Meson and Baryon Screening in Hot Moving Plasmas

EMMI – Workshop

– String Theory and Extreme Matter –

in collaboration with Carlo Ewerz

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Motivation

- Understand the dynamics of the quark-gluon plasma (QGP)
- Method needed for strongly coupled gauge theory
- Computation of dynamical quantities

→ AdS/CFT correspondence is promising candidate:
- η/s Bound
- Jet Quenching Parameter $\hat{q}$

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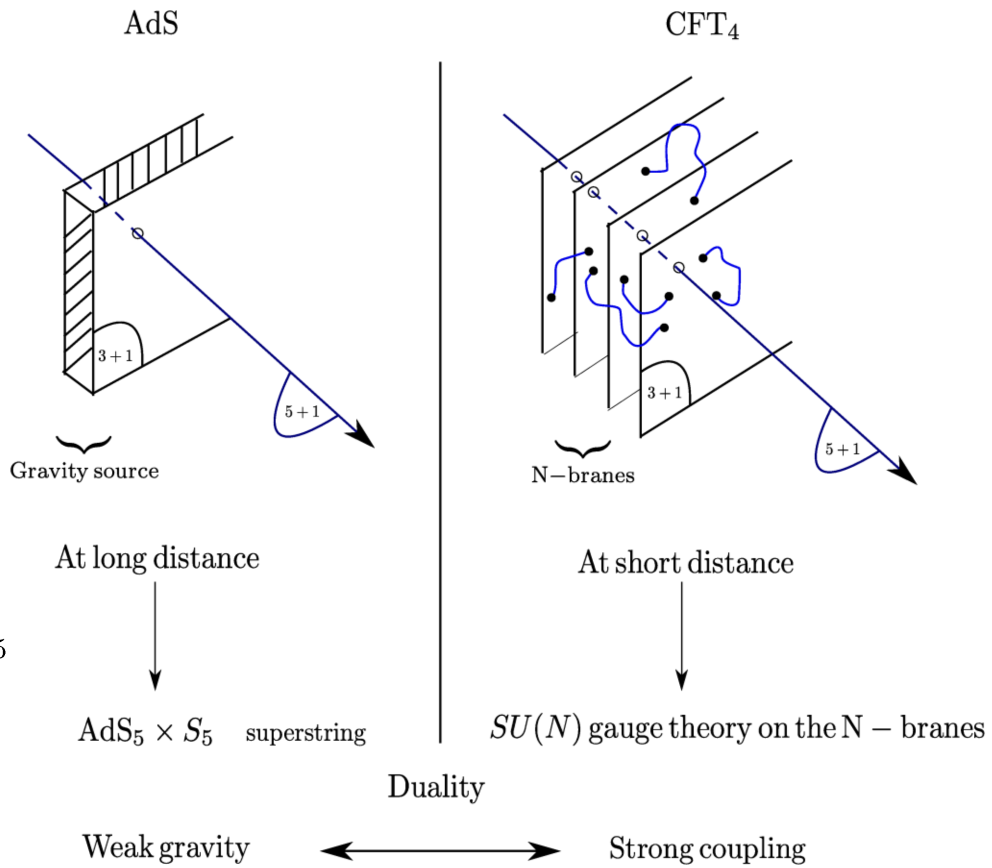
AdS/CFT correspondence

Gauge theory side:

$\mathcal{N}=4$ supersymmetric Yang-Mills theory in 3+1 dimensions in the limit of $N_c \to \infty$

String theory side:

Type IIB string theory on a $AdS_5 \times S_5$ spacetime



$$ds^2 = \frac{r^2}{R^2}\left(-dt^2 + \sum_{i=1}^{3} dx_i^2\right) + \frac{R^2}{r^2}\,dr^2 + R^2\,d\Omega_5$$ with curvature $R = 4\pi g_{\rm YM}^2\,\alpha'^2 N_c$

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$\mathcal{N} = 4$ and QCD

Properties of $\mathcal{N} = 4$ SYM:

- Conformal theory, coupling is constant
- Maximally supersymmetric
- No confinement
- No chiral symmetry breaking
- $N_c \to \infty$ for duality

At finite temperature T:

- No confinement in QCD
- Chiral symmetry restoration in QCD
- Finite T breaks supersymmetry

⟶ $\mathcal{N} = 4$ SYM is closer to QCD at finite T

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How to extract information

1. Search for gravity dual to QCD (not yet known)

2. Deformation of existing dualities to come closer to QCD
   - KTY model
   - 2 – parameter models which solve equations of motion

3. How do observables depend on deformations?
   - Robustness: small changes
   - Universality: no change, or systematically in one direction

→
- η/s Bound is famous example — Kovtun, Starinets, Son
- $c_s^2 < \frac{1}{3}$ in a wide class of theories — Cherman, Cohen, Nellore; Hohler, Stephanov

Metric models at finite temperature

AdS_5 metric at finite temperature:

$$ds^2 = -f dt^2 + \frac{r^2}{R^2}(dx_1^2 + dx_2^2 + dx_3^2) + \frac{1}{f} dr^2 = G_{\mu\nu} dx^\mu dx^\nu \, ,$$

$$f \equiv \frac{r^2}{R^2}\left(1 - \frac{r_0^4}{r^4}\right)$$ where r_0 is the black hole horizon

- T is Hawking temperature of the BH, with horizon r_0 (z_0):

$$T = \frac{r_0}{\pi R^2} \, , \quad T = \frac{1}{\pi z_0}$$ with $z = \frac{R^2}{r}$

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Metric models at finite temperature

KTY Model:

Kajantie, Tahkokallio, Yee

$$ds^2 = e^{\frac{29}{20}c\frac{R^4}{r^2}} \left[-\frac{r^2}{R^2}\left(1-\frac{r_0^4}{r^4}\right)dt^2 + \frac{r^2}{R^2}d\vec{x}^2 + \frac{R^2}{r^2}\frac{dr^2}{1-\frac{r_0^4}{r^4}} \right]$$

$$= \frac{R^2 e^{\frac{29cz^2}{20}}}{z^2}\left[-\left(1-\frac{z^4}{z_0^4}\right)dt^2 + d\vec{x}^2 + \frac{dz^2}{1-\frac{z^4}{z_0^4}} \right]$$ with parameter $\frac{c}{T^2}$

- Temperature is the same as in the conformal case

$$T = \frac{r_0}{\pi R^2}$$

- Relevant parameter range is $0 < \frac{c}{T^2} < 4$
- Realistic thermodynamics for $c = 0.127\,\mathrm{GeV}^2$
- KTY is not a solution to supergravity equations of motion

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Metric models at finite temperature

Deformed 2-parameter metric:

DeWolfe, Rosen; Gubser

- Supergravity action with dilaton potential V

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left(R - \frac{1}{2}(\partial_\mu \Phi)^2 - V(\Phi) \right)$$

- General ansatz for metric

$$ds^2 = e^{2A(\Phi)}(-h(\Phi)dt^2 + d\vec{x}^2) + \frac{e^{2B(\Phi)}}{h(\Phi)} d\Phi^2$$ with $\frac{c}{T^2}$, $\alpha \equiv \frac{c}{\phi}$ and $\Phi = \sqrt{\frac{3}{2}}\phi z^2$

  - Translation invariance, $SO(3)$ symmetry but not $SO(3,1)$

- Temperature in this case

$$T = \frac{e^{A(\Phi_h) - B(\Phi_h)} |h'(\Phi_h)|}{4\pi}$$

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Metric models at finite temperature

Deformed 2-parameter metric:

