# Holographic p-wave Superfluids

Patrick Kerner
MPI für Physik, Munich

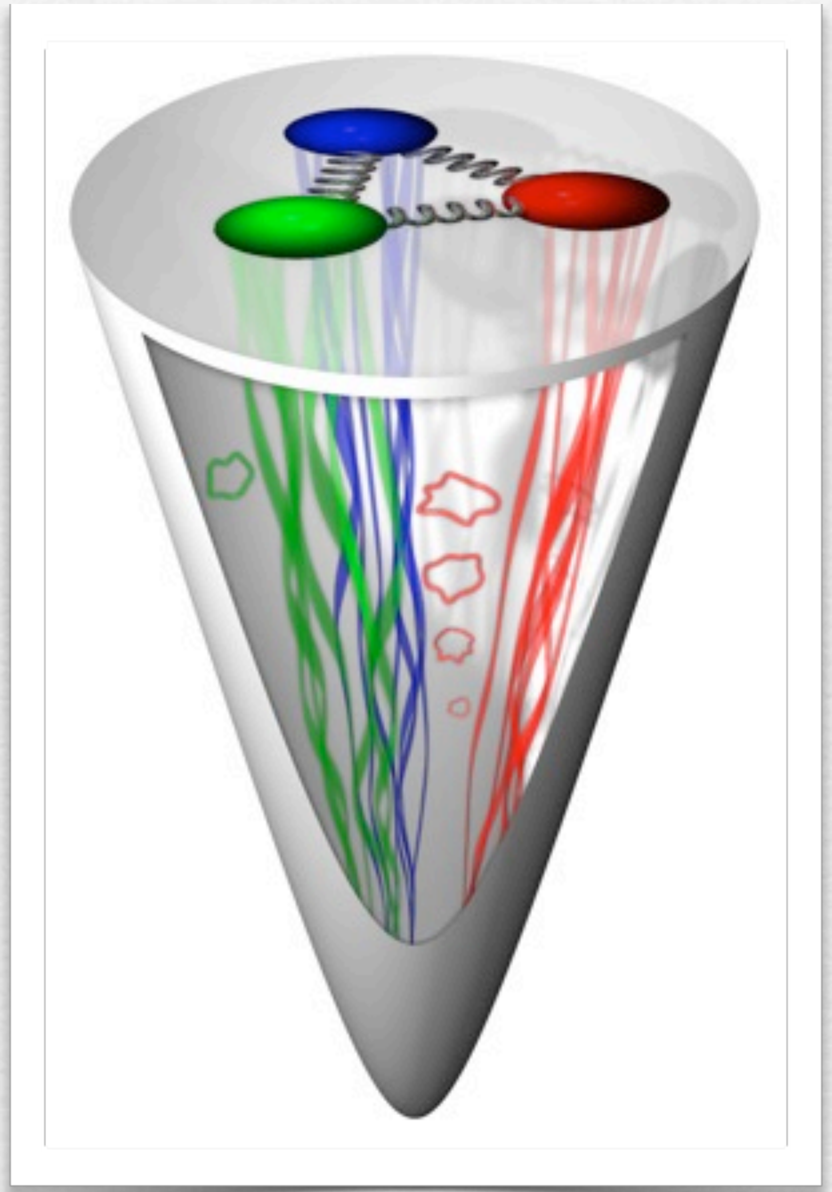
EMMI Workshop, Heidelberg

in collaboration with: M. Ammon, J. Erdmenger, V. Grass, M. Kaminski, A. O'Bannon

based on: 0810.2316, 0903.1864, 0912.3515

# What can we learn about QCD using gauge/gravity duals?

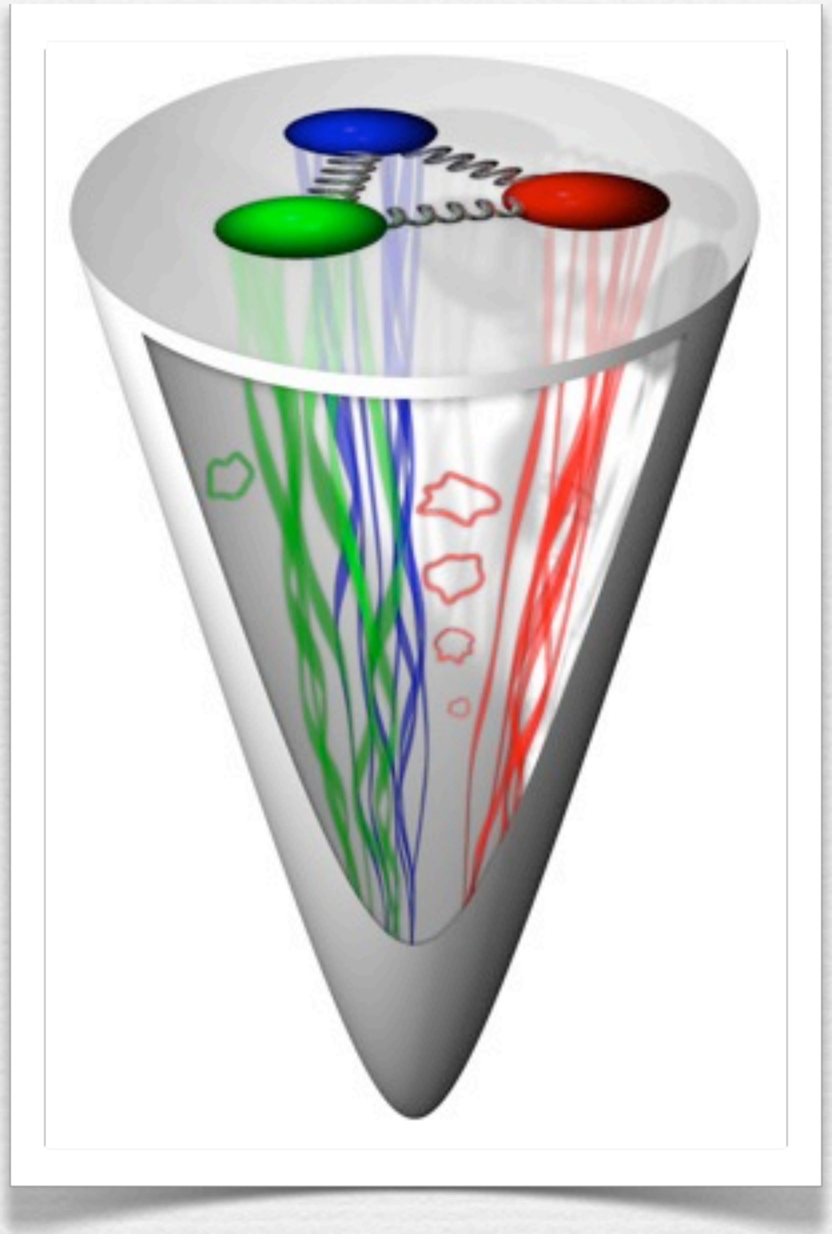
- Insides into strong coupling regime, relevant e.g. for RHIC especially hydrodynamic expansion



# What can we learn about QCD using gauge/gravity duals?

- Insides into strong coupling regime, relevant e.g. for RHIC especially hydrodynamic expansion

- Finite temperature and finite chemical potentials possible $\Rightarrow$ explore the phase diagram

