Quarks and flavour degrees of freedom in AdS/CFT from string theory

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Based on joint work with

M. Ammon, R. Apreda, J. Babington, N. Evans, Z. Guralnik, V. Graß, C. Greubel, J. Große, M. Kaminski, P. Kerner, I. Kirsch, R. Meyer, H. Ngo, A. O'Bannon, F. Rust, R. Schmidt, T. Wrase

See also talks by

Andy O'Bannon, Patrick Kerner

Outline

1. Top-down approach to AdS/CFT correspondence from string theory

2. Adding quarks (flavour degrees of freedom) to the AdS/CFT correspondence

3. Applications to elementary particle physics (strong interaction)

4. Applications to condensed matter physics (superconductivity)

Top-down approach

Use 10-dimensional (super)gravity actions obtained from string theory

to describe

Dual degrees of freedom in strongly coupled quantum field theory

Quantum Theory of Gravity and Unification of Interactions:

Give up locality at very short distances

Natural cutoff: String length

$$l_s \sim \frac{1}{M_{Planck}},$$

Open strings: Gauge interactions   Closed strings: Gravity

Higher oscillation modes may be excited $\Rightarrow$ Particles

Quantization:

Supersymmetric string theory is well-defined in $9+1$ dimensions

(no tachyons, no anomalies)

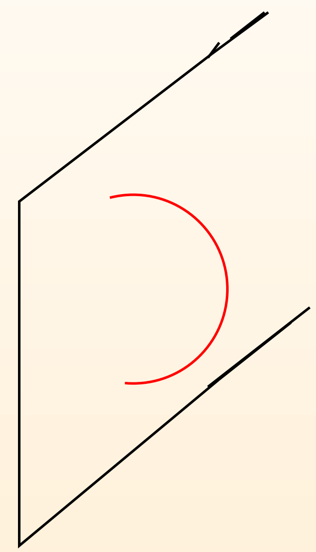
Supersymmetry: Bosons $\Leftrightarrow$ Fermions

What is the meaning of the extra dimensions?

1. Compactification

2. D-Branes

D-branes are embedded in ten-dimensional space (Hypersurfaces)



D3-Branes: (3+1)-dimensional hypersurfaces

open strings may end on D-branes $\Leftrightarrow$ dynamics

D-Branes

In low-energy limit (strings pointlike) $\Rightarrow$

Open Strings $\Leftrightarrow$ Field theory (Gauge theory) degrees of freedom on the brane

Second interpretation of D-branes:

Solitonic solutions of ten-dimensional supergravity

heavy objects which curve the space around them

Elementary excitations: closed strings

AdS/CFT correspondence

Map:

Four-dimensional quantum field theory

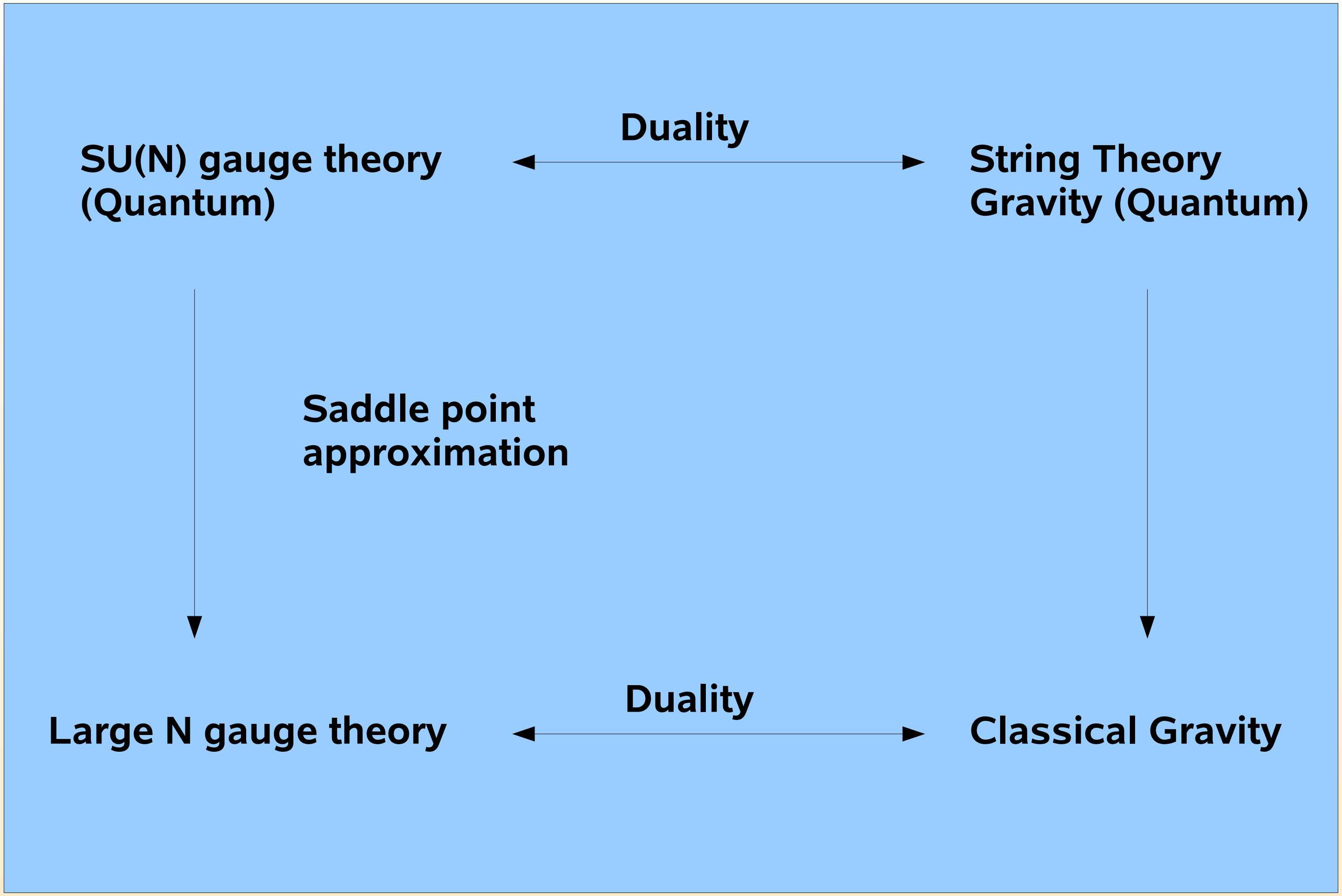
$\Leftrightarrow 5+5$-dimensional (classical) gravity theory!

arises from identifying the two different interpretations of D-branes

D3 Branes $\Rightarrow$

$\mathcal{N}=4$ Super Yang-Mills theory is dual to string theory on $AdS_5 \times S^5$

AdS/CFT correspondence (Maldacena 1997)



AdS/CFT correspondence

- Anti-de Sitter space is a curved space with constant negative curvature. It has a boundary.

  Metric: $ds^2 = e^{2r/R}\eta_{\mu\nu}dx^\mu dx^\nu + dr^2$ or $ds^2 = L^2/u^2(\eta_{\mu\nu}dx^\mu dx^\nu + du^2)$

- Isometry group of $(d+1)$-dimensional AdS space coincides with conformal group in d dimensions ($SO(d,2)$).

- $SO(6) \simeq SU(4)$: Isometry of $S^5 \Leftrightarrow \mathcal{N} = 4$ Supersymmetry

- Dictionary:

field theory operators $\Leftrightarrow$ supergravity fields

$$\mathcal{O}_\Delta \leftrightarrow \phi_m \ , \qquad \Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + R^2 m^2}$$

- Field-operator correspondence:

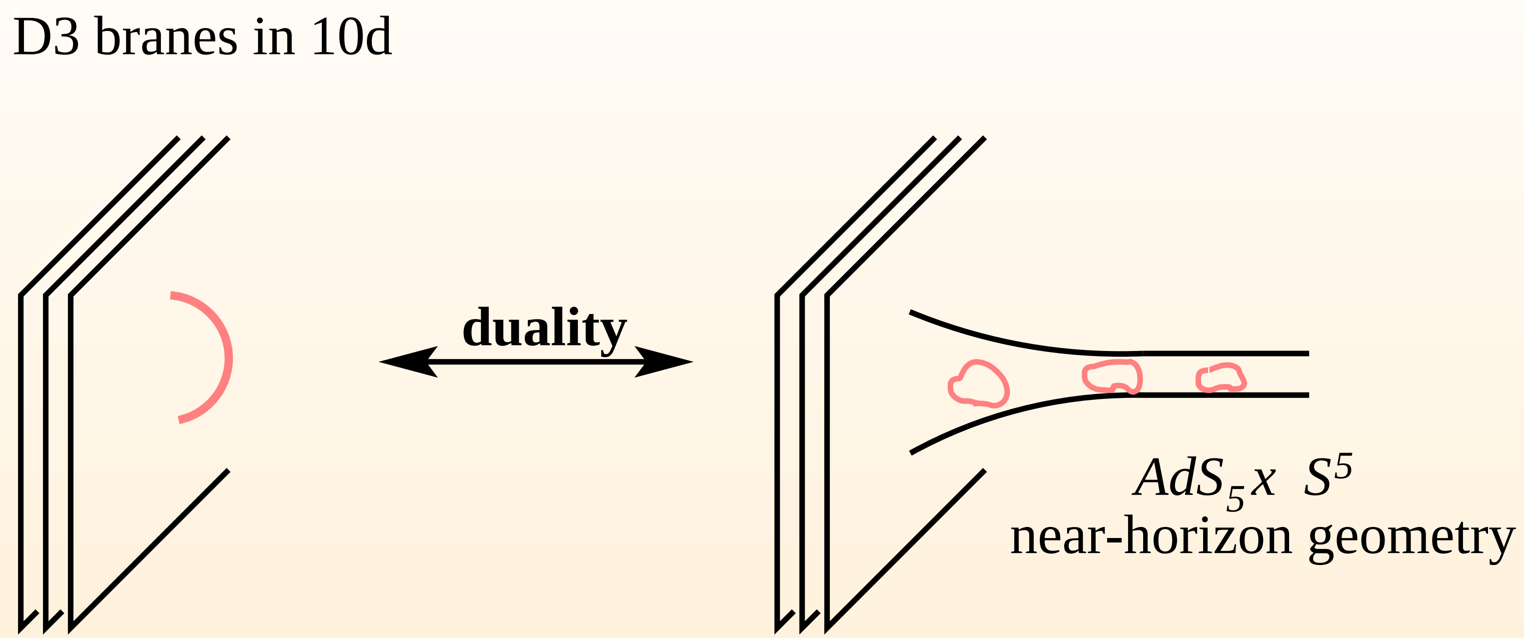
$$\langle e^{\int d^d x\, \phi_0(\vec{x}) \mathcal{O}(\vec{x})} \rangle_{CFT} = Z_{sugra} \Big|_{\phi(0,\vec{x})=\phi_0(\vec{x})}$$