- Definition of A, B :

$$A(\Phi) = \frac{1}{2}\ln\left(\sqrt{\frac{3}{2}}c\frac{R^2}{\alpha}\right) - \frac{1}{2}\ln\Phi - \frac{\alpha}{\sqrt{6}}\Phi$$

$$B(\Phi) = \ln\left(\frac{R}{2}\right) + \frac{1+2\alpha^2}{2\alpha^2}\ln\left(1+\alpha\sqrt{\frac{2}{3}}\Phi\right) - \ln\Phi - \frac{1}{\alpha\sqrt{6}}\Phi$$

- Exponential factor of KTY metric is obtained for $\alpha_{\mathrm{KTY}} = \frac{20}{49}$

- Two versions of this metric:
  - "Einstein frame": scalar Φ is not dilaton
  - "String frame": scalar Φ is dilaton

$$g^E_{\mu\nu}\, e^{\frac{4}{3}\Phi} = g^s_{\mu\nu}$$

- 2-parameter model solves supergravity equations of motion

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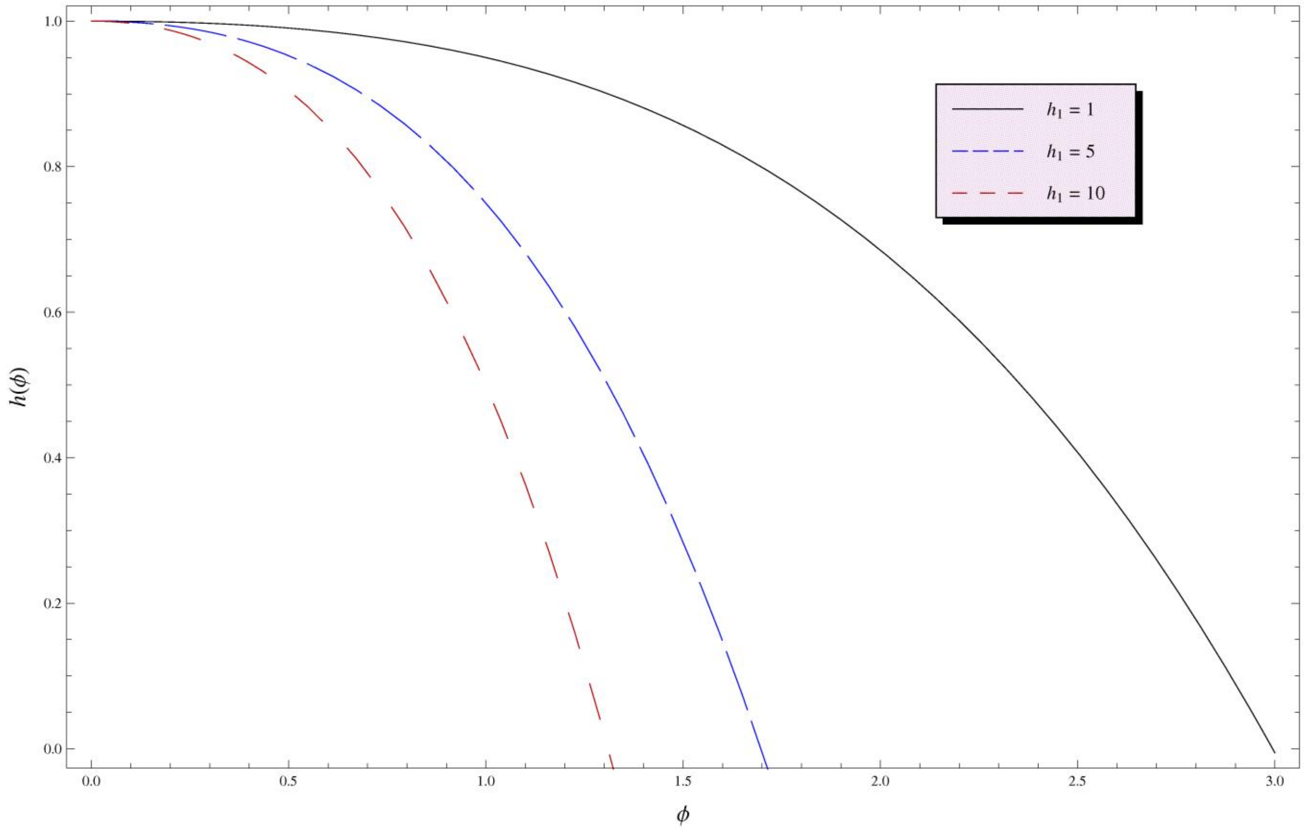
Horizon function



- AdS limit is obtained for $\phi \to 0$ in a $z = \phi$ gauge
- The horizon ϕ_0 is defined as the zero of h

$$h(\Phi) = h_0 - c^2 R^3 h_1 \int_0^{\Phi} d\Phi' \, e^{-4A(\Phi')+B(\Phi')}$$

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Screening Length in a hot moving plasma

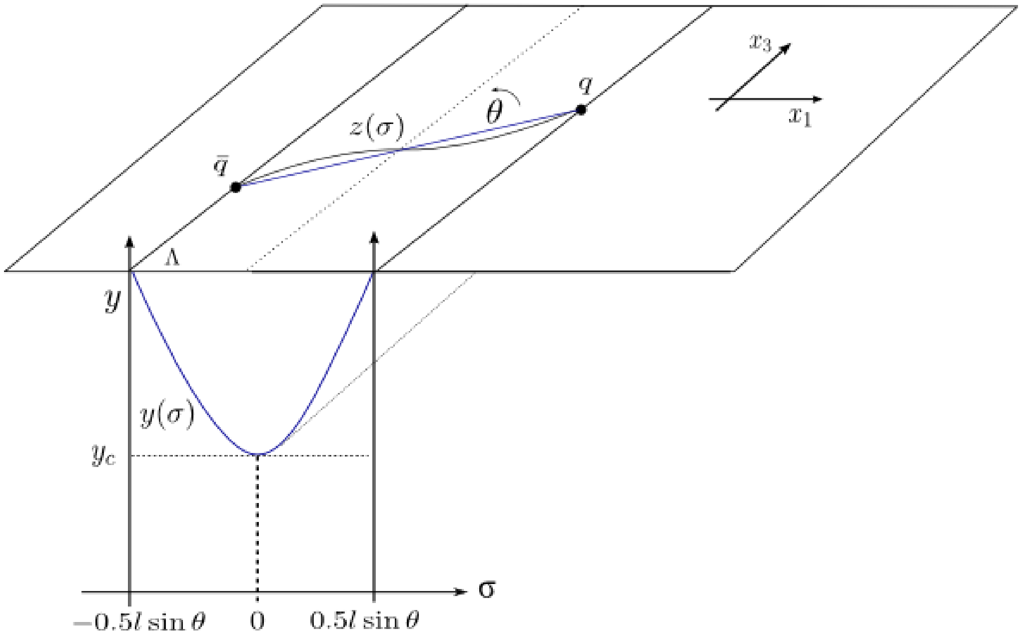
- Static quark-antiquark pair in a hot moving plasma „wind“ blowing in $-x_3$ – direction
- velocity $v = \tanh \eta$
- orientation angle θ

Nambu-Goto action:

$$S = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-\det g_{\alpha\beta}}$$

with $g_{\alpha\beta} = G_{\mu\nu}\partial_\alpha x^\mu \partial_\beta x^\nu$

Quark-antiquark configuration



by Liu, Rajagopal, Wiedemann, ‘06

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Parametrisation

- Boost into the rest frame of the quark-antiquark pair (t', x_3'):

$$dt = dt' \cosh\eta - dx_3' \sinh\eta$$
$$dx_3 = -dt' \sinh\eta + dx_3' \cosh\eta$$