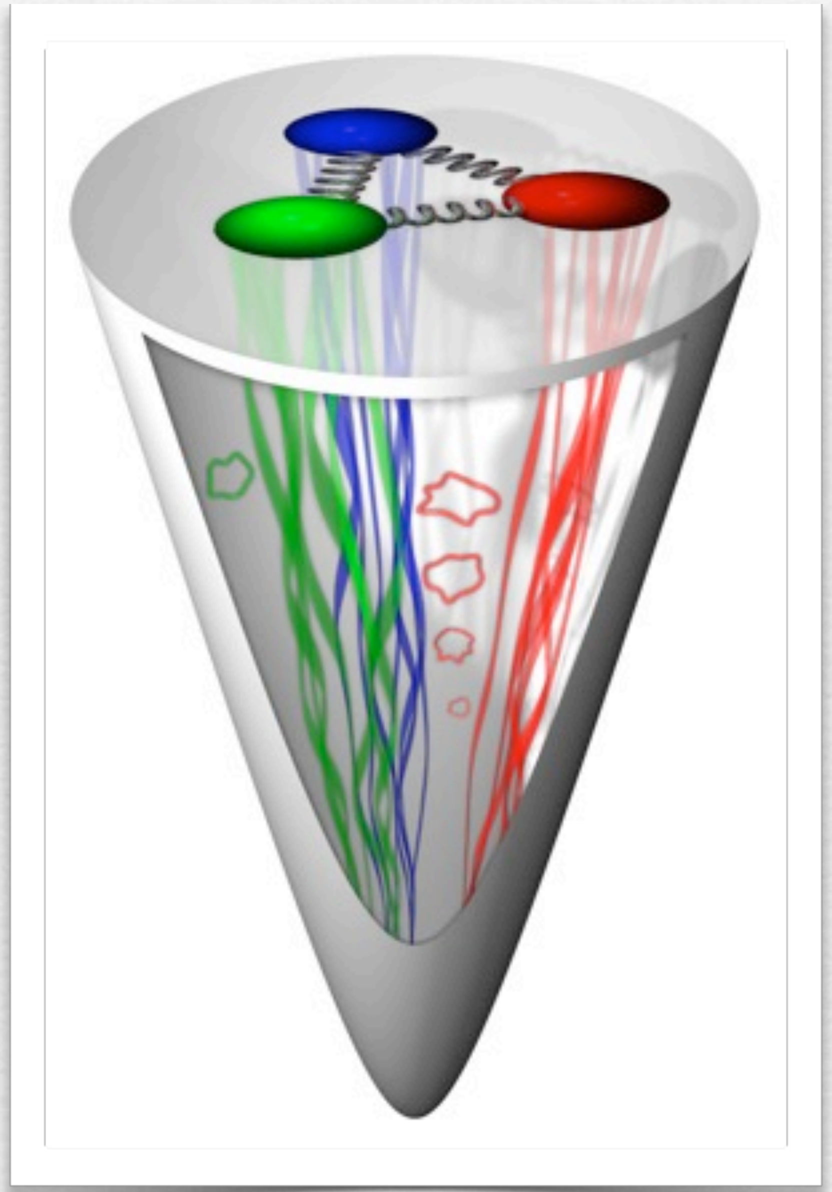


# What can we learn about QCD using gauge/gravity duals?

- Insides into strong coupling regime, relevant e.g. for RHIC especially hydrodynamic expansion

- Finite temperature and finite chemical potentials possible $\Rightarrow$ explore the phase diagram

- What kind of matter can we find?



# Meson Superfluids

- QCD has $U(N_f) = U(1) \times SU(N_f)$ global flavor symmetry. For $N_f = 2$, $SU(2)$ is isospin symmetry.

# Meson Superfluids

- QCD has $U(N_f) = U(1) \times SU(N_f)$ global flavor symmetry. For $N_f = 2, SU(2)$ is isospin symmetry.
- Introduce isospin chemical potential $\mu_I = \mu\tau^3$ which breaks $SU(2)$ down to $U(1)_3$ .

# Meson Superfluids

- QCD has $U(N_f) = U(1) \times SU(N_f)$ global flavor symmetry. For $N_f = 2, SU(2)$ is isospin symmetry.
- Introduce isospin chemical potential $\mu_I = \mu\tau^3$ which breaks $SU(2)$ down to $U(1)_3$.
- For $\mu > M_{\text{meson}}$, mesons condense due to BEC and break $U(1)_3$ spontaneously $\Rightarrow$ Meson Superfluids

# Meson Superfluids

- QCD has $U(N_f) = U(1) \times SU(N_f)$ global flavor symmetry. For $N_f = 2$, $SU(2)$ is isospin symmetry.
- Introduce isospin chemical potential $\mu_I = \mu\tau^3$ which breaks $SU(2)$ down to $U(1)_3$.
- For $\mu > M_{\text{meson}}$, mesons condense due to BEC and break $U(1)_3$ spontaneously $\Rightarrow$ Meson Superfluids
- If vector mesons condense $\Rightarrow$ rotational symmetry broken $\Rightarrow$ p-wave superfluids

QCD: Son, Stephanov; Splittorff; Sannino ...
Sakai-Sugimoto: Aharony, Peeters, Sonnenschein, Zamaklar

# p-wave Superfluids in Gauge/Gravity duals

- First: 0912.3515
  Simple gravity model with SU(2) symmetry

- Later: 0810.2316, 0903.1864
  Embedding into string theory $\Rightarrow$ superfluidity in explicit field theory:

  $\mathcal{N}=4$ $SU(N_c)$ SYM coupled to two $\mathcal{N}=2$ hypermultiplets

# Gauge/Gravity Duality

- Type IIB SUGRA on $AdS_5 \times X_5$
  is dual to
  Conformal Field Theory at large $N_c$ and large $\lambda$
  in the sense

$$Z_{\text{SUGRA}}\left[\phi(x,r)|_{r\to r_{\text{bdy}}} = \phi_0(x)\right] = \left\langle \mathrm{e}^{\int \mathrm{d}^4x \phi_0(x)\mathcal{O}(x)} \right\rangle$$

# Gauge/Gravity Duality

- Type IIB SUGRA on $AdS_5 \times X_5$
  is dual to
  Conformal Field Theory at large $N_c$ and large $\lambda$
  in the sense

$$Z_{\text{SUGRA}}\left[\phi(x,r)|_{r\to r_{\text{bdy}}} = \phi_0(x)\right] = \left\langle \mathrm{e}^{\int \mathrm{d}^4x\, \phi_0(x)\mathcal{O}(x)} \right\rangle$$

- Black holes correspond to thermal field theories

# Gauge/Gravity Duality

- Type IIB SUGRA on $AdS_5 \times X_5$
  is dual to
  Conformal Field Theory at large $N_c$ and large $\lambda$
  in the sense

$$Z_{\mathrm{SUGRA}} \left[ \phi(x,r)|_{r \to r_{\mathrm{bdy}}} = \phi_0(x) \right] = \left\langle \mathrm{e}^{\int \mathrm{d}^4 x \phi_0(x) \mathcal{O}(x)} \right\rangle$$

- Black holes correspond to thermal field theories

- Gauge fields $A_\mu^a$ are dual to global currents $J_a^\mu$
  especially vevs $A_t^0, A_t^3$ induce finite chemical
  potentials $\mu_B, \mu_I$ (source) and finite densities $\langle J_0^t \rangle, \langle J_3^t \rangle$

# Gravity model

- Einstein-Yang-Mills theory with $SU(2)$ gauge group

$$S = \int \mathrm{d}^5 x \sqrt{-g} \left[ \frac{1}{2\kappa_5^2}(R - \Lambda) - \frac{1}{4\hat{g}^2} F^a_{\mu\nu} F^{a\mu\nu} \right] \quad \alpha = \frac{\kappa_5}{\hat{g}}$$

# Gravity model

- Einstein-Yang-Mills theory with $SU(2)$ gauge group

$$S = \int \mathrm{d}^5 x \sqrt{-g} \left[ \frac{1}{2\kappa_5^2}(R - \Lambda) - \frac{1}{4\hat{g}^2} F^a_{\mu\nu} F^{a\mu\nu} \right] \quad \alpha = \frac{\kappa_5}{\hat{g}}$$

- Superfluid condensate in addition to chemical potential: ❗
  Take $\langle J^x_1 \rangle$ dual to $A^1_x$ (only the vev)

# Gravity model

- Einstein-Yang-Mills theory with $SU(2)$ gauge group

$$S = \int \mathrm{d}^5x\sqrt{-g}\left[\frac{1}{2\kappa_5^2}(R-\Lambda) - \frac{1}{4\hat{g}^2}F^a_{\mu\nu}F^{a\mu\nu}\right] \quad \alpha = \frac{\kappa_5}{\hat{g}}$$

- Superfluid condensate in addition to chemical potential: !
  Take $\langle J^x_1 \rangle$ dual to $A^1_x$ (only the vev)

- $\langle J^x_1 \rangle$ spontaneously breaks $U(1)_3$ down to $\mathbb{Z}_2$ and $SO(3)$ down to $SO(2)$ $\Rightarrow$ p-wave superfluid

# Gravity model

- Einstein-Yang-Mills theory with $SU(2)$ gauge group

$$S = \int \mathrm{d}^5 x \sqrt{-g} \left[ \frac{1}{2\kappa_5^2}(R - \Lambda) - \frac{1}{4\hat{g}^2} F^a_{\mu\nu} F^{a\mu\nu} \right] \quad \alpha = \frac{\kappa_5}{\hat{g}}$$

- Superfluid condensate in addition to chemical potential: ! Take $\langle J_1^x \rangle$ dual to $A_x^1$ (only the vev)

- $\langle J_1^x \rangle$ spontaneously breaks $U(1)_3$ down to $\mathbb{Z}_2$ and $SO(3)$ down to $SO(2)$ $\Rightarrow$ p-wave superfluid