Generating functional for correlation functions of particular composite operators in the quantum field theory

coincides with

Classical tree diagram generating functional in supergravity

String theory origin of AdS/CFT correspondence



$\Downarrow$ Low-energy limit

$\mathcal{N} = 4\ SU(N)$ theory in four dimensions ($N \to \infty$)

Supergravity on $AdS_5 \times S^5$

Generalized AdS/CFT Correspondence

Generalizations:

1. Symmetry requirements are relaxed in a controlled way

   $\Rightarrow$ Renormalization Group flows

   $\Rightarrow$ Theories with confinement similar to QCD

2. More degrees of freedom are added (Example: quarks)

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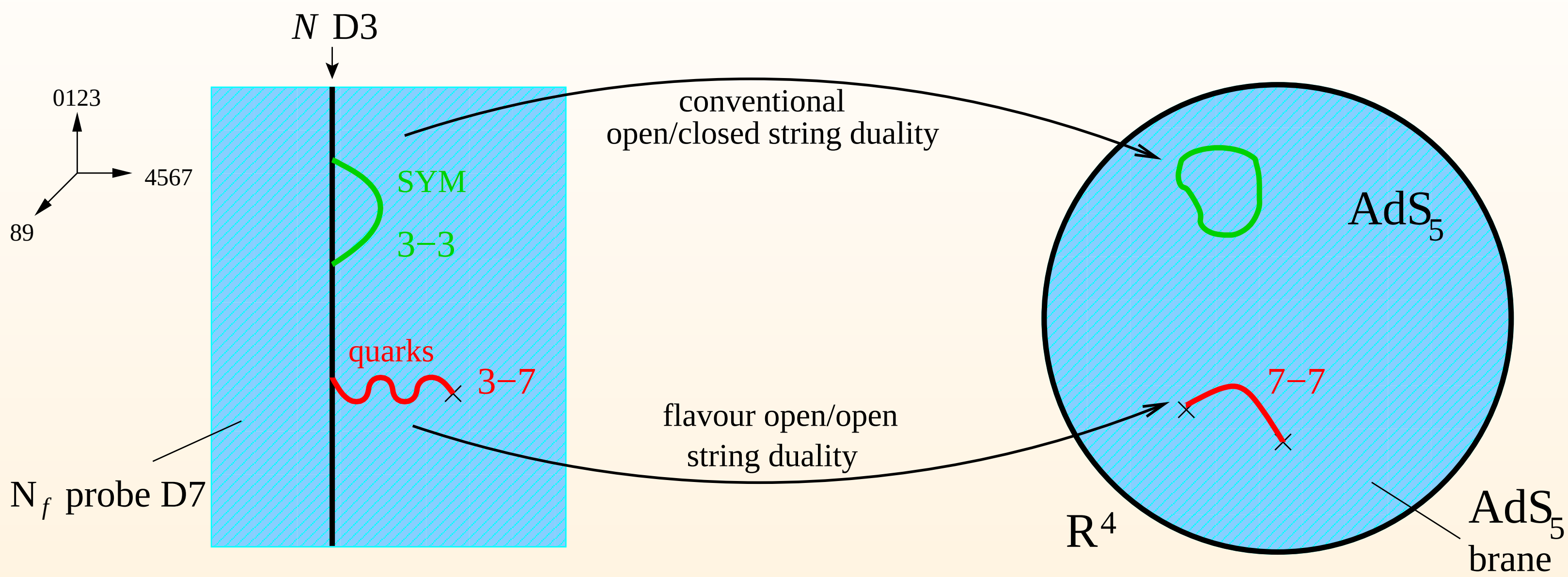
Strongly coupled quantum field theories

(difficult to solve)

are mapped to

Weakly coupled gravity theories (easy to solve)

Quarks (fundamental fields) from brane probes



$N \to \infty$ (standard Maldacena limit), N_f small (probe approximation)

duality acts twice:

$\mathcal{N} = 4$ SU(N) Super Yang-Mills theory coupled to $\mathcal{N} = 2$ fundamental hypermultiplet	$\longleftrightarrow$	IIB supergravity on $AdS_5 \times S^5$ + Probe brane DBI on $AdS_5 \times S^3$

Karch, Katz 2002

Combine the deformation of the supergravity metric

with the addition of brane probes:

Dual gravity description of chiral symmetry breaking and Goldstone bosons

J. Babington, J. E., N. Evans, Z. Guralnik and I. Kirsch,

“Chiral symmetry breaking and pions in non-SUSY gauge/gravity duals”

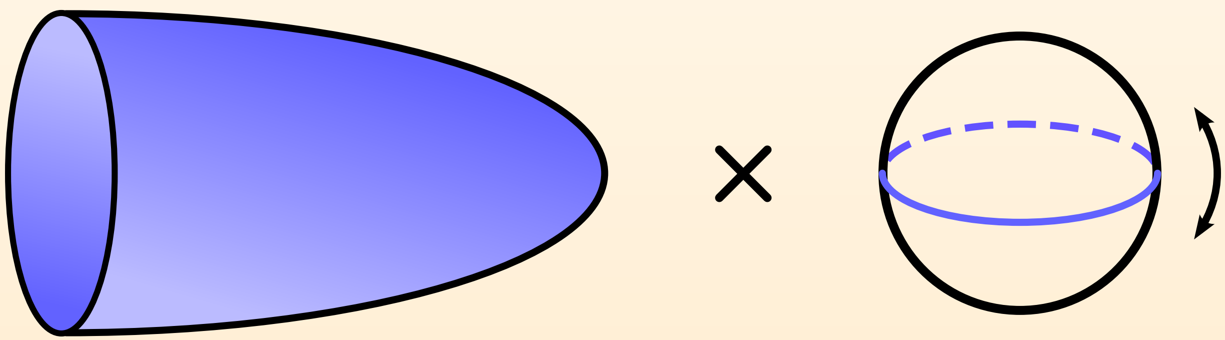
hep-th/0306018, Phys. Rev. D 69 (2004) 066007

Light mesons

Babington, J.E., Evans, Guralnik, Kirsch PRD 2004

Meson masses obtained from fluctuations of hypersurface probe D7-brane

in a confining non-supersymmetric ten-dimensional gravity background



π pseudoscalar meson mass: From fluctuations of D-brane

ρ vector meson mass: From fluctuations of gauge field on D-brane

D7 brane probe in deformed backgrounds

D7 brane probe in gravity backgrounds dual to

confining gauge theories without supersymmetry.

Example:

Constable-Myers background (particular deformation of $AdS_5 \times S^5$ metric)

- The deformation introduces a new scale into the metric.
- In UV limit, geometry returns to $AdS_5 \times S^5$ with D7 probe wrapping $AdS_5 \times S^3$.

General strategy

1. Start from Dirac-Born-Infeld action for a D7-brane embedded in deformed background

2. Derive equations of motion for transverse scalars (w_5, w_6)

3. Solve equations of motion numerically using shooting techniques

   Solution determines embedding of D7-brane (e.g. $w_5 = 0, w_6 = w_6(\rho)$)

4. Meson spectrum:

   Consider fluctuations δw_5, δw_6 around a background solution obtained in 3.

   Solve equations of motion linearized in $\delta w_5, \delta w_6$

Asymptotic behaviour of supergravity solutions

UV asymptotic behaviour of solutions to equation of motion:

$$w_6 \propto m\, e^{-r} + c\, e^{-3r}$$

Identification of the coefficients as in the standard AdS/CFT correspondence:

m quark mass, $c = \langle \bar{q} q \rangle$ quark condensate

Here:

$m \neq 0$: explicit breaking of $U(1)_A$ symmetry

$c \neq 0$: spontaneous breaking of $U(1)_A$ symmetry

The Constable-Myers deformation

$\mathcal{N} = 4$ super Yang-Mills theory deformed by VEV for $tr\, F^{\mu\nu} F_{\mu\nu}$
(R-singlet operator with $D = 4$) $\quad \rightarrow$ non-supersymmetric QCD-like field theory

The Constable-Myers background is given by the metric

$$ds^2 = H^{-1/2} \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{\delta/4} dx_4^2 + H^{1/2} \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{(2-\delta)/4} \frac{w^4 - b^4}{w^4} \sum_{i=1}^{6} dw_i^2,$$

where

$$H = \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{\delta} - 1 \qquad (\Delta^2 + \delta^2 = 10)$$

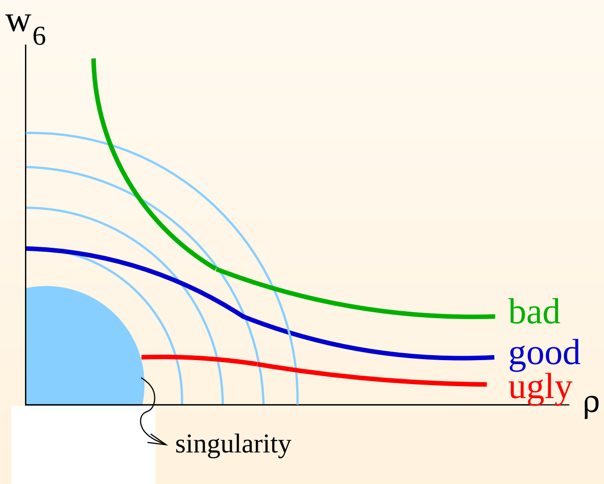
and the dilaton and four-form

$$e^{2\phi} = e^{2\phi_0} \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{\Delta}, \qquad C_{(4)} = -\frac{1}{4} H^{-1} dt \wedge dx \wedge dy \wedge dz$$

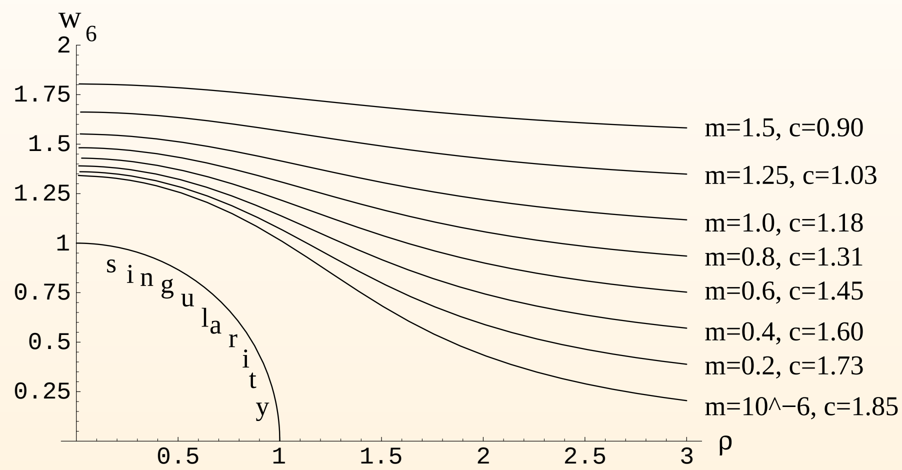
This background has a singularity at $w = b$