- The metric has then the special form given by (for $\mathcal{N} = 4$):

$$ds^2 = -Adt^2 - 2Bdtdx_3 + Cdx_3^2 + \frac{r^2}{R^2}(dx_1^2 + dx_2^2) + \frac{1}{f}dr^2$$

with $$A = \frac{r^2}{R^2}\left(1 - \frac{r_1^4}{r^4}\right), \qquad B = \frac{r_1^2 r_2^2}{r^2 R^2}, \qquad C = \frac{r^2}{R^2}\left(1 + \frac{r_2^4}{r^4}\right)$$

and $$r_1^4 = r_0^4 \cosh^2\eta\,, \qquad r_2^4 = r_0^4 \sinh^2\eta$$

- Use appropriate parametrisation for the different orientations:

For $\theta = \frac{\pi}{2}$:

$$X^\mu = \left(t, \sigma = x_1 \in \left[-\frac{L}{2}, \frac{L}{2}\right], x_2, x_3, r(\sigma)\right)$$

For $\theta = 0$:

$$X^\mu = \left(t, x_1, x_2, x_3 = \sigma \in \left[-\frac{L}{2}, \frac{L}{2}\right], r(\sigma)\right)$$

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Calculation in $\mathcal{N}=4$

- Calculate determinant of the induced metric and Nambu-Goto action (shown for $\mathcal{N}=4$case):

For $\theta = \frac{\pi}{2}$ $$S = \frac{\mathcal{T}}{2\pi\alpha'} \int_{-\frac{L}{2}}^{\frac{L}{2}} d\sigma \sqrt{A\left(\frac{(\partial_\sigma r)^2}{f} + \frac{r^2}{R^2}\right)}$$

- Calculate conserved Hamiltonian and solve for the coordinate functions (with $r \equiv r_0\, y$):

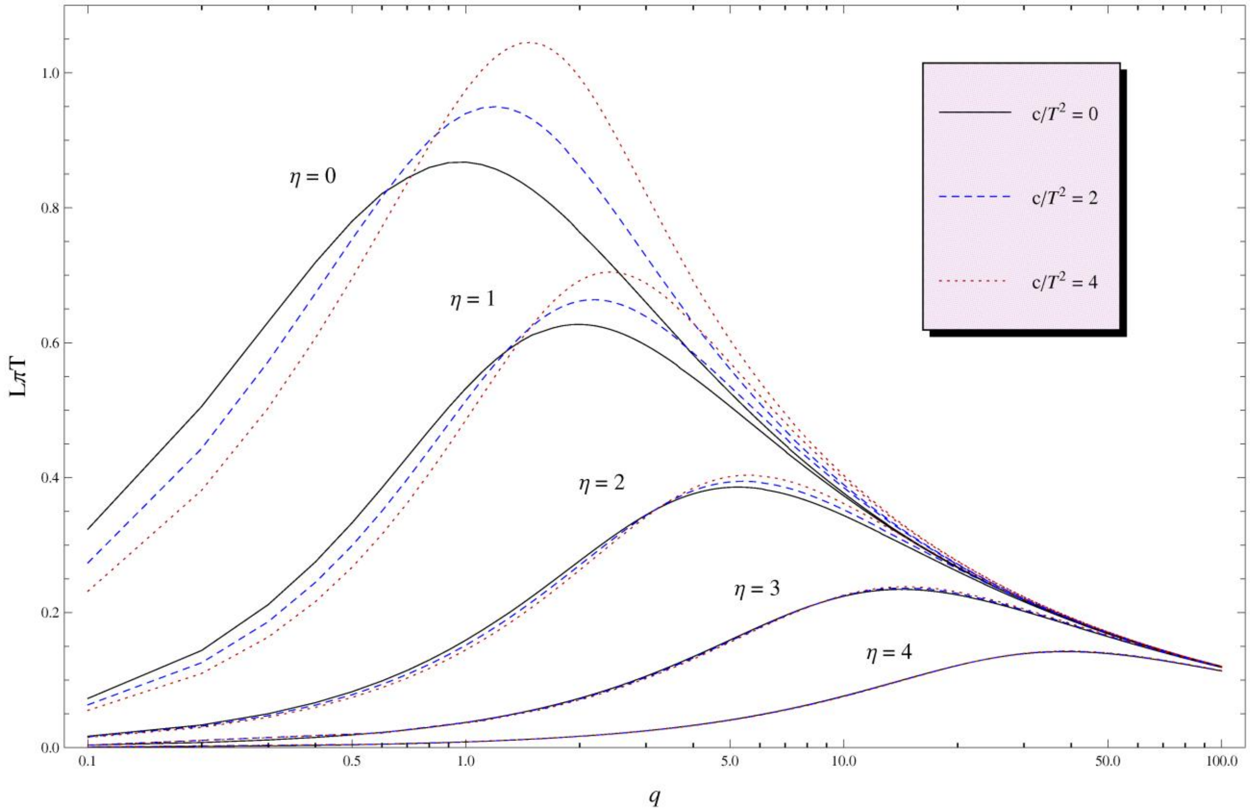
$$\mathcal{H} \equiv \mathcal{L} - y' \frac{\partial \mathcal{L}}{\partial y'} = \frac{y^4 - \cosh^2 \eta}{\mathcal{L}} = q$$

$$y' = \frac{1}{q}\sqrt{(y^4-1)(y^4-y_c^4)}$$ with $y_c^4 \equiv \cosh^2 \eta + q^2$

- Use boundary conditions to obtain the distance:

$$L\pi T = 2\int_0^{\frac{L\pi T}{2}} d\sigma = 2q\int_{y_c}^{\infty} \frac{dy}{y'} = 2q\int_{y_c}^{\infty} dy \frac{1}{\sqrt{(y^4-1)(y^4-y_c^4)}}$$

Quark-antiquark distance (KTY)