- Probe limit $\alpha = 0$ studied by Gubser and Pufu '08

# Interpretation of $\alpha$

- Holographic calculations of Weyl anomaly: $1/\kappa_5^2 \propto c$, $c$: number of degrees of freedom
- Correlators of $SU(2)$ currents proportional to $1/\hat{g}^2$ $\Rightarrow 1/\hat{g}^2$ counts degrees of freedom charged under $SU(2)$.
- Intuitively,

$$\alpha^2 = \frac{\kappa_5^2}{\hat{g}^2} \propto \frac{\#\ \text{charged degrees of freedom}}{\#\ \text{total degrees of freedom}}$$

# Behavior at finite chemical potential

- Reissner-Nordström black hole

$$ds^2 = -N(r)\mathrm{d}t^2 + \frac{1}{N(r)}\mathrm{d}r^2 + r^2\mathrm{d}\vec{x}^2$$

$$N(r) = r^2 - \frac{2m_0}{r^2} + \frac{2\alpha^2 q^2}{3r^4}$$

$$A = \left(\mu - \frac{q}{r}\right)\tau^3\mathrm{d}t$$

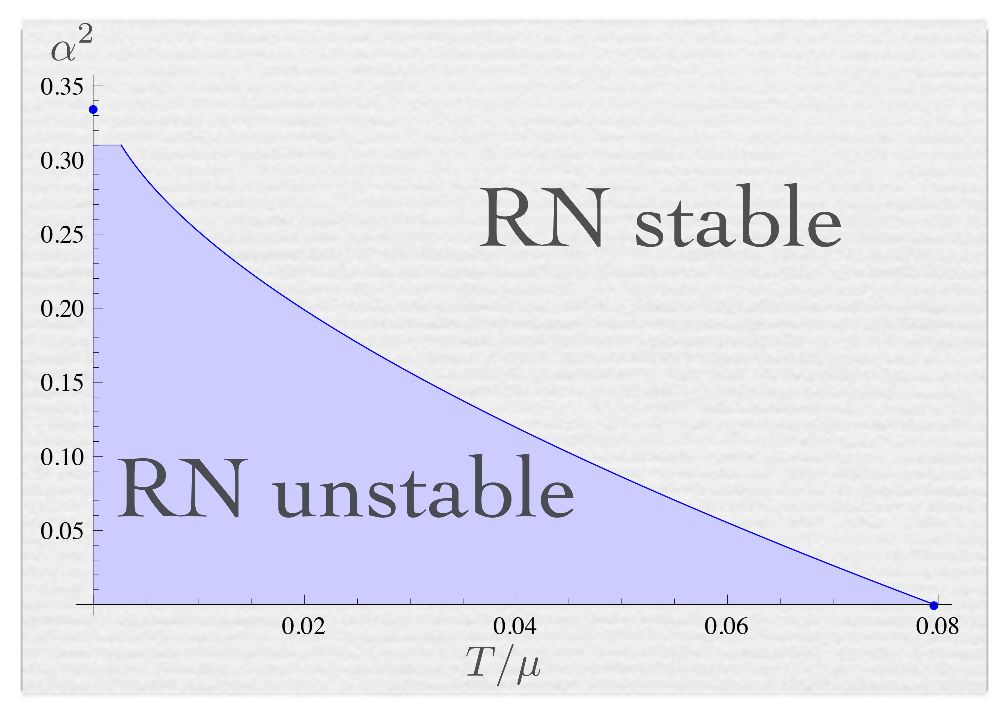
# Behavior at finite chemical potential

- Reissner-Nordström black hole

$$ds^2 = -N(r)\mathrm{d}t^2 + \frac{1}{N(r)}\mathrm{d}r^2 + r^2\mathrm{d}\vec{x}^2$$

$$N(r) = r^2 - \frac{2m_0}{r^2} + \frac{2\alpha^2 q^2}{3r^4}$$

$$A = \left(\mu - \frac{q}{r}\right)\tau^3\mathrm{d}t$$




- At low temperature the back hole unstable against fluctuations in $A_x^1$ $\Rightarrow$ Condensation

# Condensation Process

Gubser

- Sketchy action for $A_x^1$ :

$$S \sim \partial_\mu A_x^1 \partial^\mu A_x^1 + \underbrace{2g^{tt}g^{xx}\left(A_t^3\right)^2}_{=m_{\text{eff}}}\left(A_x^1\right)^2$$

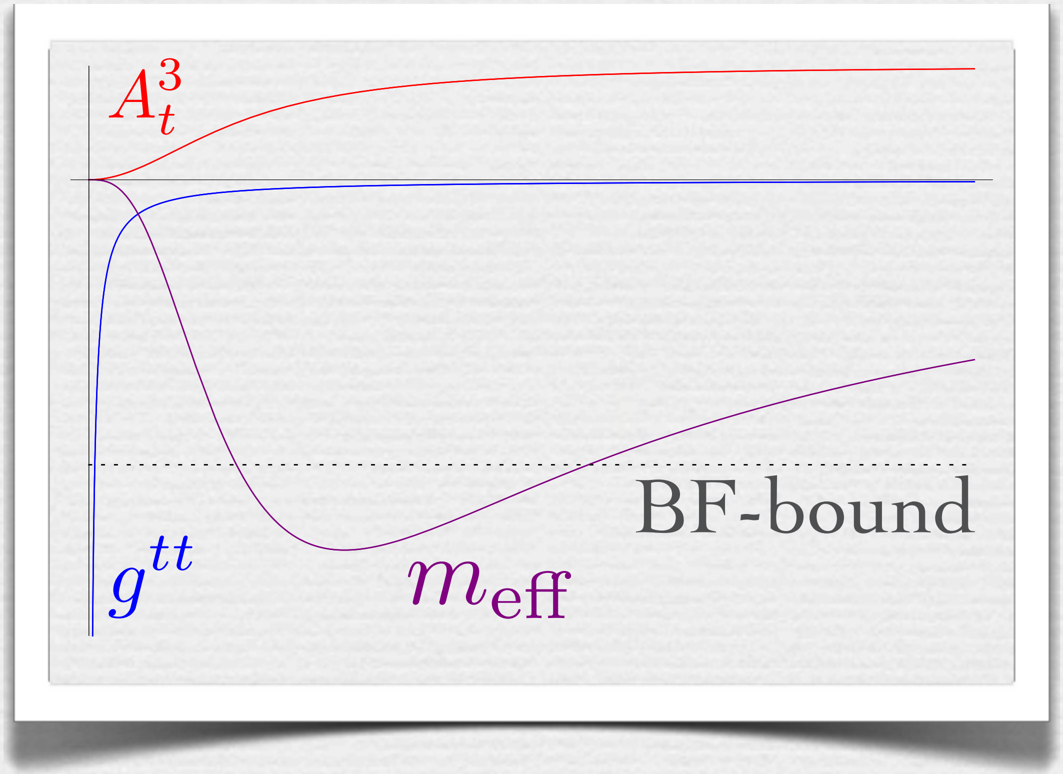
# Condensation Process

Gubser

- Sketchy action for $A_x^1$ :

$$S \sim \partial_\mu A_x^1 \partial^\mu A_x^1 + \underbrace{2g^{tt}g^{xx}\left(A_t^3\right)^2}_{=m_{\text{eff}}}\left(A_x^1\right)^2$$

- Due to black hole $g^{tt} \rightarrow -\infty$, $m_{\text{eff}}$ can be lower than BF-bound ⇒Instability ⇒Condensation⇒vector hair