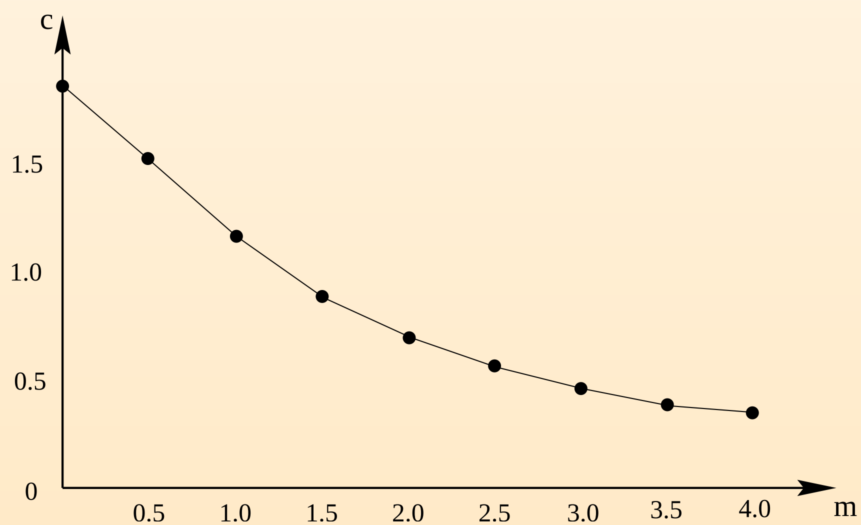
Chiral symmetry breaking

Solution of equation of motion for probe brane



Numerical Result:





Result:

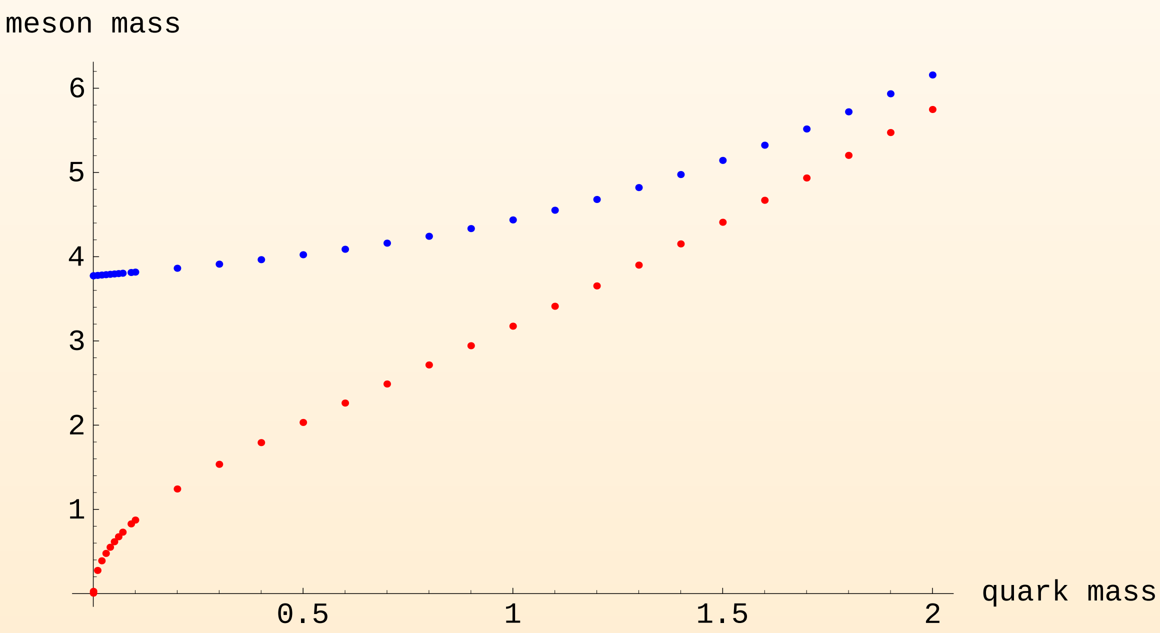
Screening effect: Regular solutions do not reach the singularity

Spontaneous breaking of $U(1)_A$ symmetry: For $m \to 0$ we have $c \equiv \langle \bar{\psi}\psi \rangle \neq 0$

Meson spectrum

From fluctuations of the probe brane

Ansatz: $\delta w_i(x,\rho) = f_i(\rho) sin(k\cdot x)\,, \qquad M^2 = -k^2$

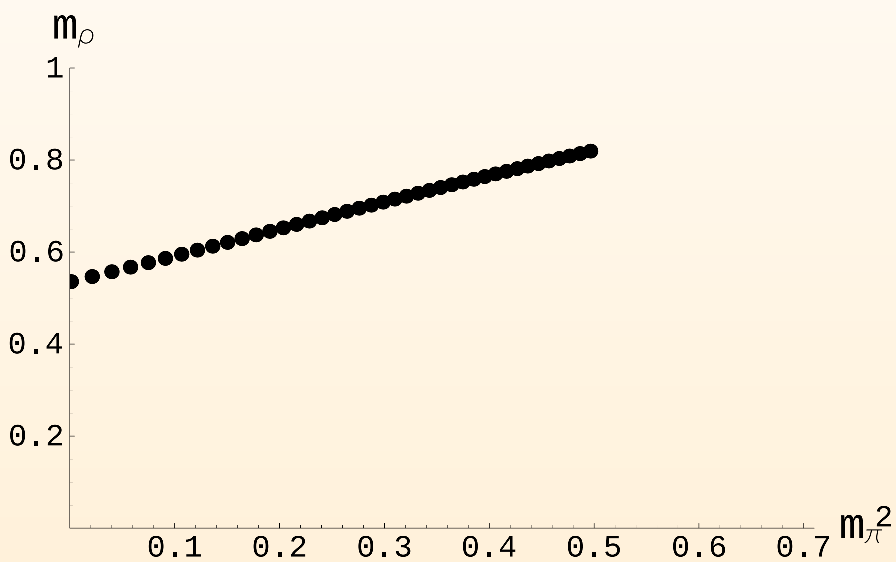


Goldstone boson (η')

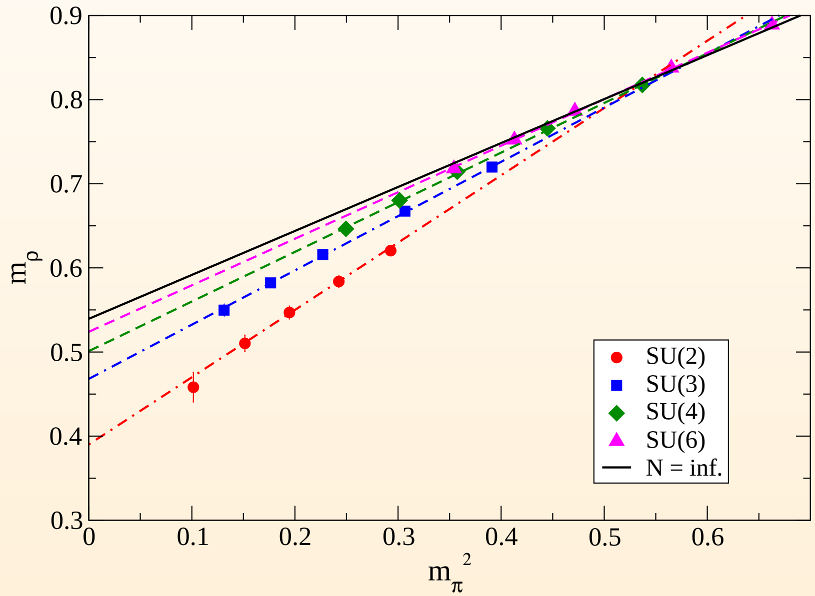
Gell-Mann-Oakes-Renner relation: $M_{Meson} \propto \sqrt{m_{Quark}}$

Comparison to lattice gauge theory

Mass of ρ meson as function of π meson mass^2 (for $N \to \infty$)



J.E., Evans, Kirsch, Threlfall '07, review EPJA



Lattice: Lucini, Del Debbio, Patella, Pica '07

AdS/CFT result:

$$\frac{m_\rho(m_\pi)}{m_\rho(0)} = 1 + 0.307 \left(\frac{m_\pi}{m_\rho(0)}\right)^2$$

Lattice result (from Bali, Bursa '08): slope 0.341 ± 0.023

$\mathcal{N}=4$ Super Yang-Mills theory at finite temperature is dual to AdS black hole

Witten 1998

$$ds^2 = \frac{1}{2}\left(\frac{\varrho}{R}\right)^2\left(-\frac{f^2}{\tilde{f}}\mathrm{d}t^2 + \tilde{f}\mathrm{d}\vec{x}^2\right) + \left(\frac{R}{\varrho}\right)^2(\mathrm{d}\varrho^2 + \varrho^2\mathrm{d}\Omega_5^2)$$

$$f(\varrho) = 1 - \frac{\varrho_H^4}{\varrho^4}, \quad \tilde{f}(\varrho) = 1 + \frac{\varrho_H^4}{\varrho^4}$$

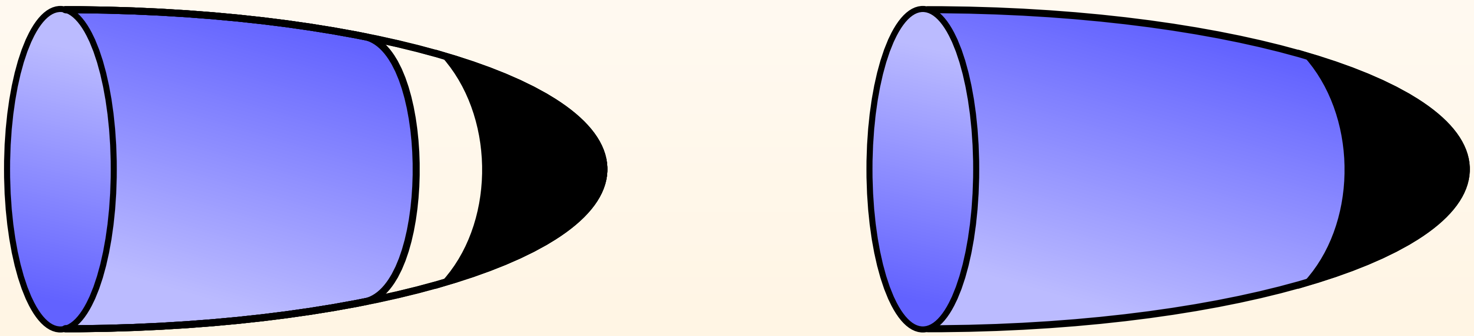
Temperature and horizon related by

$$T = \frac{\varrho_H}{\pi R^2}$$

R: AdS radius

For $\varrho_H \to 0$, metric of $AdS_5 \times S^5$ is recovered.

