rm pert} \propto g_T^4/|x|^8$ can be isolated simultaneously.

This project (to be realized through the Kubo formulae):

– Using the stochastic vacuum model at finite temperature (Yu.A. Simonov, N.O. Agasian, '95 – '08), calculate $\rho_{\rm backgr}$.
To be presented below.

– Calculate the contribution of valence gluons to ρ at $T \sim T_c$.

The Kubo formulae

A reminder on the stochastic vacuum model (SVM).

While QCD sum rules assume $\left\langle g^2(F^a_{\mu\nu})^2\right\rangle$, the SVM additionally assumes a finite correlation length of the vacuum, $\mu^{-1} < \infty$ (Pisa group, '86-'03):

$$\left\langle F^a_{\mu\nu}(x)F^b_{\lambda\rho}(0)\right\rangle \sim \mathrm{e}^{-\mu|x|} \quad \Rightarrow$$

the SVM can quantitatively describe confinement with the string tension $\sigma \propto \mu^{-2}\left\langle g^2(F^a_{\mu\nu})^2\right\rangle$.

The spatial string tension at $T > T_c$:

$$\sigma_s(T) \propto \mu_T^{-2}\left\langle g^2(F^a_{ij})^2\right\rangle_T\,,$$

i.e. the chromo-magnetic vacuum still confines.

The Kubo formulae

Since

$$\Theta_{\mu\nu} = \mathcal{O}\left(g^2 (F^a_{\alpha\beta})^2\right),$$

the expected contributions of the background fields to the viscosities are

$$\eta \propto \zeta \propto \frac{\left\langle g^2 (F^a_{ij})^2 \right\rangle_T^2}{\mu_T^5},$$

in agreement with

$$\sigma_{\rm total}^{\rm SVM} \propto \left\langle g^2 (F^a_{\mu\nu})^2 \right\rangle^2 \quad (\text{Heidelberg group}, '91 - '03)$$

$\Rightarrow$ at temperatures $T > T_*$,

$$\eta \propto \zeta \propto (g_T^2 T)^3,$$

whereas $s \propto T^3$ at $T \gtrsim 2T_c \Rightarrow$

$$\frac{\eta}{s} \propto \frac{\zeta}{s} \propto g_T^6 \quad \text{at} \quad T \gtrsim 2T_c.$$

We get the coefficients in these formulae.

Calculation of the viscosities

Notations:

$$\left\langle g^2 (F^a_{\mu\nu})^2 \right\rangle \equiv \left\langle G^2 \right\rangle, \quad \left\langle g^2 (F^a_{ij})^2 \right\rangle_T \equiv \left\langle G^2 \right\rangle_T, \quad \rho_{\rm backgr} \equiv \rho_T, \quad \omega_k = 2\pi Tk.$$

Assuming at $T = 0$ exponentially falling off Ansätze

$$U^{(s),(b)}_{T=0}(x) = N^{(s),(b)}_\alpha \left\langle G^2 \right\rangle^2 \cdot \frac{K_{2-\alpha}(M|x|)}{(M|x|)^{2-\alpha}}, \quad \text{where} \quad \alpha > 0,$$

we get at $T > T_c$ the Fourier transformed $(\sum_k {\rm e}^{i\omega_k x_4} f_k)$ Kubo formulae:

$$\int_0^\infty d\omega \, \rho^{(s),(b)}_T(\omega) \, \frac{\omega}{\omega^2 + \omega_k^2} = \pi^2 2^\alpha \Gamma(\alpha) N^{(s),(b)}_\alpha \left\langle G^2 \right\rangle^2_T \frac{M_T^{2\alpha-4}}{(\omega_k^2 + M_T^2)^\alpha}. \quad (*)$$

Calculation of the viscosities

Lorentzian-type spectral densities

$$\rho_T^{(s),(b)}(\omega) = C_T^{(s),(b)} \cdot \frac{\omega}{(\omega^2 + M_T^2)^{\alpha+\frac{1}{2}}}$$

ensure that both sides of Eq. (*) have the same large-$|k|$ behavior.
$M_T \sim \mu_T$ is the momentum scale, below which PT breaks down.