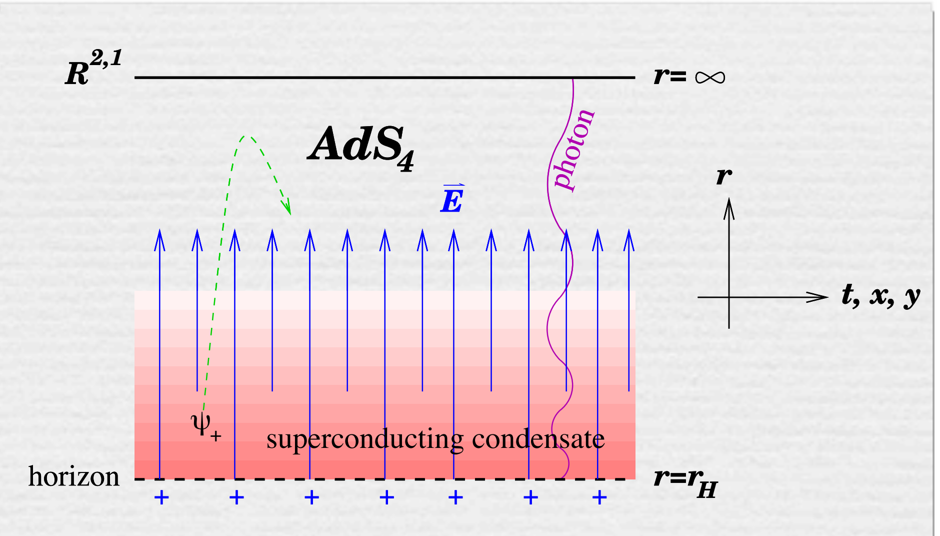
# Condensation Process

Gubser

- Sketchy action for $A_x^1$ :

$$S \sim \partial_\mu A_x^1 \partial^\mu A_x^1 + \underbrace{2g^{tt}g^{xx}\left(A_t^3\right)^2}_{=m_{\text{eff}}}\left(A_x^1\right)^2$$

- Due to black hole $g^{tt} \rightarrow -\infty$, $m_{\text{eff}}$ can be lower than BF-bound $\Rightarrow$Instability $\Rightarrow$Condensation$\Rightarrow$vector hair



Gubser, Pufu '08

- Hair is stabilized by the equilibrium of electric and gravitational force in AdS space.

# Solutions in the broken phase

- We numerically solve the Einstein-Yang-Mills equations for the ansatz

$$\mathrm{d}s^2 = -N(r)\sigma(r)^2\mathrm{d}t^2 + \frac{1}{N(r)}\mathrm{d}r^2 + r^2 f(r)^{-4}\mathrm{d}x^2 + r^2 f(r)^2\left(\mathrm{d}y^2 + \mathrm{d}z^2\right)$$

$$A = \phi(r)\tau^3\mathrm{d}t + w(r)\tau^1\mathrm{d}x \quad N(r) = r^2 - \frac{2m(r)}{r^2}$$

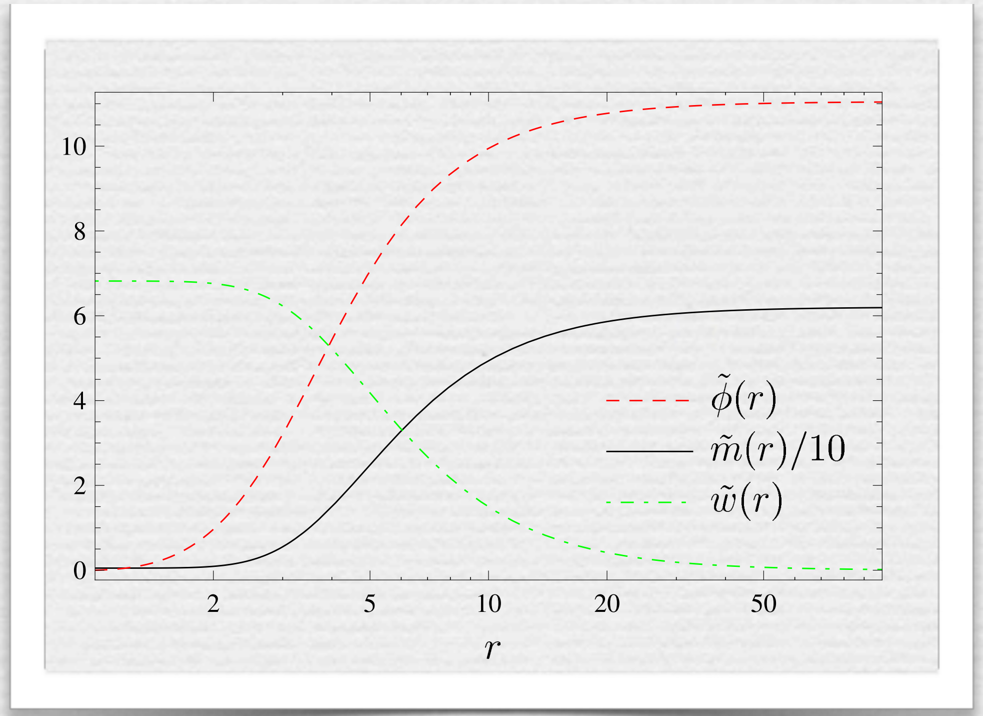
# Solutions in the broken phase

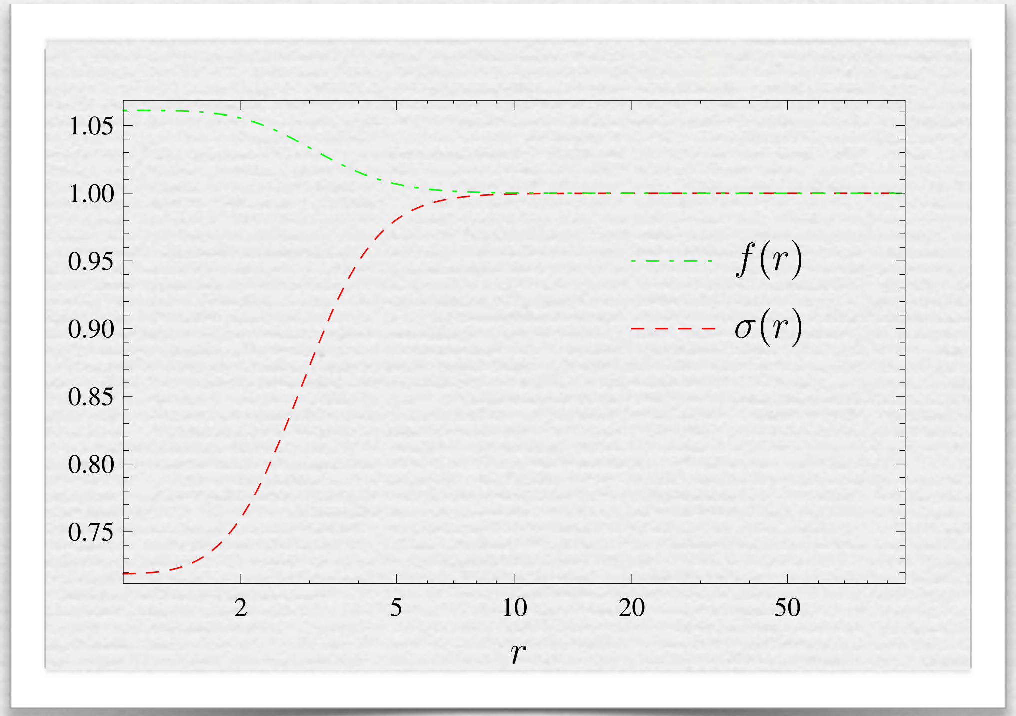
- We numerically solve the Einstein-Yang-Mills equations for the ansatz

$$ds^2 = -N(r)\sigma(r)^2 \mathrm{d}t^2 + \frac{1}{N(r)}\mathrm{d}r^2 + r^2 f(r)^{-4}\mathrm{d}x^2 + r^2 f(r)^2\left(\mathrm{d}y^2 + \mathrm{d}z^2\right)$$

$$A = \phi(r)\tau^3 \mathrm{d}t + w(r)\tau^1 \mathrm{d}x \quad N(r) = r^2 - \frac{2m(r)}{r^2}$$