D7 brane embedding in black hole background



First order phase transition

Babington, J.E., Evans, Guralnik, Kirsch
Mateos, Myers, Thomson

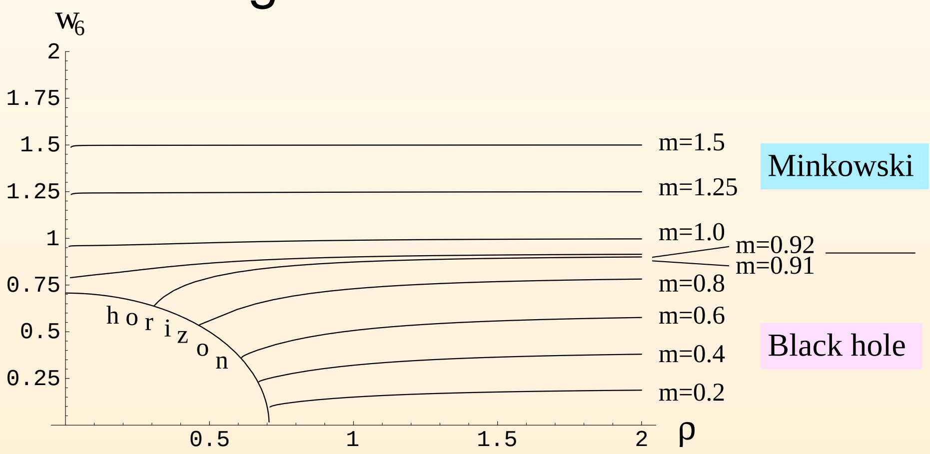
D7 brane embedding in black hole background

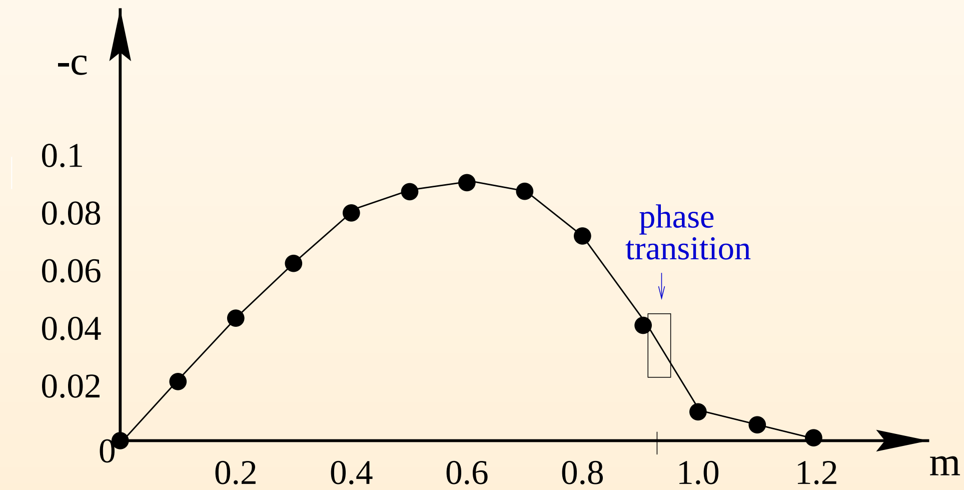
Babington, J.E., Evans, Guralnik, Kirsch 0306018

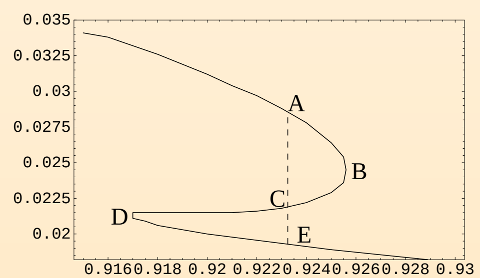
Condensate $c \equiv \langle\bar{\psi}\psi\rangle$ vs. quark mass m

m in units of T

Embeddings







Phase transition at $m_c \approx 0.92$

(1st order)

Kirsch 2004

Masses and decay widths of mesons - Spectral functions

Standard procedure in D3/D7: Mateos, Myers et al 2003

Meson masses calculated from linearized fluctuations of D7 embedding

Fluctuations: $\delta w(x,\rho) = f(\rho) e^{i(\vec{k}\cdot\vec{x}-\omega t)}$, $M^2 = -k^2$

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For black hole embeddings, ω develops negative imaginary part

$\Rightarrow$ damping $\Rightarrow$ decay width

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Make contact with hydrodynamics: Starinets, Kovtun ....

Spectral function determined by poles of retarded Green function

Quasinormal modes

Identify mesons with resonances in spectral function

Landsteiner, Hoyos, Montero

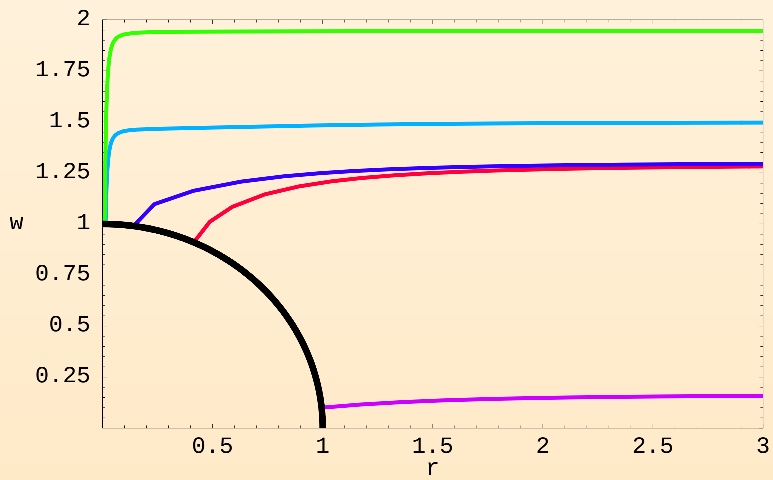
Finite $U(1)$ baryon density

Mateos, Myers, Matsuura et al

Baryon density n_B and $U(1)$ chemical potential μ
from VEV for gauge field time component:

$$\bar{A}_0(\rho) \sim \mu + \frac{\tilde{d}}{\rho^2}\,, \qquad \tilde{d} = \frac{2^{5/2}}{N_f\sqrt{\lambda}T^3}\, n_B$$

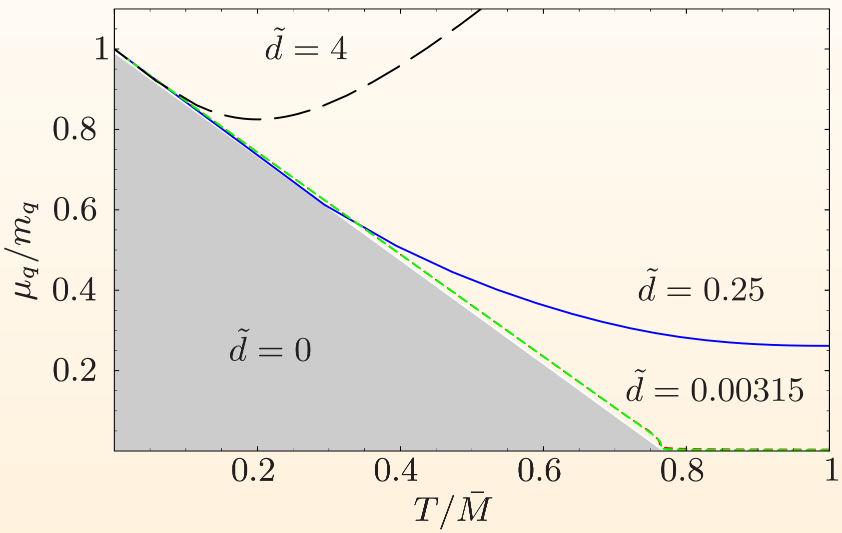
At finite baryon density, all embeddings are black hole embeddings



Phase diagram with finite $U(1)$ baryon density

Phase diagram:

grey region: $n_B = 0$
white region: $n_B \neq 0$



Sin, Yogendran et al; Mateos, Myers et al; Karch, O'Bannon; . . .

Isospin chemical potential and density

- Embed two coincident D7-branes into AdS-Schwarzschild
  gauge fields $A_\mu = A^a_\mu \sigma^a \in u(2) = u(1)_B \oplus su(2)_I$
- Finite isospin density: $A^3_0 \neq 0 \Rightarrow$ Explicit breaking to $u(1)_3$
- Dynamics of Flavour degrees is described by non-abelian DBI action

Field theory described:

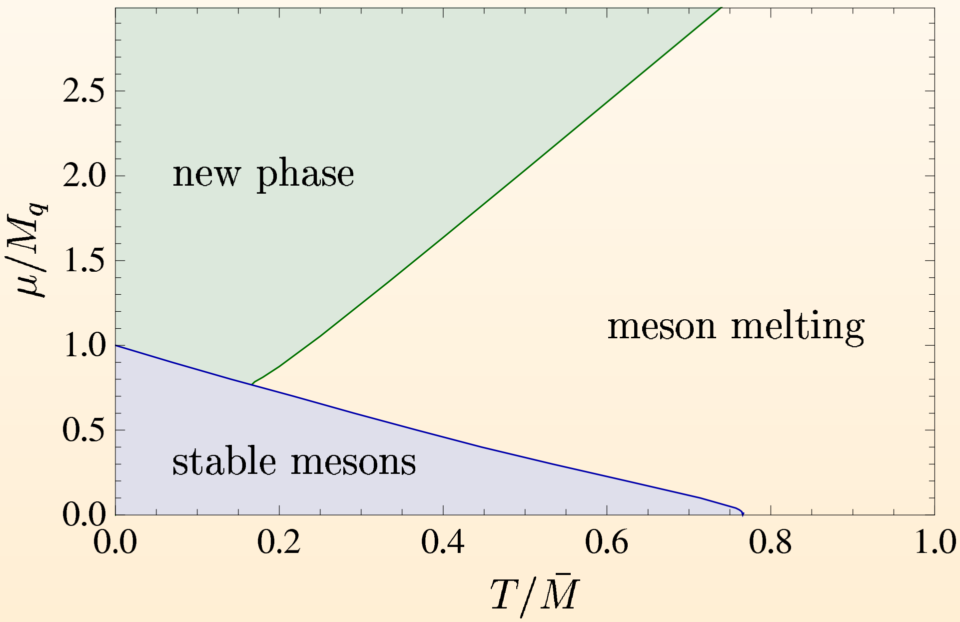
$\mathcal{N} = 4$ Super Yang-Mills plus two flavors of fundamental matter