- For $|k| \gg 1$,

$$\text{LHS of Eq. } (*) = \frac{C_T^{(s),(b)}}{\omega_k^{2\alpha}} \left[\frac{\pi}{2\sin(\pi\alpha)} + \mathcal{O}\left(\frac{M_T^2}{\omega_k^2}\right) + \sum_{i=2}^{\infty} c_i \left(\frac{M_T}{\omega_k}\right)^{i-2\alpha} \right]$$

$\Rightarrow$ the leading term in the brackets is k-independent only for $\alpha < 1$.

$$\text{RHS of Eq. } (*) = \pi^2 2^\alpha \Gamma(\alpha) N_\alpha^{(s),(b)} \frac{\left\langle G^2 \right\rangle_T^2}{\omega_k^{2\alpha}} M_T^{2\alpha-4} \cdot \left[1 + \mathcal{O}\left(\frac{M_T^2}{\omega_k^2}\right) \right].$$

Calculation of the viscosities

$$\Rightarrow \quad \eta\Big|_{|k|\gg 1} = \pi^2 2^{\alpha+1}\Gamma(\alpha)N_\alpha^{(s)}\sin(\pi\alpha)\frac{\left\langle G^2\right\rangle_T^2}{M_T^5}.$$

Note: $|k| \gg 1$ means $|k| \geq 3$, since $\frac{M_T}{\omega_3} < 0.35$ for any $T > T_c$.

• For $|k| \sim 1$ (e.g. $k = 0$ for $T > T_*$), $\mathcal{O}\left(\frac{\omega_k^2}{M_T^2}\right)$-terms and higher can be disregarded $\Rightarrow$

$$\eta\Big|_{|k|\sim 1} = \pi^{5/2}2^{\alpha+1}\Gamma\left(\alpha+\frac{1}{2}\right)N_\alpha^{(s)}\frac{\left\langle G^2\right\rangle_T^2}{M_T^5}.$$

The ratio

$$\frac{\eta\Big|_{|k|\gg 1}}{\eta\Big|_{|k|\sim 1}} = \frac{\Gamma(\alpha)\sin(\pi\alpha)}{\sqrt{\pi}\Gamma\left(\alpha+\frac{1}{2}\right)} \quad \text{for} \quad 0<\alpha<1$$

is equal to 1 at $\alpha = \frac{1}{2}$, i.e. $\eta\Big|_{\alpha=\frac{1}{2}}$ is k-independent.

Calculation of the viscosities

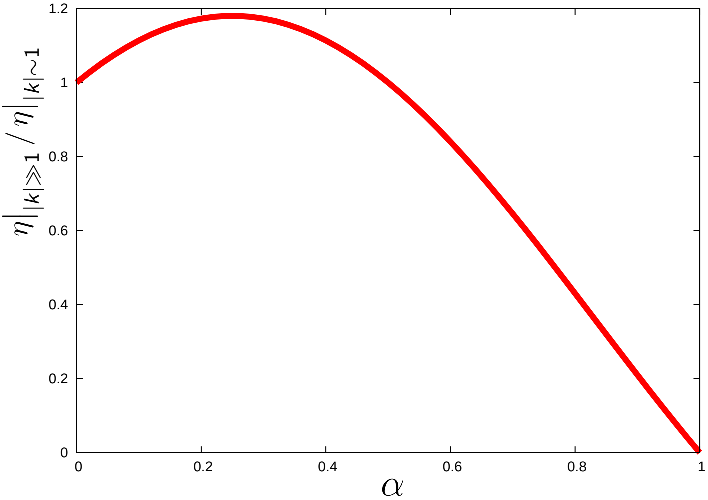

Figure: The ratio $\frac{\eta\big|_{|k|\gg1}}{\eta\big|_{|k|\sim1}}$.

Calculation of the viscosities

For $\alpha = \frac{1}{2}$, $\rho_T^{(s),(b)}(\omega)$ take the purely Lorentzian form, with

$$C_T^{(s),(b)} = (2\pi)^{3/2} N_{1/2}^{(s),(b)} \cdot \frac{\left\langle G^2 \right\rangle_T^2}{M_T^3}.$$

The coefficients $N_{1/2}^{(s),(b)}$ can be determined via the Gaussian-dominance hypothesis, which disregards the connected parts of $\left\langle \Theta_{\mu\nu}(0)\Theta_{\lambda\rho}(x) \right\rangle$.

The SVM parametrizes the remaining two-point functions.