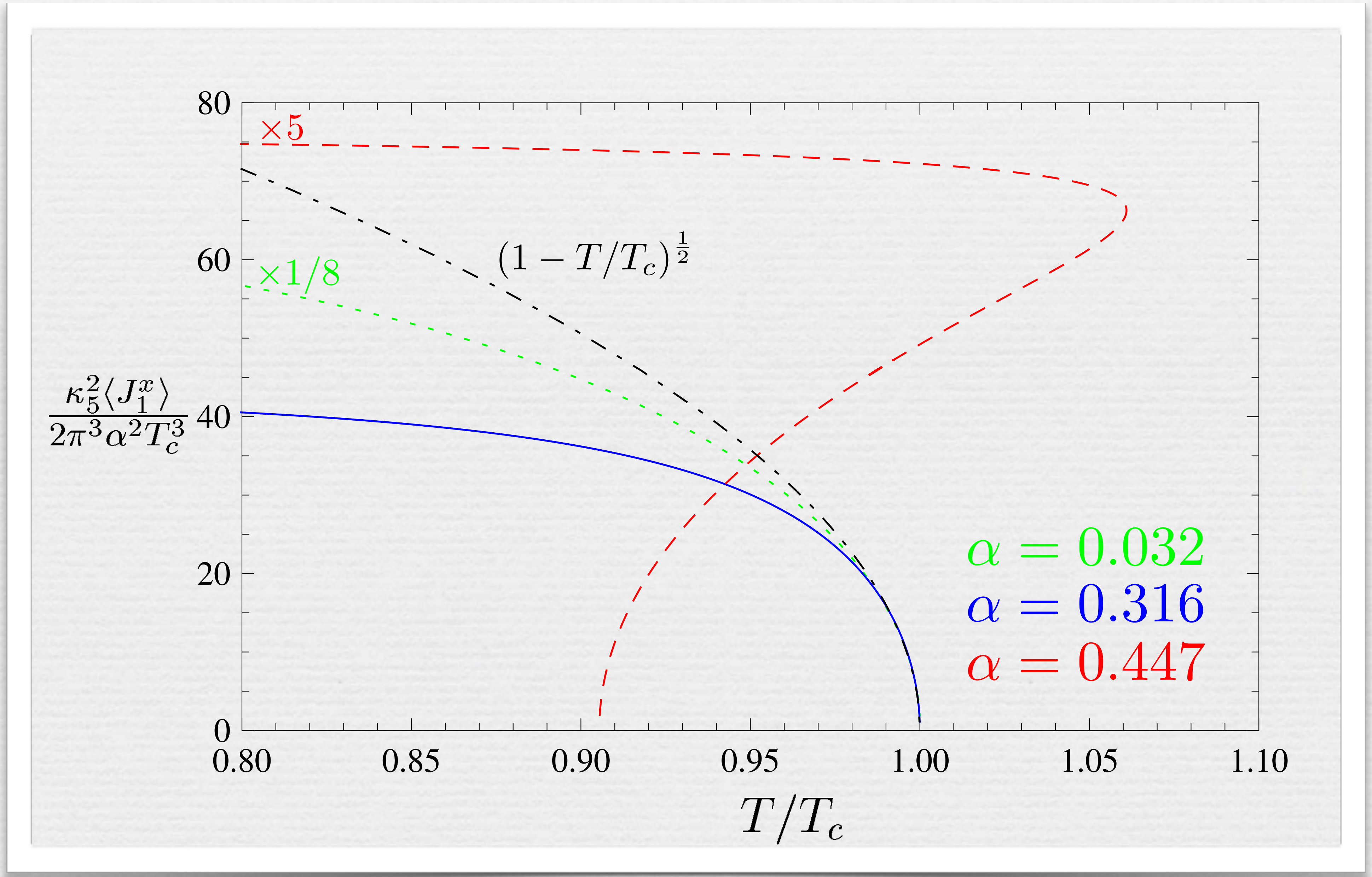
- Typical solution for $w \neq 0$ :



# Phase transition

- Order parameter $\langle J_1^x \rangle$ determined by boundary behavior of $A_x^1$.

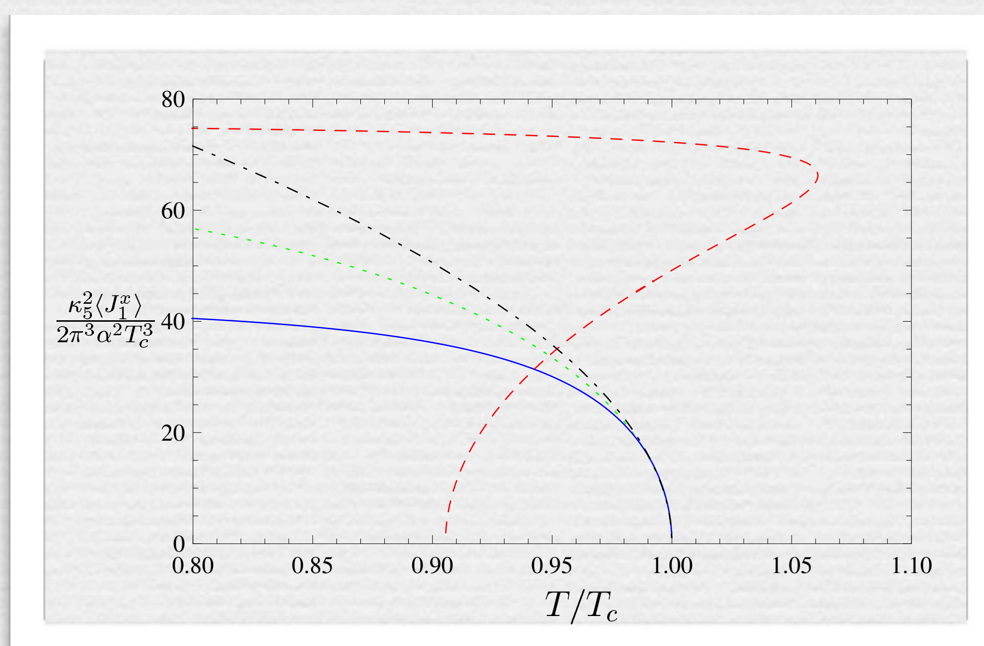


# Phase transition

- Order parameter $\langle J_1^x \rangle$ determined by boundary behavior of $A_x^1$.

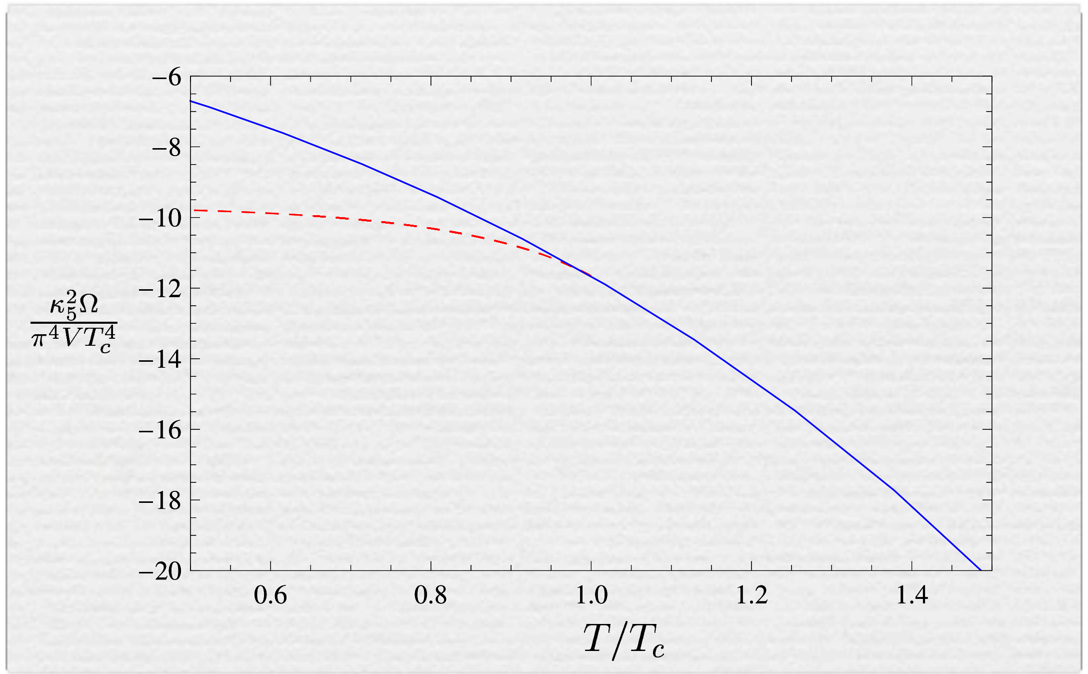
- For $\alpha < \alpha_c$ order parameter increases monotonically $\Rightarrow$ 2nd order transition (mean field)

- For large $\alpha > \alpha_c$ order parameter becomes multivalued $\Rightarrow$1st order transition