at finite temperature and finite isospin density

ρ meson condensation

J.E., Kaminski, Kerner, Rust 0807.2663

Above a critical isospin density, a new phase forms



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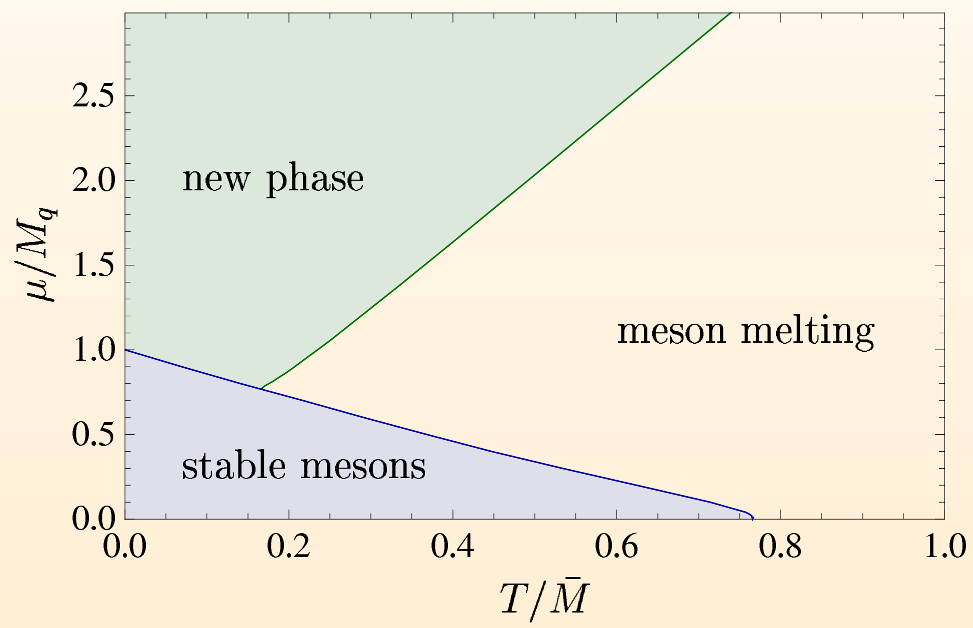
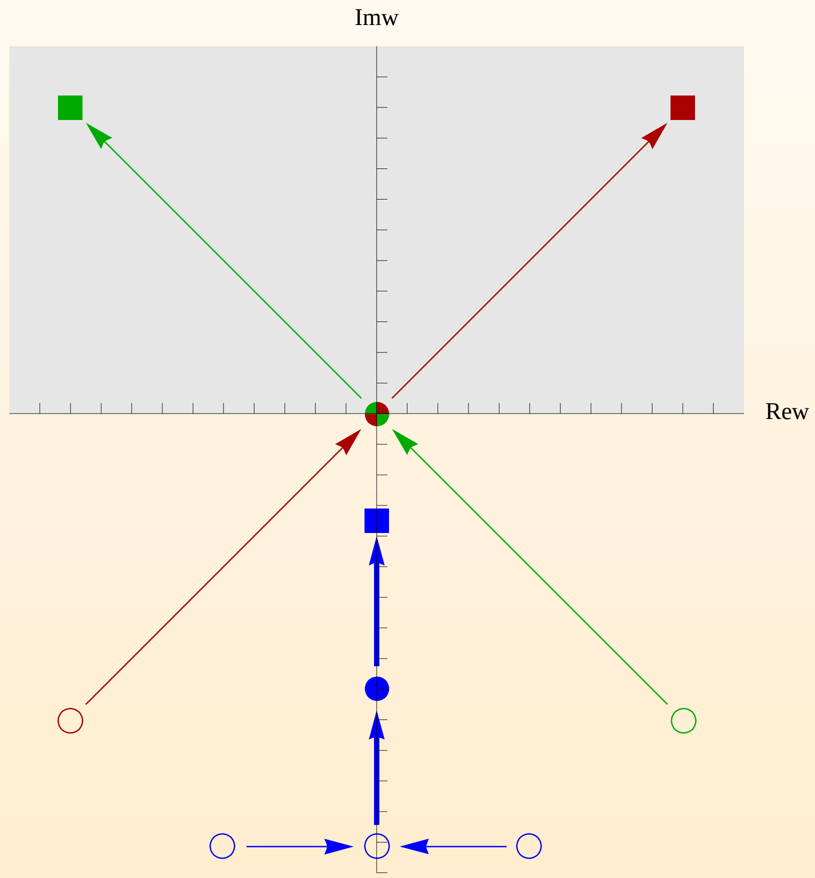


New phase is unstable

Instability:



A new ground state forms

There is a new solution to the equations of motion

with non-zero vev for $A_3^1\sigma^1$ in addition to the non-zero $A_0^3\sigma^3$

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$$A_0^3 = \mu - \frac{\tilde{d}_0^3}{2\pi\alpha'} \frac{\rho_H}{\rho^2} + \ldots \,, \qquad A_3^1 = -\frac{\tilde{d}_1^3}{2\pi\alpha'} \frac{\rho_H}{\rho^2} + \ldots$$

A new ground state forms

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with non-zero vev for $A_3^1\sigma^1$ in addition to the non-zero $A_0^3\sigma^3$

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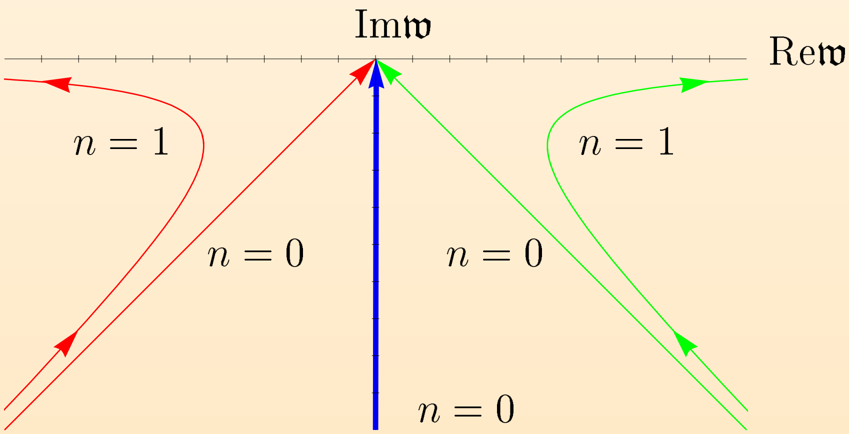
Pole structure:



Superconductivity

Ammon, J.E., Kaminski, Kerner 0810.2316, 0903.1864

The new ground state has properties known from superconductors:

- infinite DC conductivity, gap in the AC conductivity
- second order phase transition, critical exponent of 1/2 (mean field)
- a remnant of the Meissner–Ochsenfeld effect

Order parameter $\tilde{d}_3^1 \propto \langle \bar{\psi}_u \gamma_3 \psi_d + \bar{\psi}_d \gamma_3 \psi_u + \, bosons \,\rangle \neq 0$

Dual to $A_3^1 \sigma^1$ in gravity theory

Superfluidity and Superconductivity

Order parameter $\tilde{d}_3^1 \propto \langle \bar{\psi}_u \gamma_3 \psi_d + \bar{\psi}_d \gamma_3 \psi_u + \, bosons \, \rangle \neq 0$

Dual to $A_3^1 \sigma^1$ in gravity theory

Spontaneous breaking of (global) $U(1)_3$

Flavor superfluid

Superfluidity and Superconductivity

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Dual to $A_3^1 \sigma^1$ in gravity theory

Spontaneous breaking of (global) $U(1)_3$

Flavor superfluid

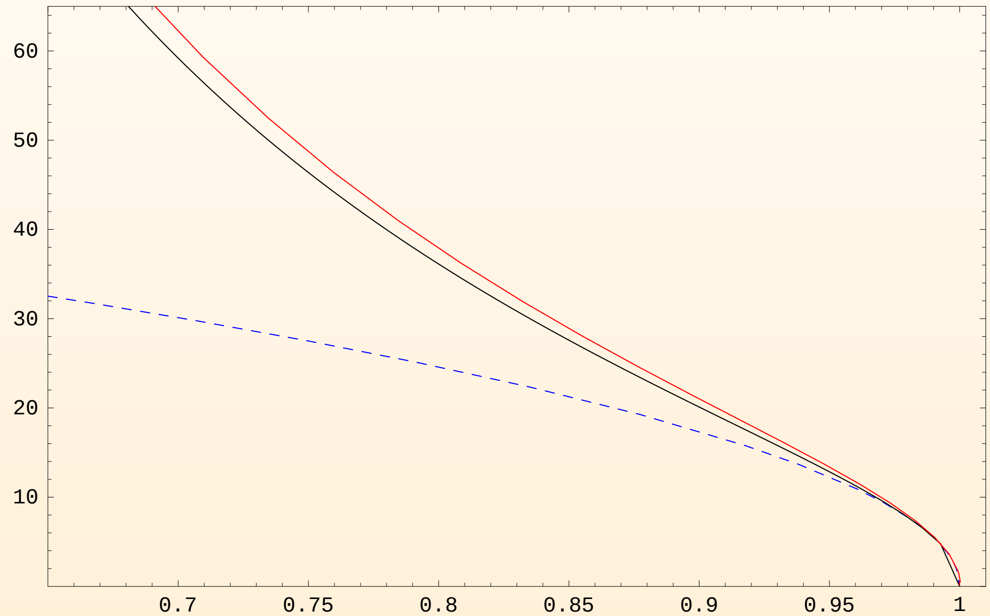
Condensate corresponds to ρ meson superfluid

discussed in QCD literature Son, Stephanov; Splittorff; Sannino ...