- Two solutions for each distance L up to a maximal $L_{\max}$

→ $L_{\max}$ is the screening length

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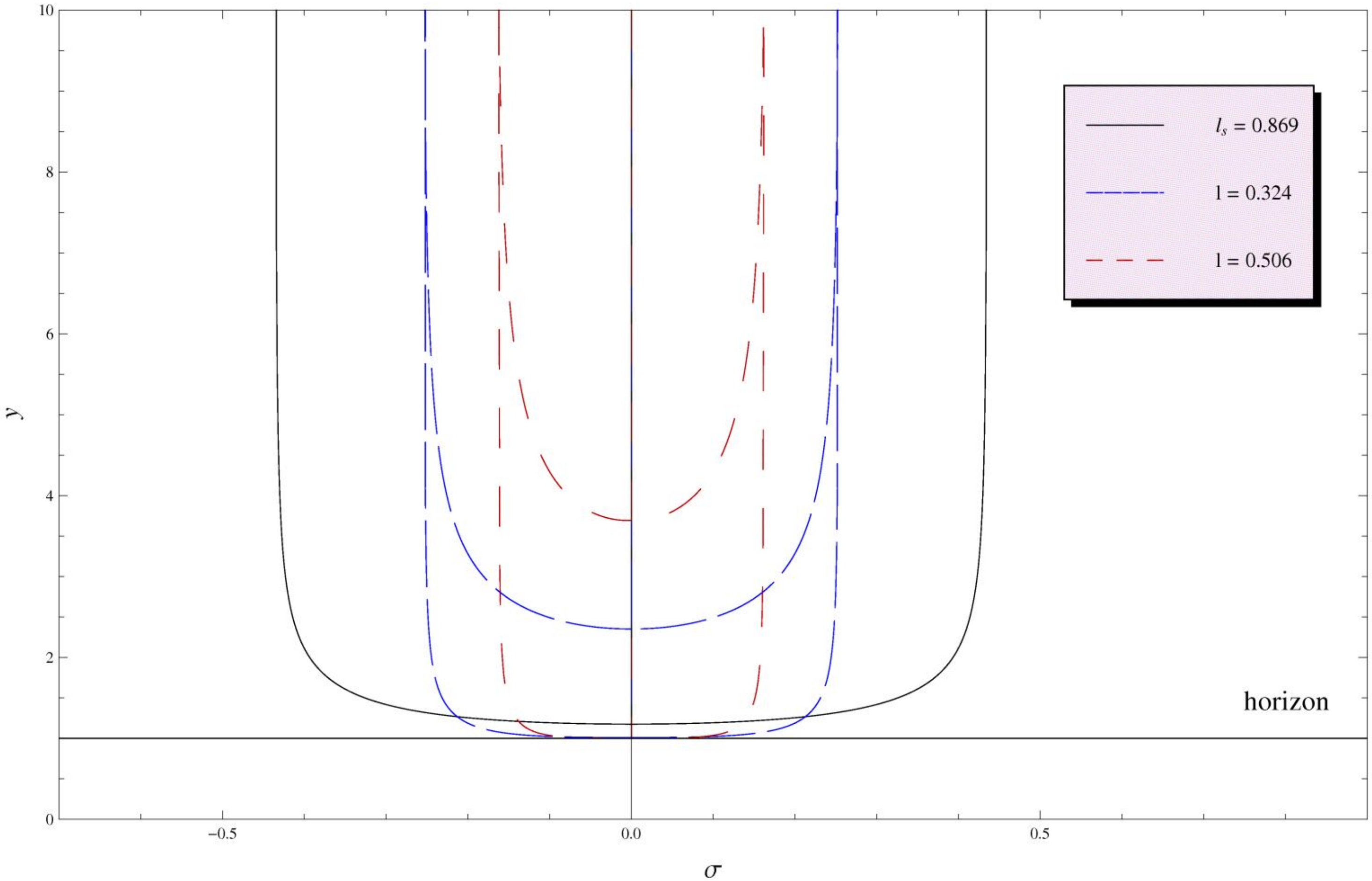
Configuration of the strings



- The string coming closer to the horizon is unstable

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Free energy

- General formulation of the quark-antiquark free energy:

$$E(L,\eta,\theta)\mathcal{T} = S(L,\eta,\theta) - S_0$$

- The string action reads:

$$S = \sqrt{\lambda}\mathcal{T}T \int_{y_c}^{\infty} \frac{d\sigma}{y'} \sqrt{(y^4 - \cosh^2\eta)\left(1 + \frac{y'^2}{y^4-1}\right)} \quad \Longrightarrow \quad \frac{S}{\sqrt{\lambda}\mathcal{T}T} = \int_{y_c}^{\infty} dy \frac{y^4 - \cosh^2\eta}{\sqrt{(y^4 - y_c)(y^4-1)}}$$

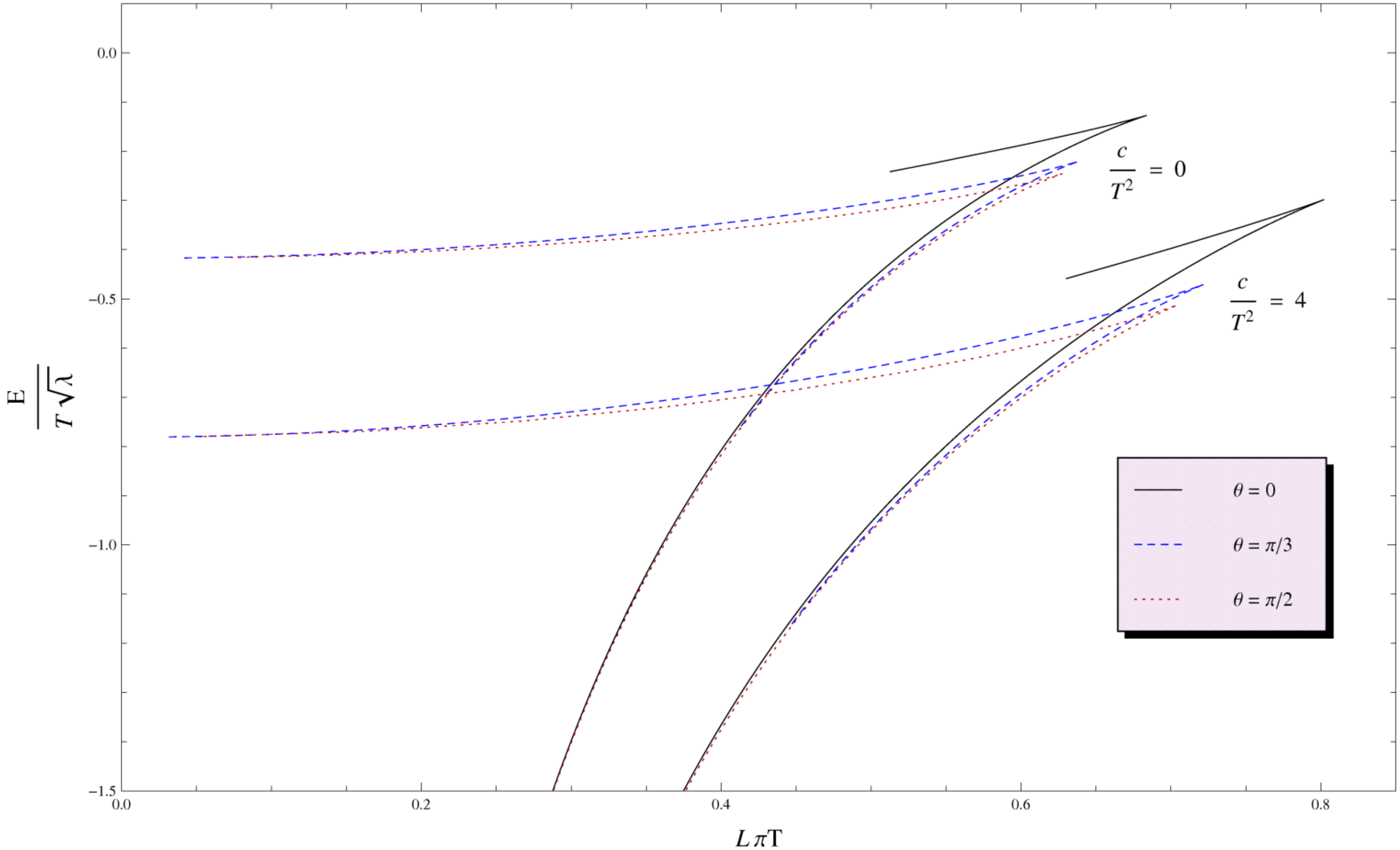
- Drag solution for a moving heavy quark:

$$S_0 = \sqrt{\lambda}\mathcal{T}T \int_1^{\infty} dy$$

- Equation for the free energy:

$$E\left(q(L),\eta,\frac{\pi}{2}\right)\mathcal{T} = \sqrt{\lambda}\mathcal{T}T \int_{y_c}^{\infty} \left[\frac{y^4 - \cosh^2\eta}{\sqrt{(y^4-y_c)(y^4-1)}} - 1\right] dy - \sqrt{\lambda}\mathcal{T}T(y_c - 1)$$