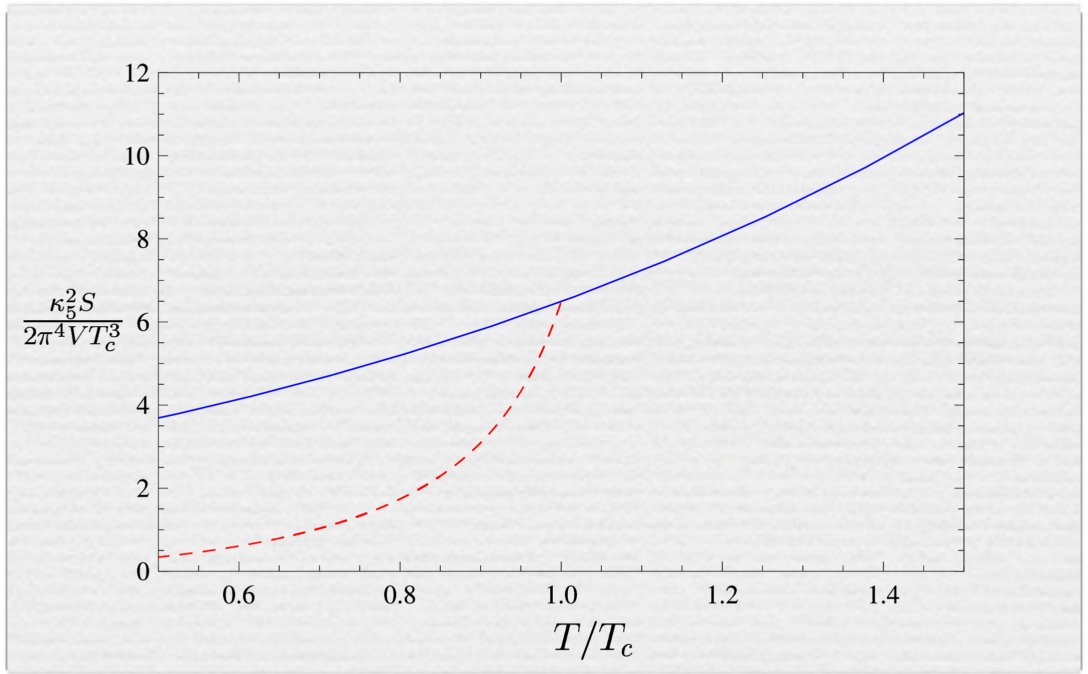



# Thermodynamics

$$\alpha < \alpha_c$$




Grand potential



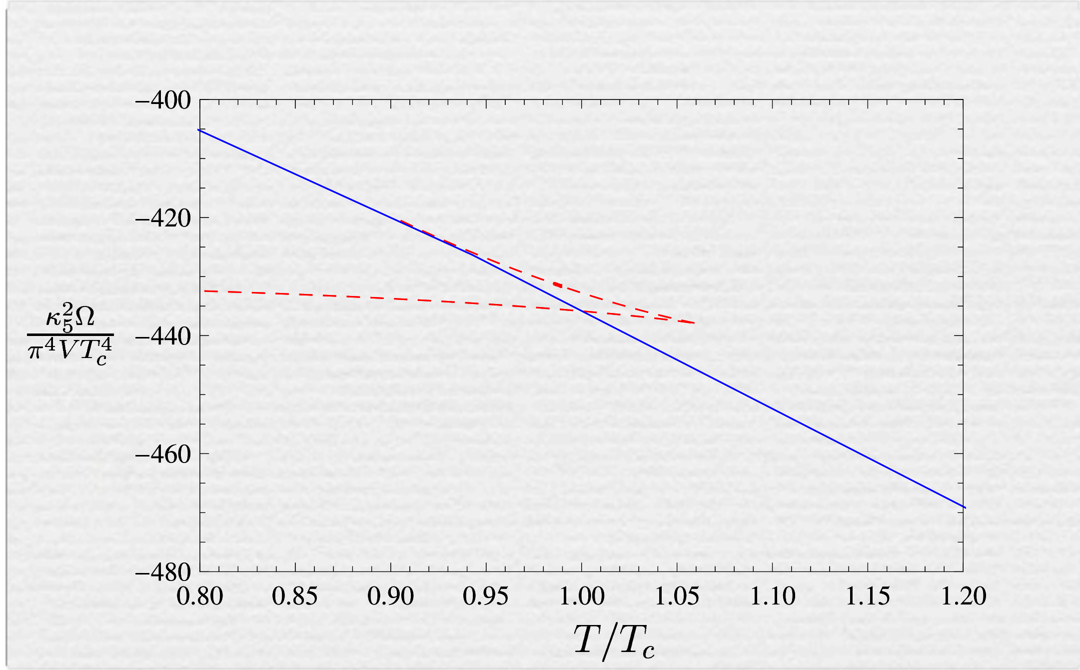

Entropy

normal phase

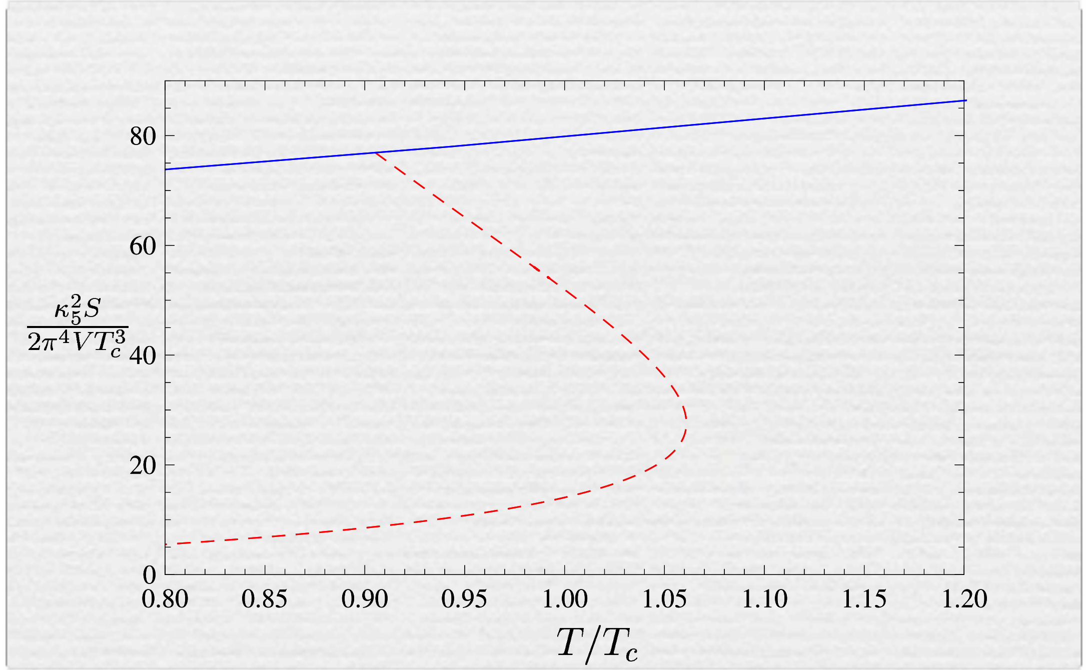
superfluid phase

# Thermodynamics

$$\alpha > \alpha_c$$




Grand potential

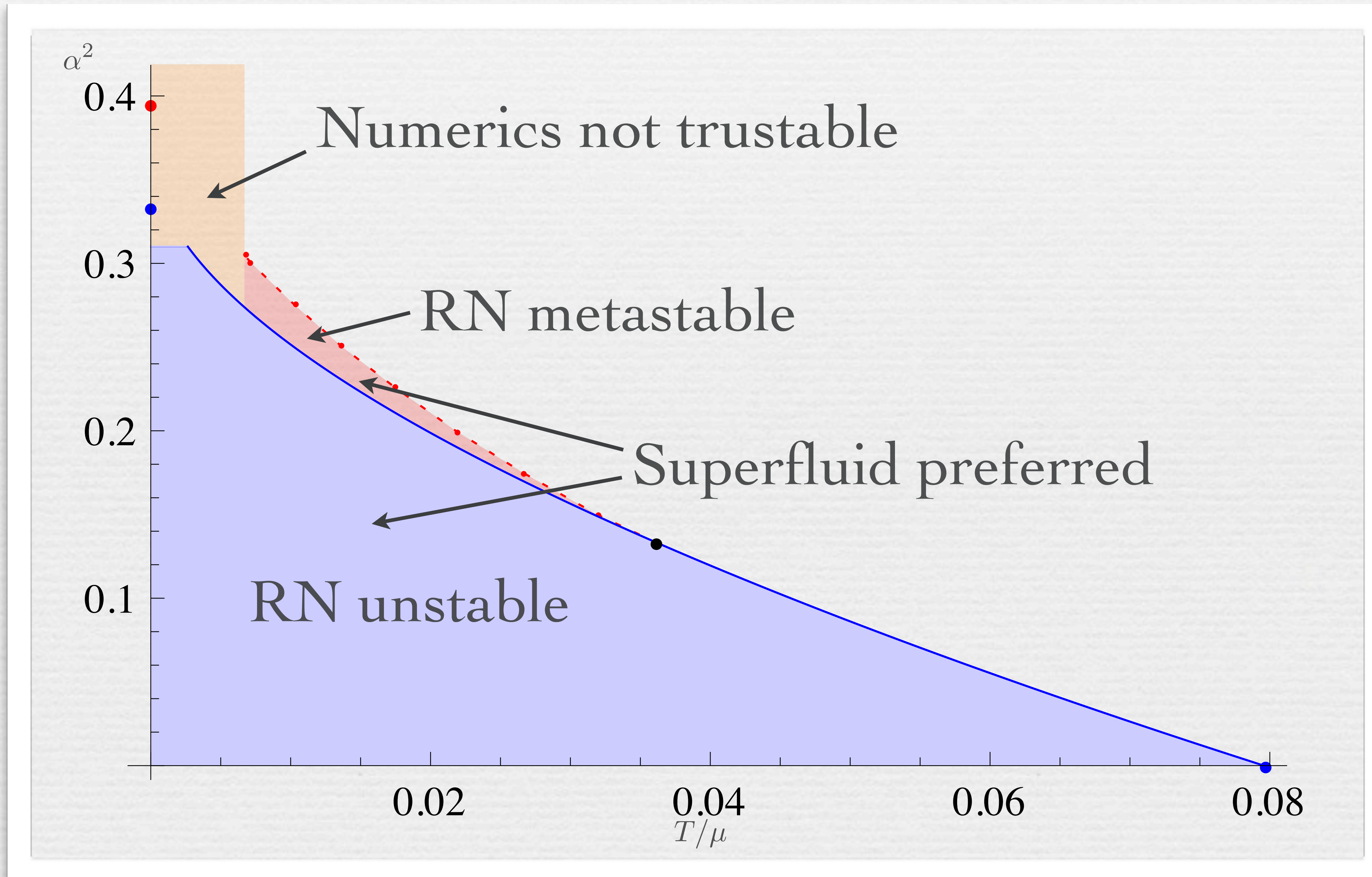



Entropy

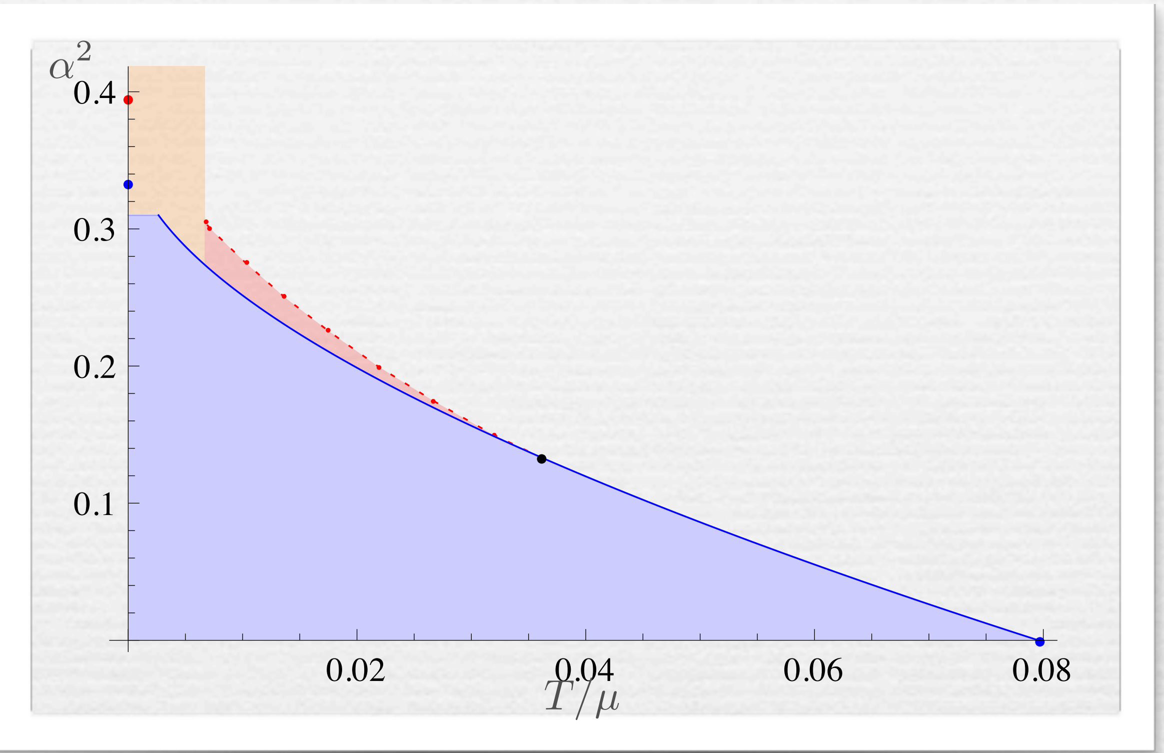
normal phase

superfluid phase

# “Phase diagram”



# “Phase diagram”




- Superfluid thermodynamically preferred in red and blue region
- Phase transition second order for $\alpha < \alpha_c$ and first order for $\alpha > \alpha_c$ with $\alpha_c = 0.365$
- For $\alpha > 0.628$ no superfluid phase available

# Embedding in String Theory Setup

- Gravity model can be embedded into D3/D7 brane setup (see Johanna's lecture)
- D3/D7 brane setup dual to $\mathcal{N}=4$ $SU(N_c)$ SYM coupled to $\mathcal{N}=2$ hypermultiplets
- The D7-branes provide a non-Abelian gauge field $\Rightarrow$ Global flavor symmetry

# Embedding in String Theory non-Abelian DBI action

- Need non-Abelian DBI action (best guess, correct up to $(\alpha')^4$ )

$$S_{\rm DBI} = T_{D7}\,{\rm Str}\int {\rm d}^8\xi \sqrt{\det Q}\sqrt{\det\left(P_{ab}\left[E_{MN} + E_{Mi}\left(Q^{-1}-\delta\right)^{ij} E_{jN}\right] + 2\pi\alpha' F_{ab}\right)}$$