ρ condensation in Sakai-Sugimoto model: Aharony, Peeters, Sonnenschein, Zamaklar

Order parameter: p wave condensate

$-\tilde{d}_3^1$

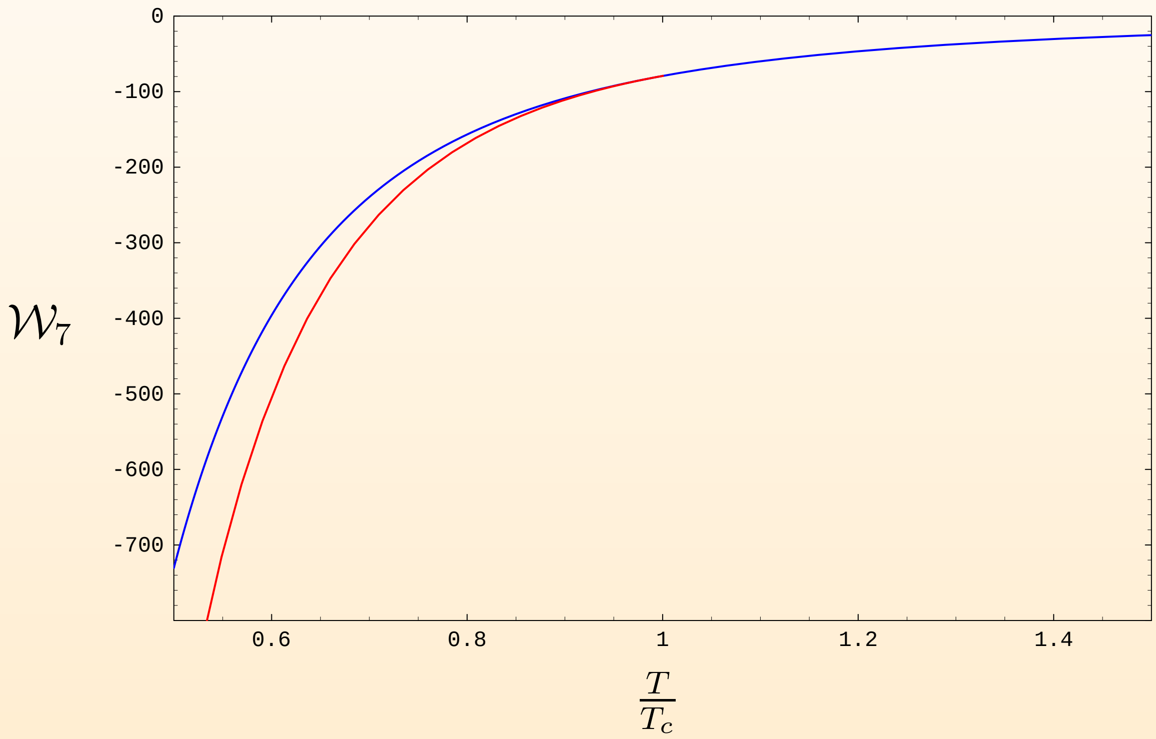


T/T_c

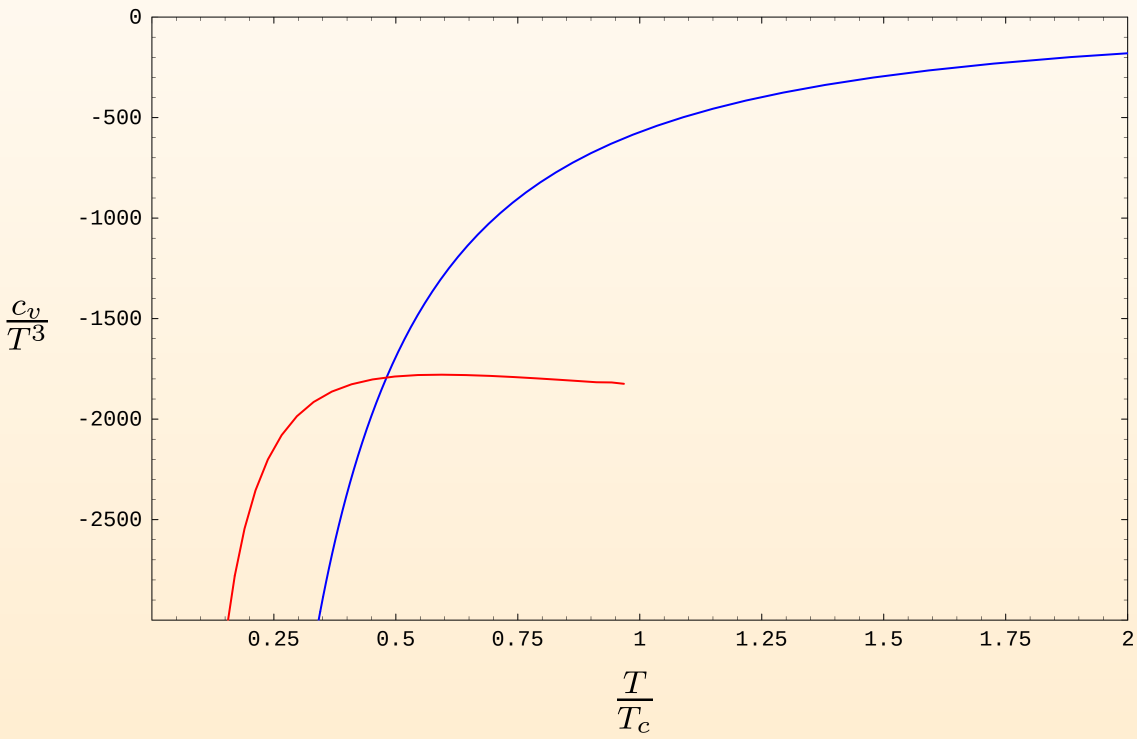
Red: Vanishing quark mass; Black: Finite quark mass, $\mu/M_q = 3$

Blue: Fit displaying critical exponent 1/2

Flavor contribution to Grand potential vs. temperature



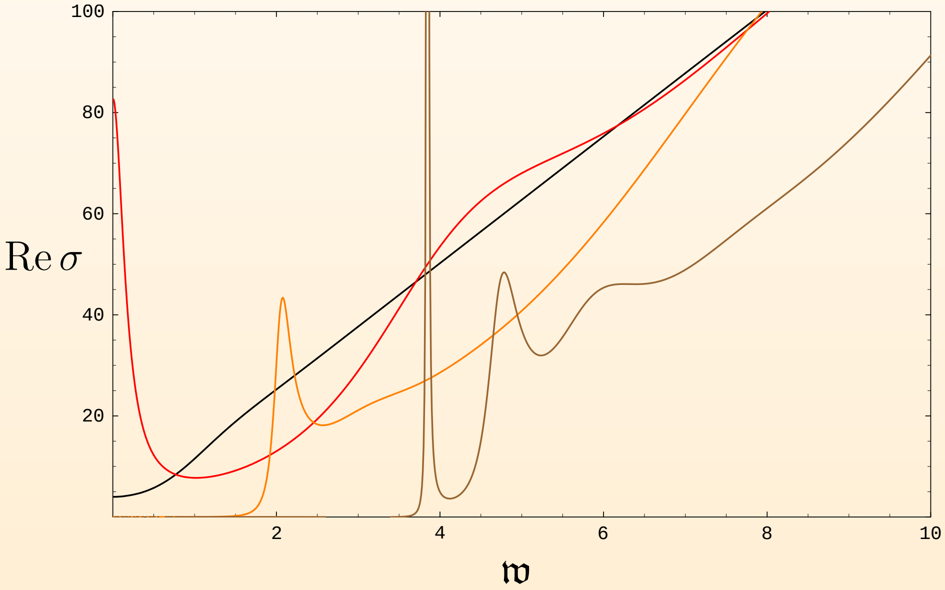
Flavor contribution to heat capacity



Conductivity

Frequency-dependent conductivity $\sigma(\omega) = \frac{i}{\omega} G^R(\omega)$

G^R retarded Green function for fluctuation a_2^3

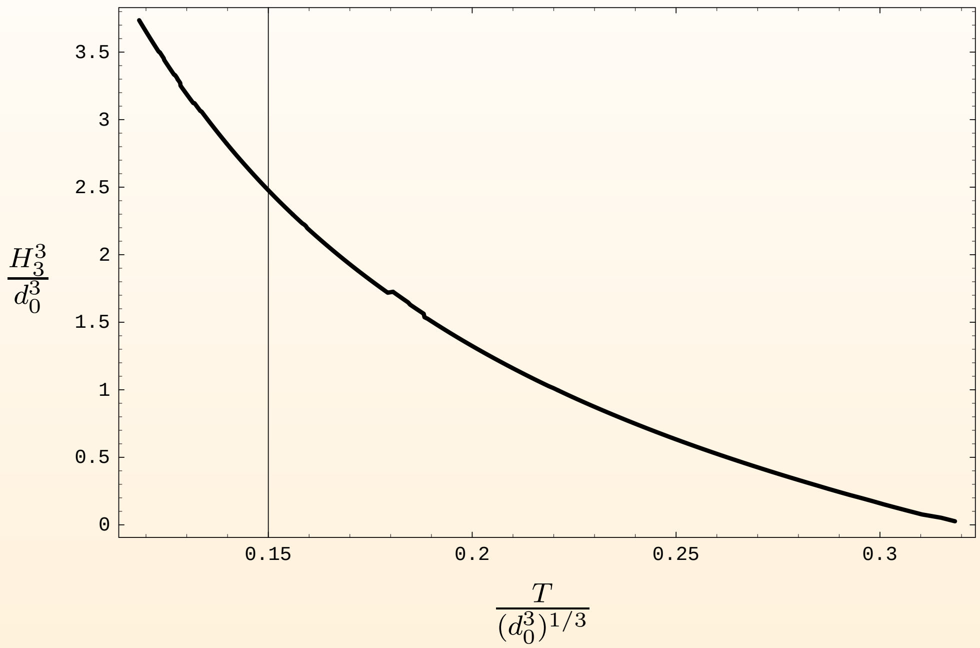


$\mathfrak{w} = \omega/(2\pi T)$

T/T_c: Black: ∞, Red: 1, Orange: 0.5, Brown: 0.28.

(Vanishing quark mass)

Meissner effect



Lower phase: magnetic field and condensate coexist

Upper phase: condensate vanishes

Non-abelian DBI action

Evaluation non-trivial in presence of both σ^0, σ^1

Two evaluation methods:

1) Expansion to fourth order

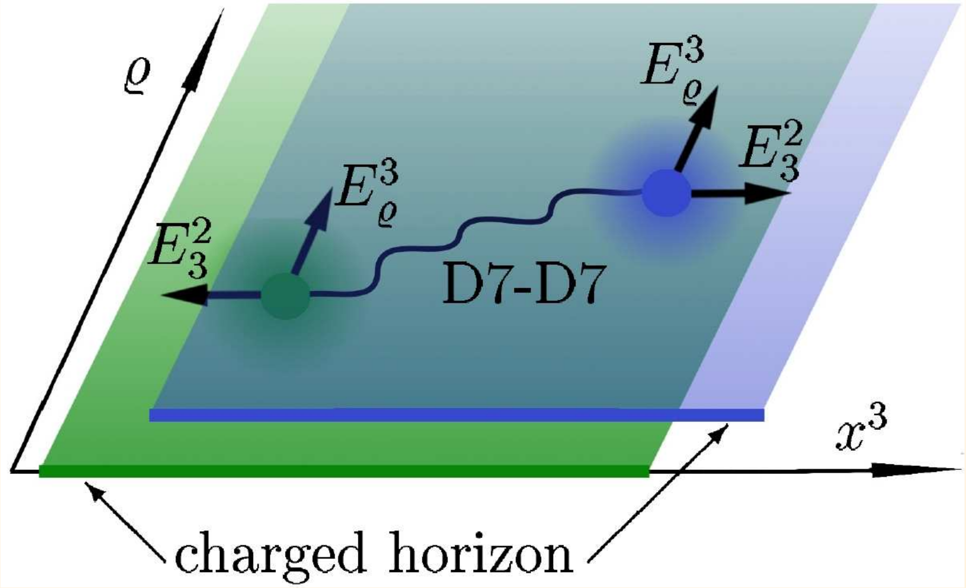
2) Simplification: Omitting commutators of Pauli matrices
Modified prescription for symmetrized trace