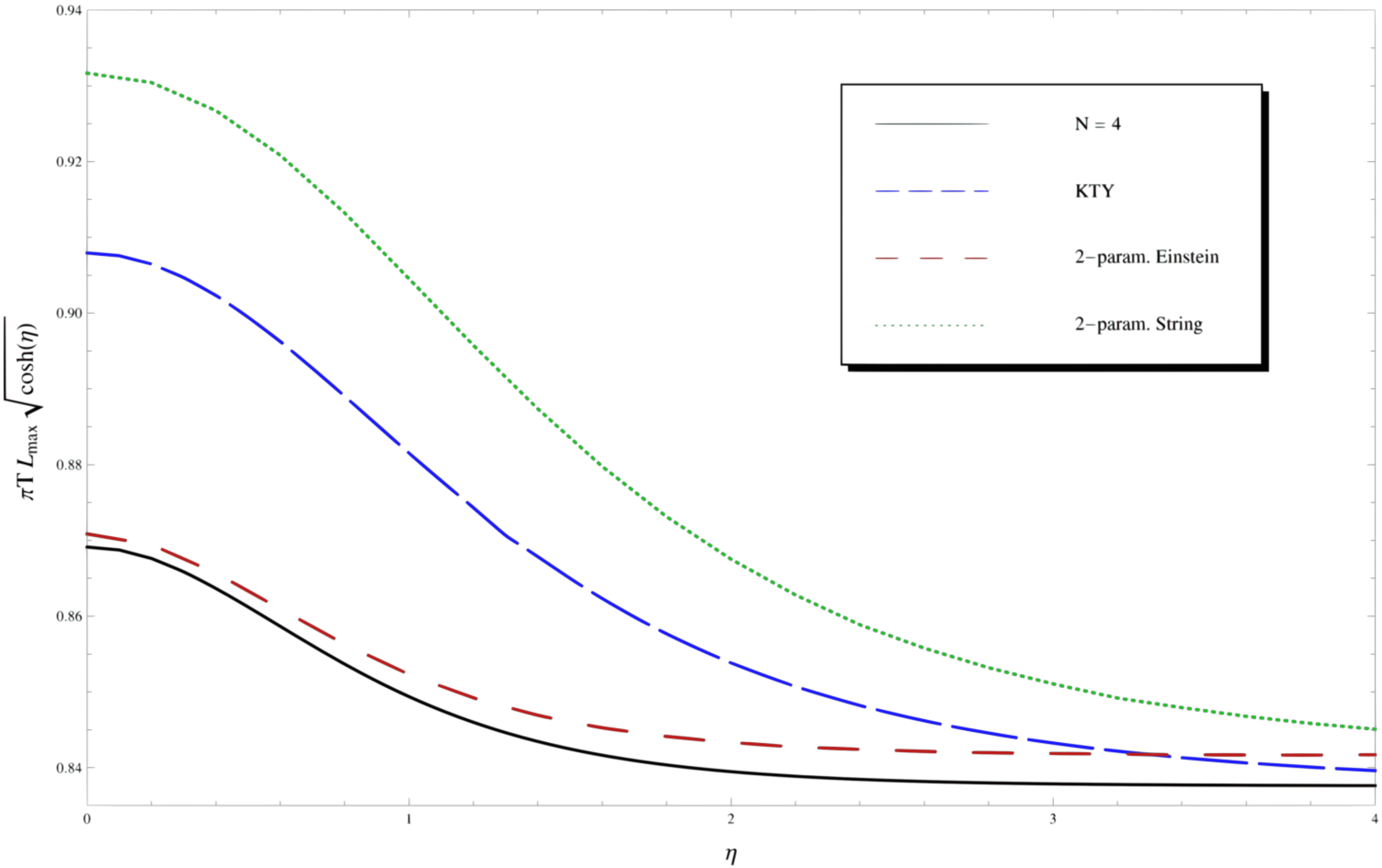
Free energy of a quark-antiquark pair



- Free energy for different orientation angles and $\eta = 1$

Screening length bound

- $L_{\max}$ is minimal for $\mathcal{N} = 4$



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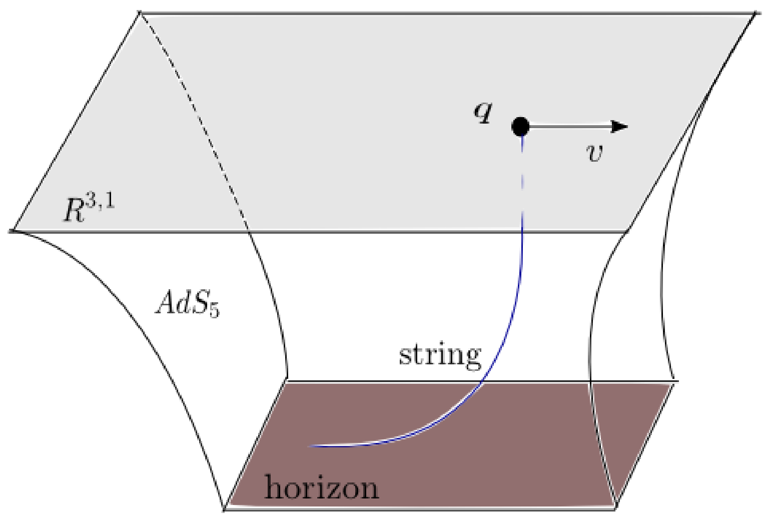
Single quark drag force

- Consider single quark being pulled through medium with velocity v



- Drag force is defined as:

$$\frac{dp_1}{dt} = -\frac{\pi_z}{2\pi\alpha'} \cdot \frac{r_0^2}{R^2} = -\frac{q}{2\pi\alpha'} \cdot \frac{r_0^2}{R^2}$$

- For $\mathcal{N} = 4$ we have

$$\frac{dp_1}{dt} = -\frac{\pi\sqrt{\lambda}T^2}{2} \cdot \frac{v}{\sqrt{1-v^2}}$$

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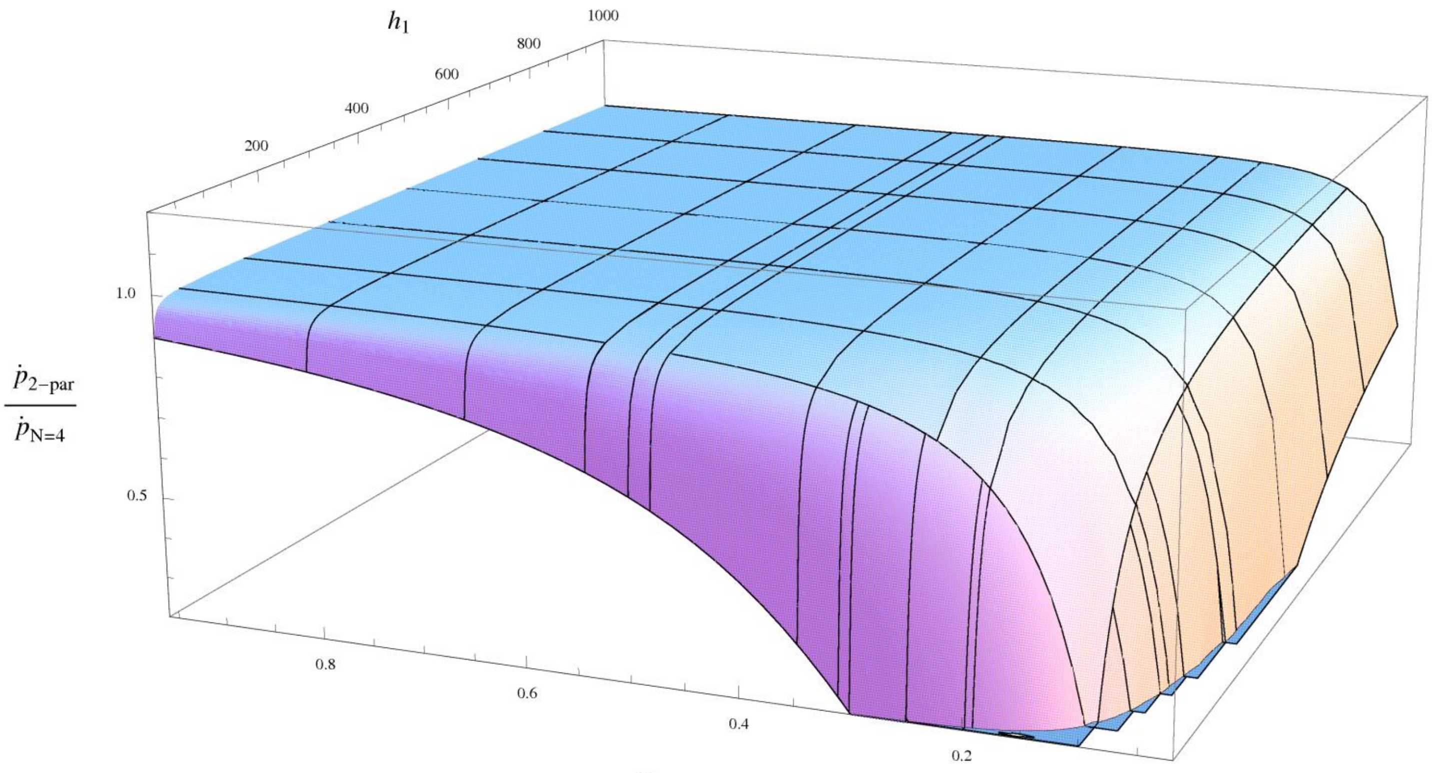
Drag Force bound