$$E_{MN} = g_{MN} + B_{MN} \qquad Q^i_j = \delta^i_j + {\rm i}2\pi\alpha'\left[\Phi^i, \Phi^k\right] E_{kj} \qquad \begin{array}{l} M, N = 0, \ldots, 9 \\ a, b = 0, \ldots, 7 \\ i, j = 8, 9 \end{array}$$

# Embedding in String Theory non-Abelian DBI action

- Need n ??? belian DBI action (best guess, correct up to (

$$S_{\mathrm{DBI}} = T_{D7}\,\mathrm{Str}\int \mathrm{d}^8\xi \sqrt{\det Q}\sqrt{\det\left(P_{ab}\left[E_{MN} + E_{Mi}\left(Q^{-1}-\delta\right)^{ij} E_{jN}\right] + 2\pi\alpha' F_{ab}\right)}$$

$$E_{MN} = g_{MN} + B_{MN} \qquad Q^i_j = \delta^i_j + \mathrm{i}2\pi\alpha'\left[\Phi^i, \Phi^k\right] E_{kj}$$

$M, N = 0, \ldots, 9$
$a, b = 0, \ldots, 7$
$i, j = 8, 9$

# Embedding in String Theory non-Abelian DBI action

- Need n… belian DBI action (best guess, correct up to (

???

$$S_{\text{DBI}} = T_{D7}\,\text{Str}\int \mathrm{d}^8\xi\,\sqrt{\det Q}\,\sqrt{\det\left(P_{ab}\left[E_{MN} + E_{Mi}\left(Q^{-1}-\delta\right)^{ij}E_{jN}\right] + 2\pi\alpha' F_{ab}\right)}$$

$$E_{MN} = g_{MN} + \cancel{B_{MN}} \qquad Q^i_j = \delta^i_j + \mathrm{i}2\pi\alpha'\left[\Phi^i,\Phi^k\right]E_{kj}$$

$M, N = 0,\ldots,9$
$a, b = 0,\ldots,7$
$i, j = 8, 9$

# Embedding in String Theory non-Abelian DBI action

- Need n[...]belian DBI action (best guess, correct up to ([...]