Allows for all-order calculation of the non-abelian DBI

Error of order $1/N_f$

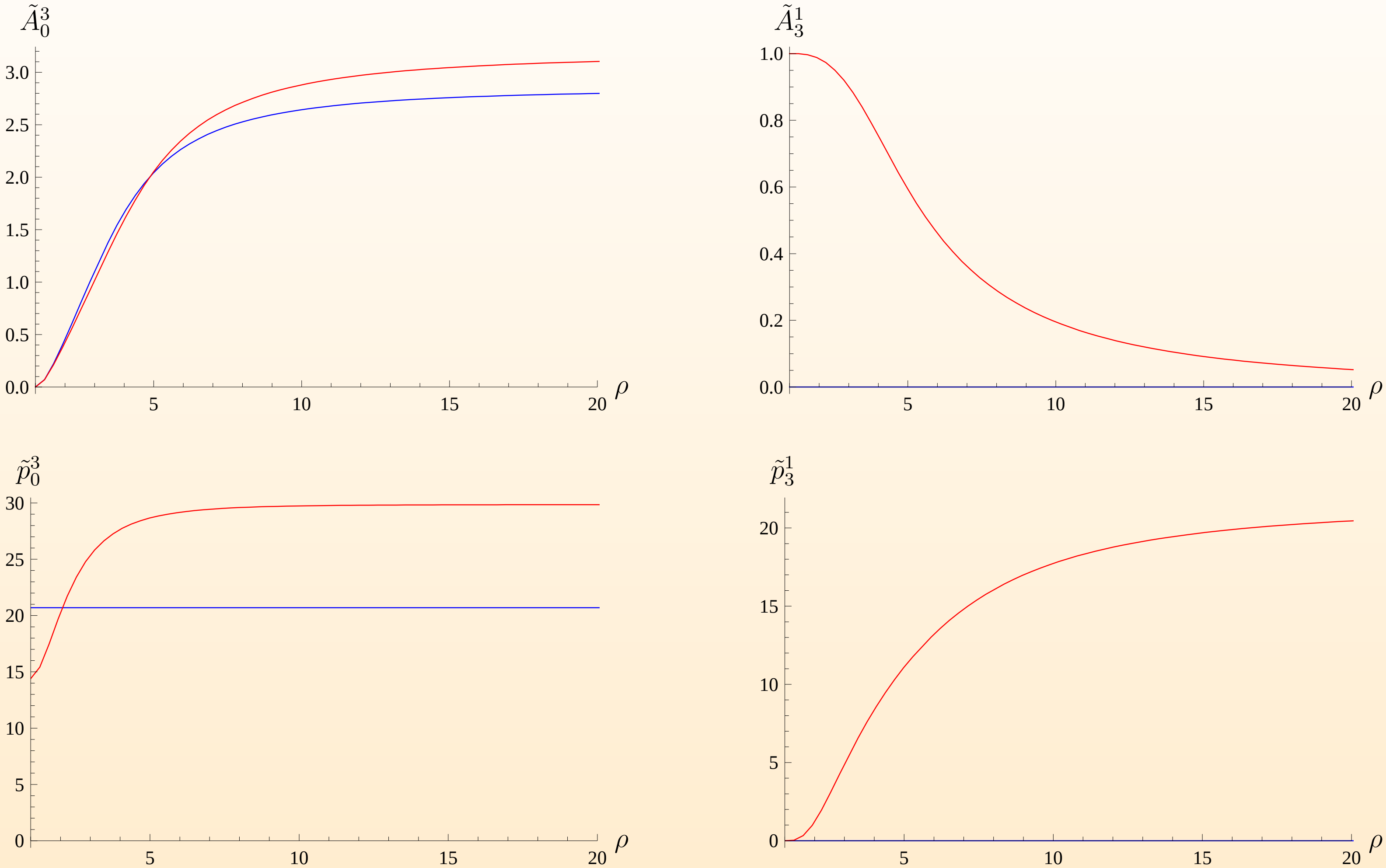
cf. Myers, Constable, Tafjord 1999

String picture



- Strings stretched between D7 branes and horizon induce a charge near the horizon
- System unstable above a critical charge density
- Horizon strings recombine to $D7 - D7$ strings
- $D7 - D7$ strings propagate into the bulk, balancing flavorelectric and gravitational forces
- $D7 - D7$ strings distribute isospin charge into the bulk $\rightarrow$ condensate

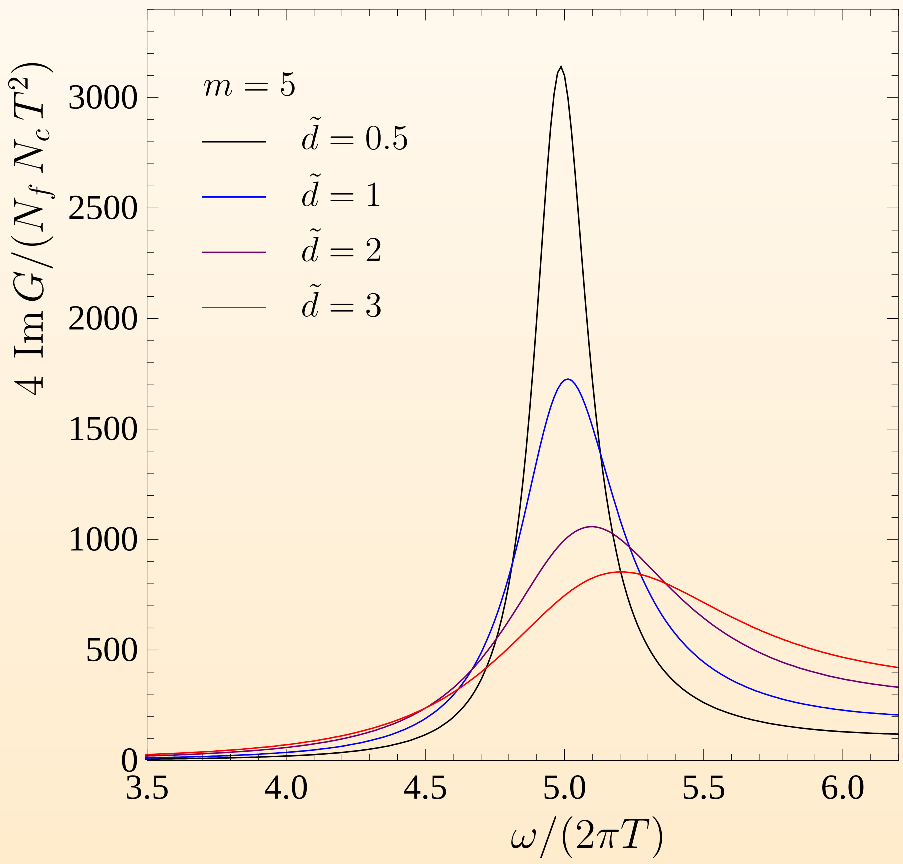
Charge distributions



Finite baryon chemical potential

Easy to introduce: VEV for time component of gauge field

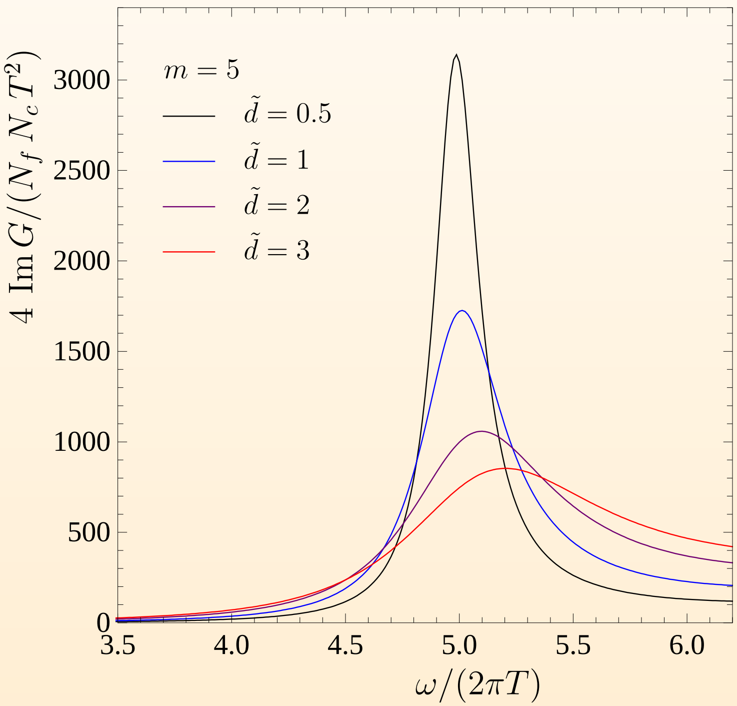
ρ vector meson spectral function in dense hadronic medium



AdS/CFT result (J.E., Kaminski, Kerner, Rust 2008)

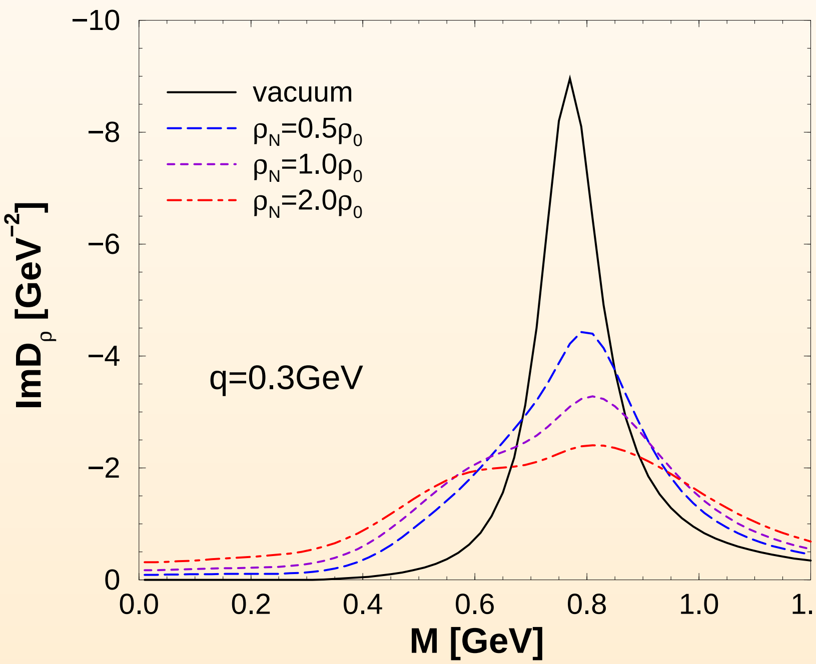
Spectral function at finite baryon density