- Retaining only confining self-interactions of the background fields:

$$\left\langle g^2 F_{\mu\nu}^a(x) F_{\lambda\rho}^b(0) \right\rangle = \frac{\left\langle G^2 \right\rangle}{12} \cdot (\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda}) \cdot \frac{\delta^{ab}}{N_c^2 - 1} \cdot D(x),$$

where $D(x) \to \mathrm{e}^{-\mu|x|}$. The compatibility with $U_{T=0,\alpha=1/2}^{(s)}$ is achieved by

$$D(x) = \mathcal{A} \cdot \sqrt{\frac{K_{3/2}(2\mu|x|)}{(2\mu|x|)^{3/2}}} \ \text{and} \ \ M = 2\mu.$$

Calculation of the viscosities

$$\Rightarrow \quad N_{1/2}^{(s)} = \frac{\mathcal{A}^2}{576}.$$

The constant $\mathcal{A}$ is fixed by $\sigma_{\rm f} = \frac{\langle G^2 \rangle}{144} \int d^2x\, D(x) \quad \Rightarrow$

$$\mathcal{A} = \frac{4}{\int_0^\infty dz \cdot z^{1/4} \cdot \sqrt{K_{3/2}(z)}} \simeq 1.05 \quad \Rightarrow$$

the shear viscosity

$$\eta = \frac{\pi^{5/2}\mathcal{A}^2}{4608\sqrt{2}} \cdot \frac{\langle G^2 \rangle_T^2}{\mu_T^5}.$$

Similarly, in the one-loop approximation where $\frac{\beta(g)}{2g} \simeq -\frac{11}{32\pi^2} g^2$, the bulk viscosity

$$\zeta = \frac{\mathcal{A}^2}{1728\sqrt{2\pi^3}} \left(\frac{11}{32}\right)^2 \cdot \frac{\langle G^2 \rangle_T^2}{\mu_T^5}.$$

Calculation of the viscosities

• Accounting also for the nonconfining nonperturbative self-interactions of the background fields:

$$\left\langle g^2 F^a_{\mu\nu}(0) F^b_{\lambda\rho}(x)\right\rangle = \frac{\left\langle G^2\right\rangle}{12}\cdot\frac{\delta^{ab}}{N_c^2-1}\cdot\Big\{\kappa(\delta_{\mu\lambda}\delta_{\nu\rho}-\delta_{\mu\rho}\delta_{\nu\lambda})D(x)+$$

$$+\frac{1-\kappa}{2}\left[\partial_\mu(x_\lambda\delta_{\nu\rho}-x_\rho\delta_{\nu\lambda})+\partial_\nu(x_\rho\delta_{\mu\lambda}-x_\lambda\delta_{\mu\rho})\right]D_1(x)\Big\},\ \text{where}\ \kappa\in[0,1].$$

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Cf. the Wilson loop $\langle W(C)\rangle = \left\langle \mathrm{tr}\, P\, \exp\left(ig\oint_C dx_\mu T^a A^a_\mu\right)\right\rangle$:

$$\langle W(C)\rangle = \exp\left\{-\frac{C_2\left\langle G^2\right\rangle}{96(N_c^2-1)}\left[2\kappa\int_{\Sigma_{\min}} d\sigma_{\mu\nu}(x)\int_{\Sigma_{\min}} d\sigma_{\mu\nu}(x')D(|x-x'|)+\right.\right.$$

$$\left.\left.+(1-\kappa)\oint_C dx_\mu\oint_C dx'_\mu\int_{(x-x')^2}^{\infty} d\xi\, D_1(\sqrt{\xi})\right]\right\}.$$

Lattice data (Pisa group) suggest that $D_1(x)=D(x)$, and $\kappa\simeq 0.83$.

Calculation of the viscosities

$U^{(s),(b)}_{T=0}$ contain terms through $\mathcal{O}\left((1-\kappa)^2\right) \Rightarrow$ seeking $D(x)$ in the form

$$D(x) = \mathcal{A}_\kappa \cdot f_\kappa(\mu|x|), \quad \text{where} \quad f_\kappa = f_{\kappa=1} + (1-\kappa)f^{(1)} + (1-\kappa)^2 f^{(2)},$$

$$\mathcal{A}_{\kappa=1} = \mathcal{A} \quad \text{and} \quad f_{\kappa=1}(z) = \sqrt{\frac{K_{3/2}(2z)}{(2z)^{3/2}}} = \frac{\pi^{1/4}}{2^{7/4}} \cdot \frac{\mathrm{e}^{-z}}{z^{3/2}} \cdot (1+2z)^{1/2}.$$