- Among 2-parameter models drag force has a maximum in $\mathcal{N} = 4$



- α is the conformal breaking parameter
- h_1 is a integration constant related to the temperature

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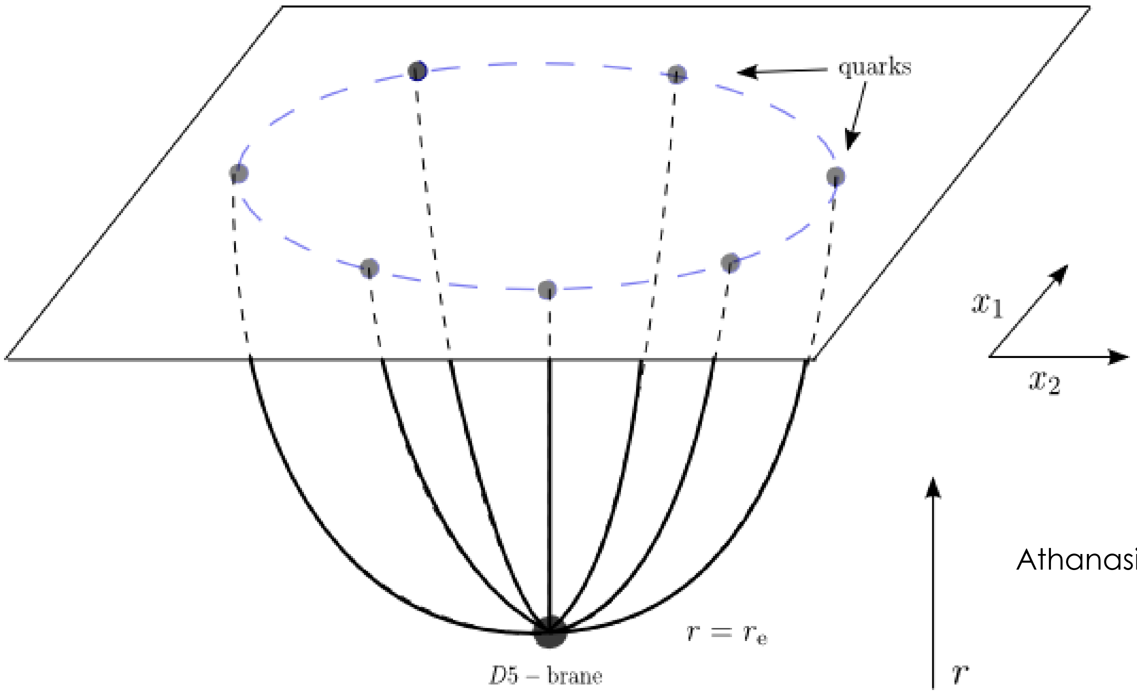
Heavy baryon screening