ρ vector meson spectral function in dense hadronic medium



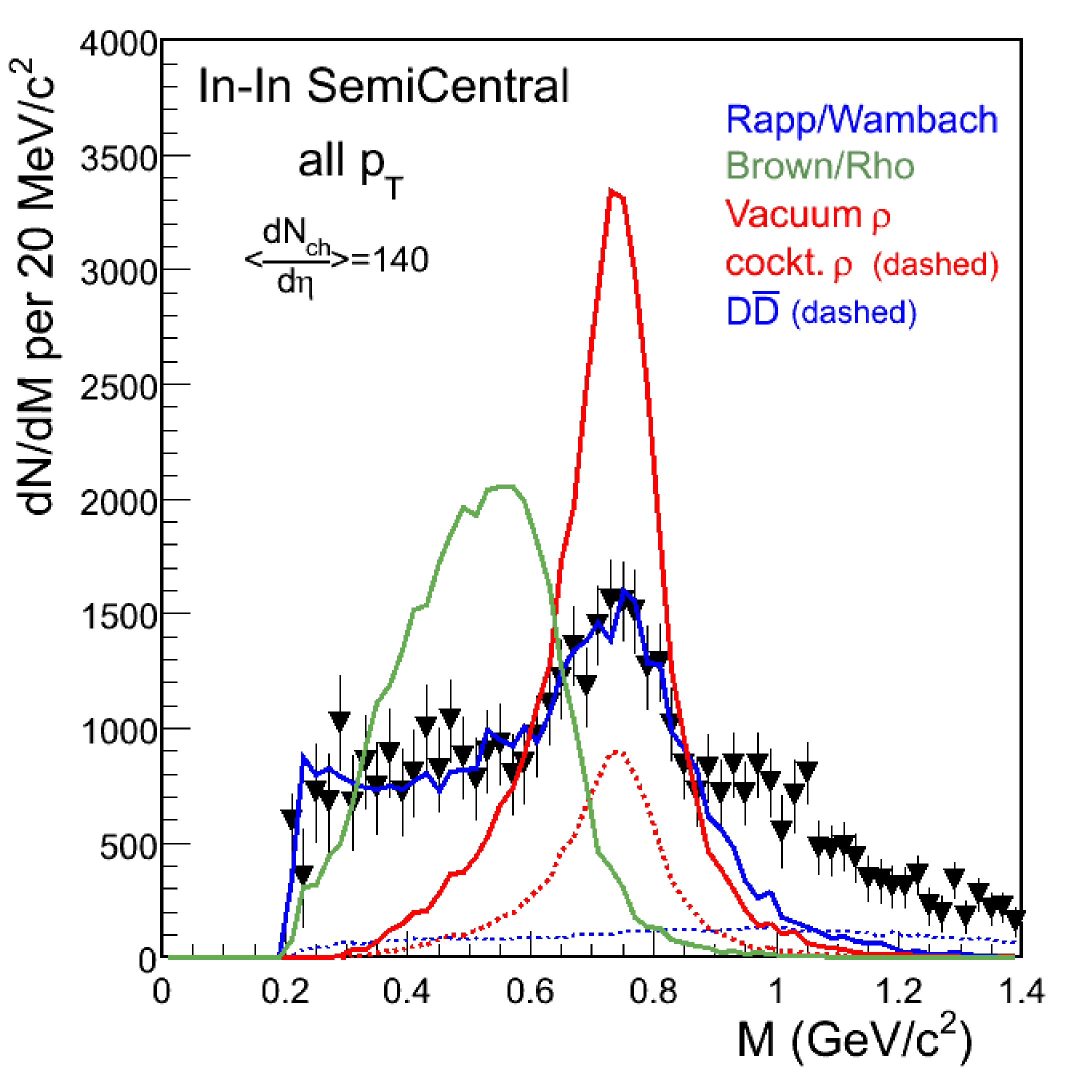
AdS/CFT result

(J.E., Kaminski, Kerner, Rust 2008)



Field theory (Rapp, Wambach 2000)

Application to NA 60 data



From NA 60 collaboration (EPJC 49 (2007) 235)

Theory: R. Rapp (2003)

(also Renk,Ruppert Dusling, Zahed)

Example III: Superconductivity

Generalized AdS/CFT models for superconductivity and superfluidity since 2007

Hartnoll, Herzog, Horowitz, Son ...

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Flavour probe brane model for superconductivity

Ammon, J.E., Kaminski, Kerner 2008

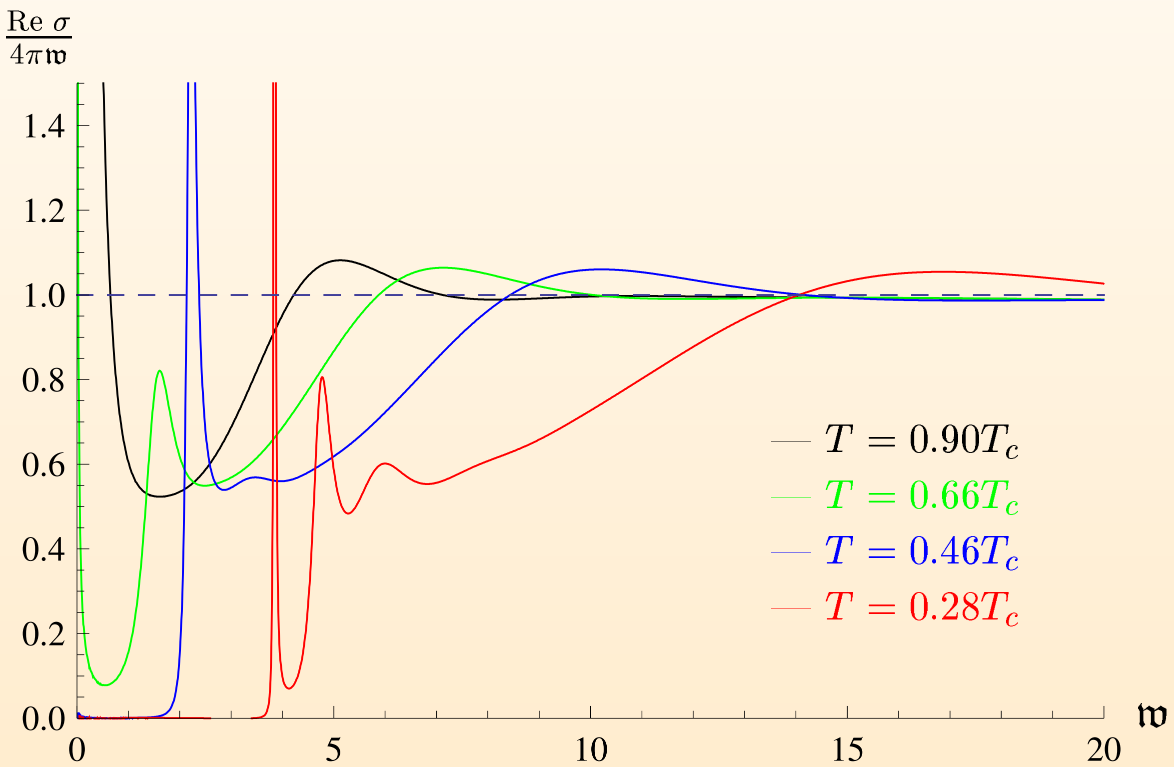
Lagrangian of dual field theory known explicitly

ρ meson condensate at finite isospin density (p-wave pairing)

Example III: Flavour Superconductivity

ρ meson condensate at finite isospin density

Frequency-dependent conductivity



$\mathfrak{w} = \omega/(2\pi T)$

Prediction: Frictionless motion of mesons through plasma

Comparison with cold atoms

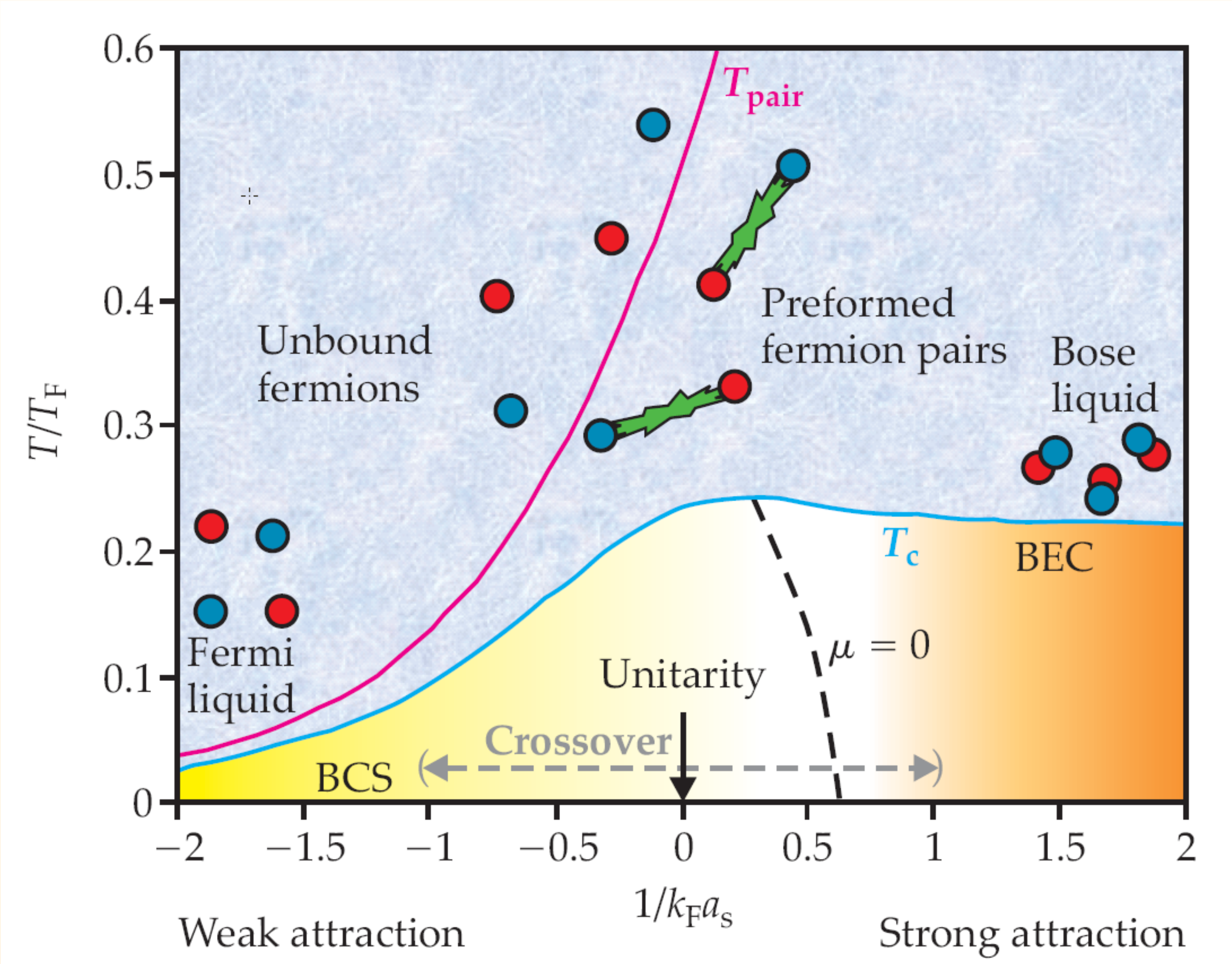


Figure: C. Sá de Melo, Physics Today Oct. 08

Conclusion

New applications of string theory methods to strongly coupled systems

In particular:

- QCD-like theories at low energies
- Quark-gluon plasma
- Condensed matter:
  Ultracold atoms, Superconductivity, Quantum Hall Effect...

A new tool for strongly coupled systems which is useful when other tools fail!