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Then

$$f_\kappa(z) = \frac{\pi^{1/4}}{256 \cdot 2^{3/4}} \cdot \frac{\mathrm{e}^{-z}}{[z(1+2z)]^{3/2}} \cdot \left\{128(1+2z)^2 + \right.$$

$$\left. +(1-\kappa)\cdot 16(1+2z)(3+6z+4z^2) + (1-\kappa)^2 \cdot [9 + 4z \cdot (9 + z \cdot (7 + 4z(1+z)))]\right\}$$

Calculation of the viscosities

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$$+(1-\kappa) \cdot 16(1+2z)(3+6z+4z^2) + (1-\kappa)^2 \cdot [9+4z \cdot (9+z \cdot (7+4z(1+z)))]\Big\}$$

The constant $\mathcal{A}_\kappa$ is again fixed via the string tension:

$$\mathcal{A}_\kappa = \frac{1}{\int_0^\infty dz \cdot z \cdot f_\kappa(z)}.$$

Calculation of the viscosities

In particular, $\mathcal{A}_{\kappa=0.83} \simeq 0.97 \Rightarrow \left(\frac{\mathcal{A}_{\kappa=0.83}}{\mathcal{A}}\right)^2 \simeq 0.85 \Rightarrow$

$\eta^{\kappa=0.83}$ and $\zeta^{\kappa=0.83}$ are by 15% (that is close to 17%) smaller than, respectively, $\eta^{\kappa=1}$ and $\zeta^{\kappa=1}$.

Parameters for the numerical calculation:

- $T_c = 270\,\mathrm{MeV}$.
- The two-loop running coupling in SU(3) YM:

$$g^{-2}(T) = 2b_0 \ln\frac{T}{\Lambda} + \frac{b_1}{b_0}\ln\left(2\ln\frac{T}{\Lambda}\right),$$

$$b_0 = \frac{11N_c}{48\pi^2},\ b_1 = \frac{34}{3}\left(\frac{N_c}{16\pi^2}\right)^2,\ N_c = 3,\ \Lambda = 0.104\,T_c.$$

Calculation of the viscosities

- Temperature dependence:

$$f(T) \equiv \begin{cases} 1 \ \ \text{at} \ \ T_c < T < T_*, \\ \frac{g_T^2 \cdot T}{g_{T_*}^2 \cdot T_*} \ \ \text{at} \ \ T > T_*, \end{cases}$$

$$\Rightarrow \mu_T = \mu \cdot f(T), \ \sigma_{\rm f}(T) = \sigma_{\rm f} \cdot f^2(T), \ \left\langle G^2 \right\rangle_T = \left\langle G^2 \right\rangle \cdot f^4(T).$$

where $\mu = 894\,\mathrm{MeV}$ (Pisa group, '97), $\sigma_{\rm f} = (440\,\mathrm{MeV})^2$, $\left\langle G^2 \right\rangle = \frac{72}{\pi}\sigma_{\rm f}\mu^2$.

- Determining T_* from the equation $\sigma_{\rm f}(T_*) = \sigma_{\rm f}$, where $\sigma_{\rm f}(T) = [0.566 g^2(T) T]^2$ (Bielefeld group, '93, '96) $\Rightarrow$ $T_* = 1.28\, T_c$.

- Entropy density $s(T) = \frac{dp_{\rm lat}}{dT} \Rightarrow s(T)/T^3$ is nearly constant at $T \gtrsim 2T_c$.

Calculation of the viscosities

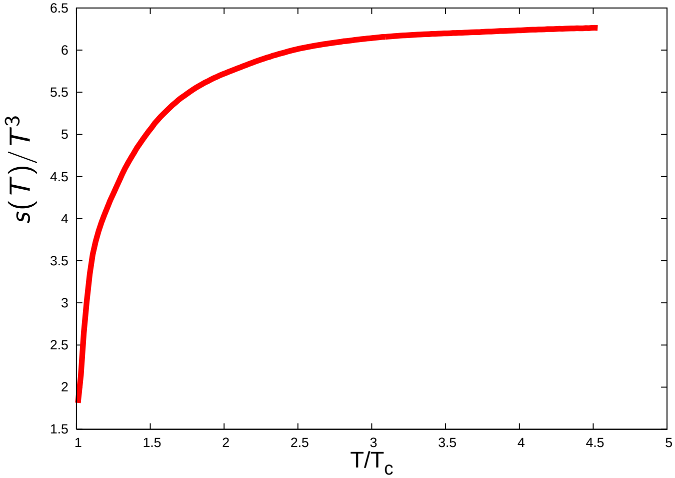

Figure: Entropy density $s(T)$, in the units of T^3, derived from the lattice values for the pressure (G. Boyd et al., 1996; courtesy of F. Karsch).