- Consider baryon configuration with N_c quarks arranged on a circle

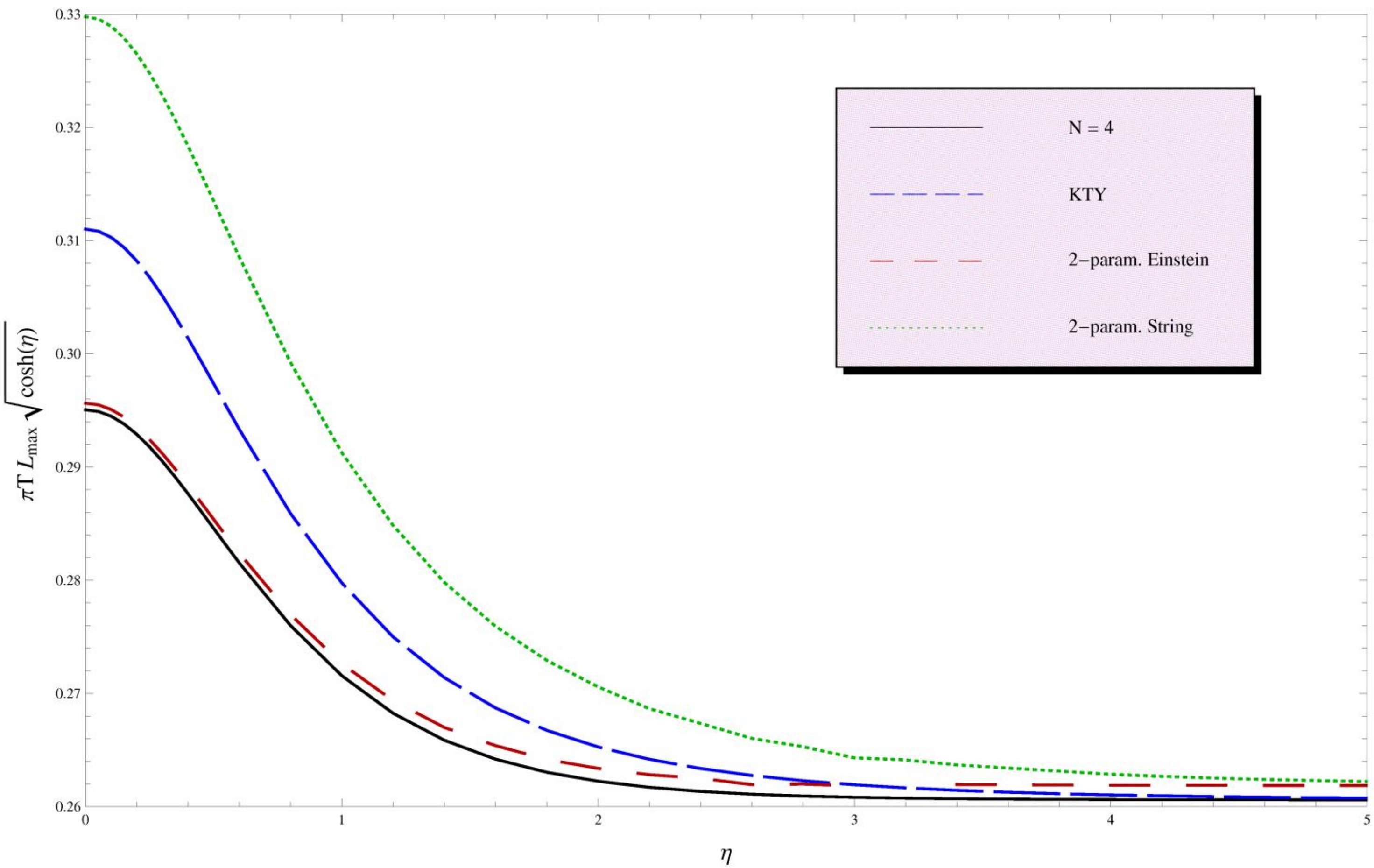


Athanasiou, Liu, Rajagopal

- Introduce density of quarks along the cirlce

Screening length bound for Baryons

- Again $L_{\max}$ is minimal for $\mathcal{N} = 4$



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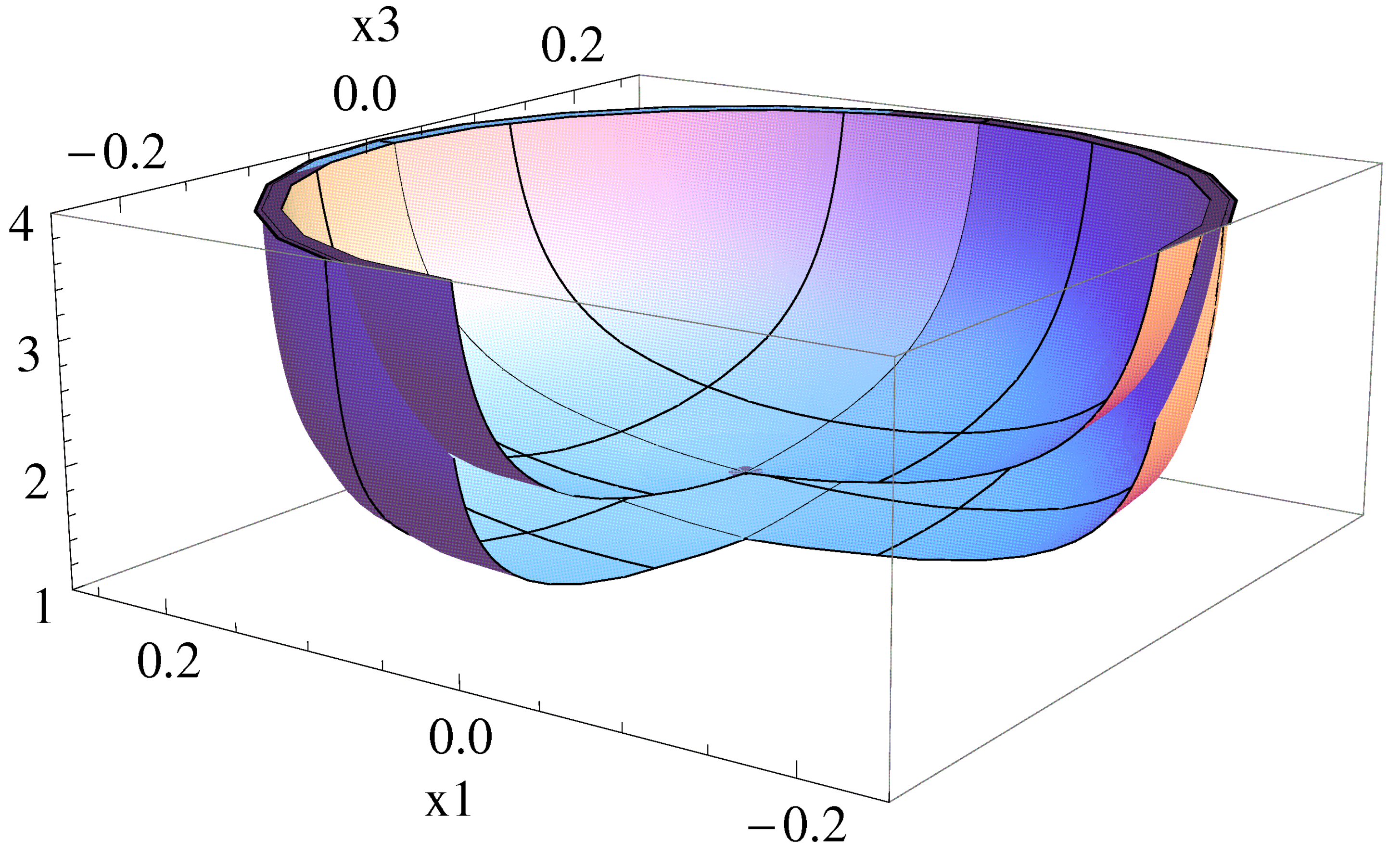
Baryon string configuration



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Summary

- We have calculated heavy meson and baryon screening in the wind of hot strongly coupled plasmas for two kinematic parameters (velocity $v = \tanh\eta$ and orientation angle θ)

- The screening length is a robust quantity

- The screening length in $\mathcal{N} = 4$ is minimal for all kinematic parameters in large class of theories

- We conjecture that it is a universal lower bound for an even wider range of theories.

Outlook

- General analytic study of small perturbations of the $AdS_5 \times S_5$ metric

- For KTY model a first order correction has the form:

$$L\pi T = L\pi T\big|_{\mathcal{N}=4} + c \cdot \frac{\sqrt{y_c^4 - 1}}{\pi^2 y_c^9} \left((3 + 4y_c^4) \frac{\sqrt{\pi}\,\Gamma(\frac{5}{4})}{\Gamma(\frac{3}{4})} \cdot {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{1}{y_c^4}\right) + \ldots \right) + \mathcal{O}(c^2)$$

- For the 2-parameter model an analytic computation is more difficult

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Thank you for your attention!

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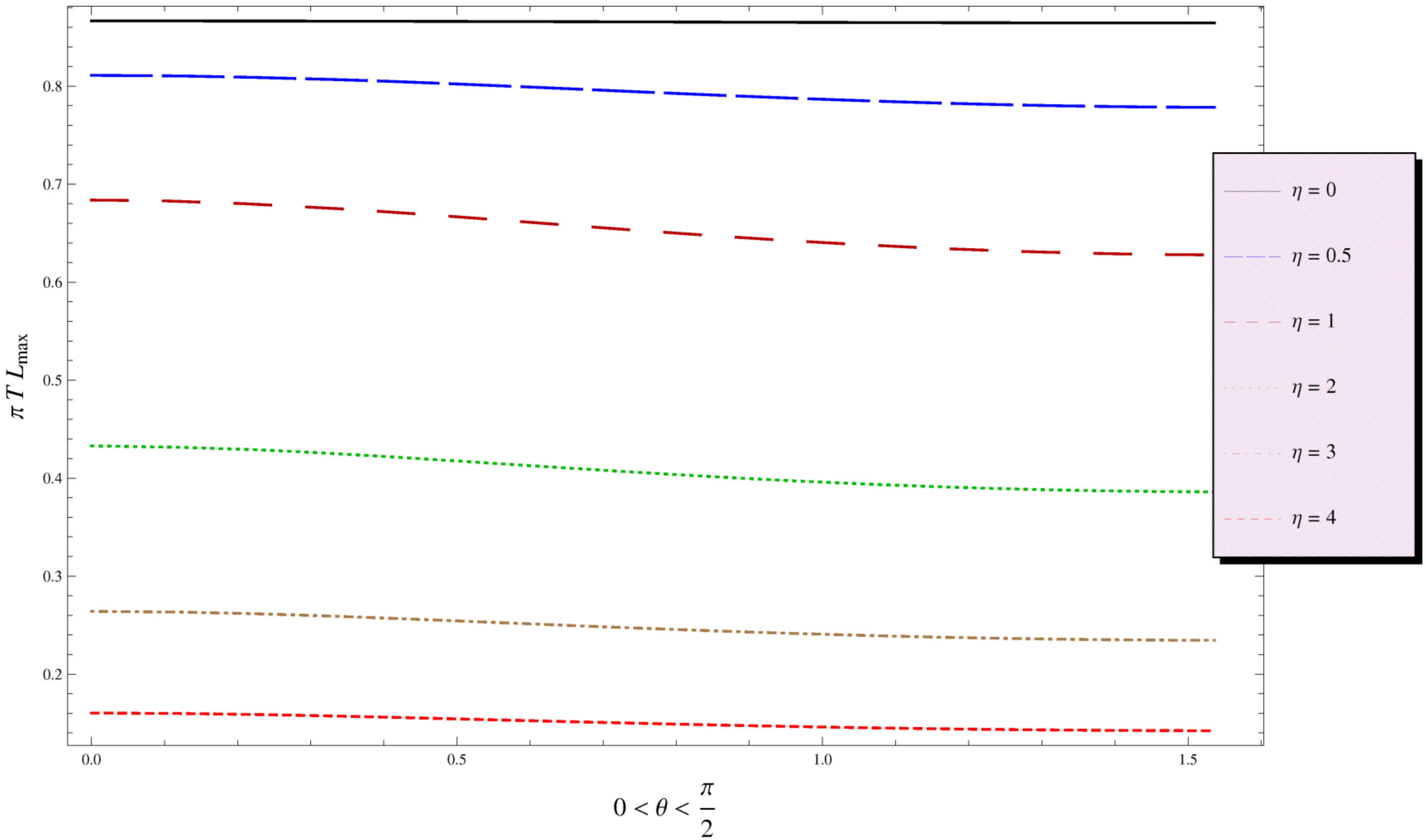