The results

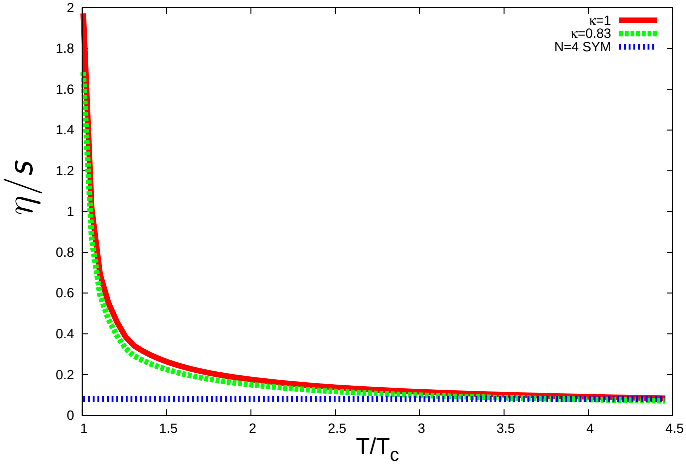

Figure: Calculated ratios of η/s. Also shown is the conjectured lower bound of $1/(4\pi)$ realized in $\mathcal{N}=4$ SYM.

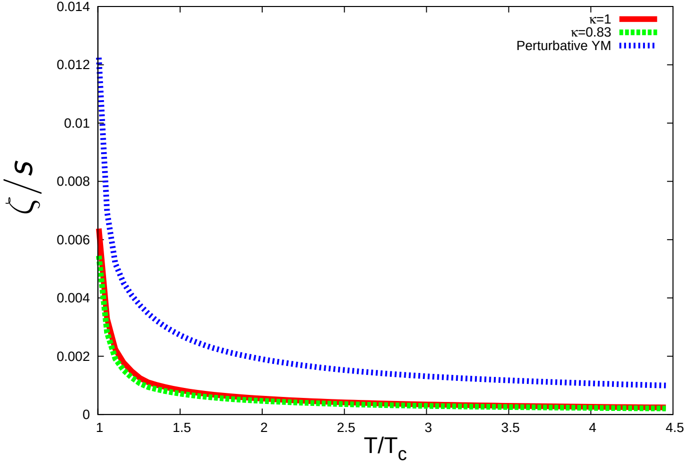

Figure: Calculated ratios of ζ/s. Also shown are the perturbative values $\zeta_{\rm pert}/s$, where $\zeta_{\rm pert} = \frac{0.443\alpha_s^2 T^3}{\ln(7.14/g_T)}$ (P. Arnold et al., '06) is extrapolated down to $T = T_c$.

Concluding remarks

• The calculated soft contribution to η/s falls off rapidly at $T_c < T \lesssim 2T_c$, and further as $\mathcal{O}(g_T^6)$, gradually crossing $1/(4\pi)$.

• However, at $T \gg T_c$, the perturbative result, with

$$\eta_{\rm NLL} = \frac{T^3}{g_T^4} \cdot \frac{27.126}{\ln \frac{2.765}{g_T}}$$

(P. Arnold et al., '03), takes it over $\Rightarrow$ A minimum of the full η/s should exist at intermediate temperatures (cf. other liquids), yielding the temperature of a possible liquid-gas phase transition.

• For ζ/s, perturbative contributions only enhance the $\mathcal{O}(g_T^6)$-behavior to the $\mathcal{O}(g_T^4)$-one.

• At $T \sim T_c$, an interference of nonperturbative contributions, produced by the background fields and by valence gluons, will be studied.

D.A., arXiv:1002.2406 (Annals Phys., in press).

Many thanks to the organizers for a very nice and interesting workshop.