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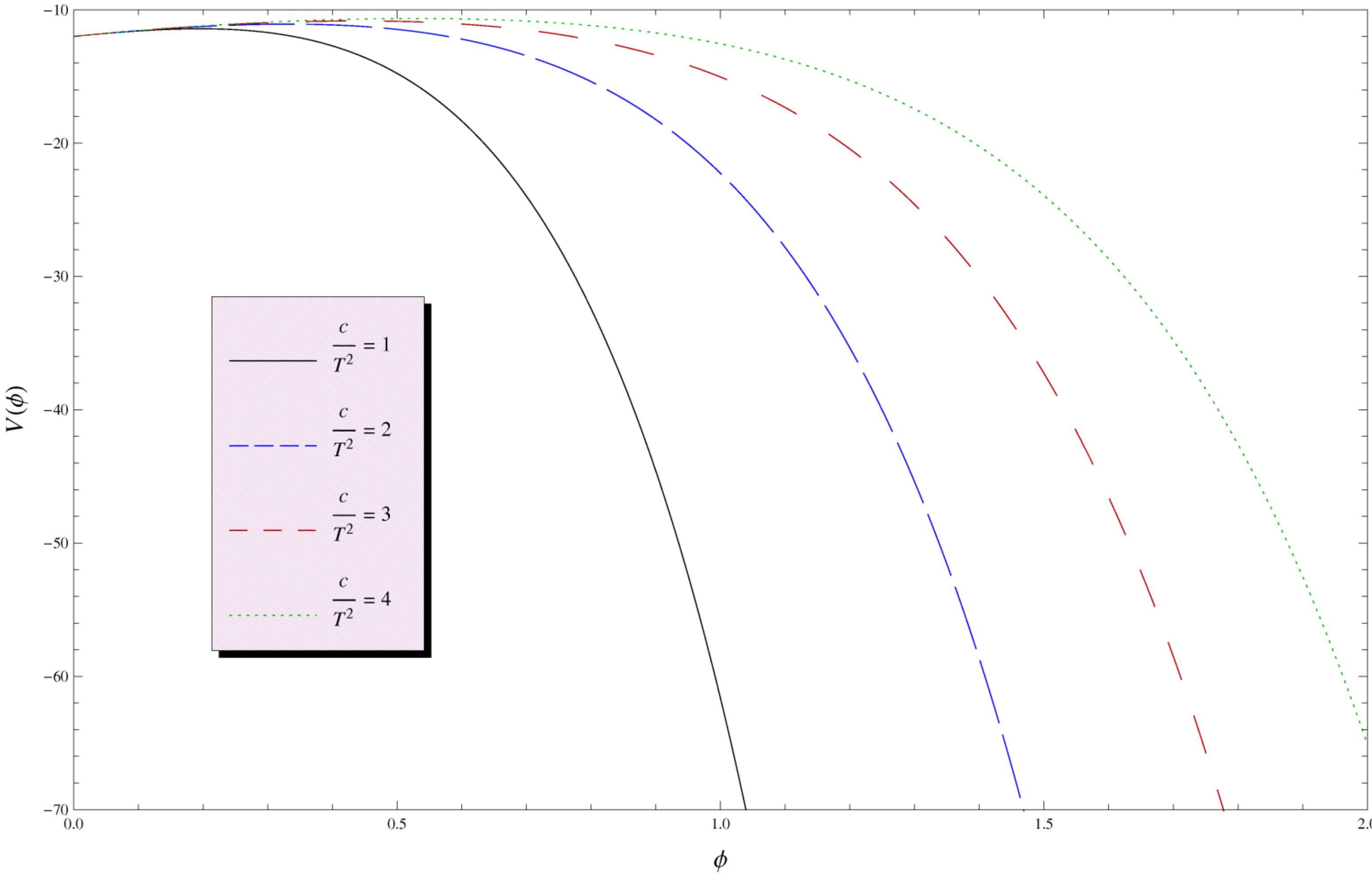

Analytic verification: Computation

- Easy for KTY. Use first order expansion and analytic computation of the screening length in $\mathcal{N}=4$

$$L\pi T = L\pi T\big|_{\mathcal{N}=4} + c\cdot\frac{\sqrt{y_c^4-1}}{\pi^2 y_c^9}\left((3+4y_c^4)\frac{\sqrt{\pi}\,\Gamma(\frac{5}{4})}{\Gamma(\frac{3}{4})}\cdot {}_2F_1\left(\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{1}{y_c^4}\right)+\ldots\right)+\mathcal{O}(c^2)$$

- Not possible for 2-parameter model (intractable equations)

→ Consider small perturbations of the $AdS_5\times S_5$ metric

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu} \quad \text{with} \quad h_{\mu\nu} \ll g^0_{\mu\nu}$$

1. Derivation of the linearized Einstein equations
2. Derivation of the equations of motion for general metric ansatz

$$\delta A(z),\ \ \delta B(z),\ \ \delta h(z)$$

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Analytic verification: Results

3. Use polynomial ansatz for $\delta A(z)$ and calculate $\delta B(z)$ and $\delta h(z)$:

$$\delta A = \phi z^k$$

- For $k = \frac{26}{9}$ an analytical calculation is possible:
  - The Drag Force is maximal in $\mathcal{N} = 4$

$$\frac{dp_{\text{pert}}/dt}{dp_{\mathcal{N}=4}/dt} = 1 + \left(\text{Sech}(\eta)^{22/9} - \frac{7}{3} \right) \phi + \mathcal{O}(\phi^2)$$

  - For the Screening length in $\mathcal{N} = 4$ an analogue result can be obtained

$$L\pi T = L\pi T\big|_{\mathcal{N}=4} + \phi \left(\frac{\sqrt{y_c^4 - 1}}{18\sqrt{\pi} y_c^{71/9}} \cdot (7y_c^{44/9} - 3) \frac{\Gamma(\frac{7}{4})}{\Gamma(\frac{5}{4})} \cdot {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{5}{4}, \frac{1}{y_c^4}\right) + \ldots \right) + \mathcal{O}(c^2)$$

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