???

$$S_{\text{DBI}} = T_{D7}\,\text{Str}\int \mathrm{d}^8\xi\,\sqrt{\det Q}\,\sqrt{\det\left(P_{ab}\left[E_{MN} + E_{Mi}\left(Q^{-1}-\delta\right)^{ij} E_{jN}\right] + 2\pi\alpha' F_{ab}\right)}$$

$$E_{MN} = g_{MN} + B_{MN} \qquad Q^i_j = \delta^i_j + \mathrm{i}2\pi\alpha'\left[\Phi^i,\Phi^k\right]E_{kj}$$

$M,N = 0,\ldots,9$
$a,b = 0,\ldots,7$
$i,j = 8,9$

- Use symmetries of our setup

Erdmenger, Kamiski, PK, Rust 0807.2663

$$\Phi^9 = 0 \quad\Rightarrow\quad \left[\Phi^i,\Phi^j\right] = 0 \quad\Rightarrow\quad Q^i_j = \delta^i_j$$

# Embedding in String Theory
# non-Abelian DBI action

- Need n… Abelian DBI action (best guess, correct up to (…

???

$$S_{\rm DBI} = T_{D7}\,\mathrm{Str}\int \mathrm{d}^8\xi \sqrt{\det Q}\sqrt{\det\left(P_{ab}\left[E_{MN}+E_{Mi}\left(Q^{-1}-\delta\right)^{ij}E_{jN}\right]+2\pi\alpha' F_{ab}\right)}$$

$$E_{MN} = g_{MN} + B_{MN} \qquad Q^i_j = \delta^i_j + \mathrm{i}2\pi\alpha'\left[\Phi^i,\Phi^k\right]E_{kj}$$

$M, N = 0, \ldots, 9$
$a, b = 0, \ldots, 7$
$i, j = 8, 9$

- Use symmetries of our setup  Erdmenger, Kamiski, PK, Rust 0807.2663

$$\Phi^9 = 0 \quad \Rightarrow \quad \left[\Phi^i,\Phi^j\right] = 0 \quad \Rightarrow \quad Q^i_j = \delta^i_j$$

# Embedding in String Theory non-Abelian DBI action

- Need n... ...belian DBI action (best guess, correct up to (

???

$$S_{\mathrm{DBI}} = T_{D7}\,\mathrm{Str}\int \mathrm{d}^8\xi\,\sqrt{\det Q}\sqrt{\det\left(P_{ab}\left[E_{MN}+E_{Mi}\left(Q^{-1}-\delta\right)^{ij}E_{jN}\right]+2\pi\alpha' F_{ab}\right)}$$

$$E_{MN}=g_{MN}+B_{MN} \qquad Q^i_j=\delta^i_j+\mathrm{i}2\pi\alpha'\left[\Phi^i,\Phi^k\right]E_{kj}$$

$M,N=0,\ldots,9$
$a,b=0,\ldots,7$
$i,j=8,9$

- Use symmetries of our setup

Erdmenger, Kamiski, PK, Rust 0807.2663

$$\Phi^9=0 \quad\Rightarrow\quad \left[\Phi^i,\Phi^j\right]=0 \quad\Rightarrow\quad Q^i_j=\delta^i_j$$

- left $$P_{ab}[g_{MN}]=g_{ab}+(2\pi\alpha')^2 g_{ij}\big(\partial_a\Phi^i\partial_b\Phi^j+\mathrm{i}\partial_a\Phi^i\left[A_b,\Phi^j\right]+\mathrm{i}\left[A_a,\Phi^i\right]\partial_b\Phi^j-\left[A_a,\Phi^i\right]\left[A_b,\Phi^j\right]\big)$$

# Embedding in String Theory non-Abelian DBI action

- Need n[???]Abelian DBI action (best guess, correct up to (

$$S_{\mathrm{DBI}} = T_{D7}\,\mathrm{Str}\int \mathrm{d}^8\xi\,\sqrt{\det Q}\sqrt{\det\left(P_{ab}\left[E_{MN} + E_{Mi}\left(Q^{-1}-\delta\right)^{ij}E_{jN}\right] + 2\pi\alpha' F_{ab}\right)}$$

$$E_{MN} = g_{MN} + B_{MN} \qquad Q^i_j = \delta^i_j + \mathrm{i}2\pi\alpha'\left[\Phi^i,\Phi^k\right]E_{kj}$$

$M, N = 0, \ldots, 9$
$a, b = 0, \ldots, 7$
$i, j = 8, 9$

- Use symmetries of our setup

Erdmenger, Kamiski, PK, Rust 0807.2663

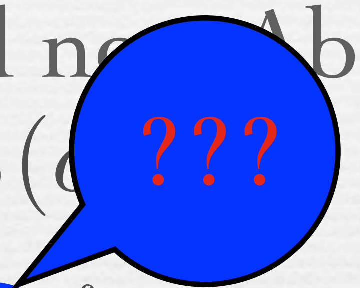
$$\Phi^9 = 0 \quad \Rightarrow \quad \left[\Phi^i,\Phi^j\right] = 0 \quad \Rightarrow \quad Q^i_j = \delta^i_j$$

- left $P_{ab}[g_{MN}] = g_{ab} + (2\pi\alpha')^2 g_{ij}\big(\partial_a\Phi^i\partial_b\Phi^j + \mathrm{i}\partial_a\Phi^i\left[A_b,\Phi^j\right] + \mathrm{i}\left[A_a,\Phi^i\right]\partial_b\Phi^j - \left[A_a,\Phi^i\right]\left[A_b,\Phi^j\right]\big)$

- choose $\Phi^8 = \Phi^{8,0}\tau^0 \quad \Rightarrow \quad \left[A_a,\Phi^i\right] = 0$

# Embedding in String Theory non-Abelian DBI action

- Need n belian DBI action (best guess, correct up to (




$$S_{\text{DBI}} = T_{D7}\,\text{Str}\int \mathrm{d}^8\xi\,\sqrt{\det Q}\sqrt{\det\left(P_{ab}\left[E_{MN} + E_{Mi}\left(Q^{-1}-\delta\right)^{ij}E_{jN}\right] + 2\pi\alpha' F_{ab}\right)}$$

$$E_{MN} = g_{MN} + B_{MN} \qquad Q^i_j = \delta^i_j + \mathrm{i}2\pi\alpha'\left[\Phi^i,\Phi^k\right]E_{kj}$$

$M, N = 0, \ldots, 9$
$a, b = 0, \ldots, 7$
$i, j = 8, 9$

- Use symmetries of our setup   Erdmenger, Kamiski, PK, Rust 0807.2663

$$\Phi^9 = 0 \quad \Rightarrow \quad \left[\Phi^i,\Phi^j\right] = 0 \quad \Rightarrow \quad Q^i_j = \delta^i_j$$

- left $P_{ab}[g_{MN}] = g_{ab} + (2\pi\alpha')^2 g_{ij}\left(\partial_a\Phi^i\partial_b\Phi^j + \mathrm{i}\partial_a\Phi^i\left[A_b,\Phi^j\right] + \mathrm{i}\left[A_a,\Phi^i\right]\partial_b\Phi^j - \left[A_a,\Phi^i\right]\left[A_b,\Phi^j\right]\right)$

- choose $\Phi^8 = \Phi^{8,0}\tau^0 \quad \Rightarrow \quad \left[A_a,\Phi^i\right] = 0$

# Embedding in String Theory non-Abelian DBI action

- Need n… belian DBI action (best guess, correct up to (…

???

$$S_{\mathrm{DBI}} = T_{D7}\,\mathrm{Str}\int \mathrm{d}^8\xi \sqrt{\det Q}\sqrt{\det\left(P_{ab}\left[E_{MN} + E_{Mi}\left(Q^{-1}-\delta\right)^{ij}E_{jN}\right] + 2\pi\alpha' F_{ab}\right)}$$

$$E_{MN} = g_{MN} + B_{MN} \qquad Q^i_j = \delta^i_j + \mathrm{i}2\pi\alpha'\left[\Phi^i,\Phi^k\right]E_{kj}$$

$M, N = 0, \ldots, 9$
$a, b = 0, \ldots, 7$
$i, j = 8, 9$

- Use symmetries of our setup

Erdmenger, Kamiski, PK, Rust 0807.2663

$$\Phi^9 = 0 \quad\Rightarrow\quad \left[\Phi^i,\Phi^j\right] = 0 \quad\Rightarrow\quad Q^i_j = \delta^i_j$$

- left $P_{ab}[g_{MN}] = g_{ab} + (2\pi\alpha')^2 g_{ij}\left(\partial_a\Phi^i\partial_b\Phi^j + \mathrm{i}\partial_a\Phi^i\left[A_b,\Phi^j\right]\right.$
$\left. + \mathrm{i}\left[A_a,\Phi^i\right]\partial_b\Phi^j - \left[A_a,\Phi^i\right]\left[A_b,\Phi^j\right]\right)$

- choose $\Phi^8 = \Phi^{8,0}\tau^0 \quad\Rightarrow\quad \left[A_a,\Phi^i\right] = 0$

Looks like Abelian action

!

# String Theory Embedding Evaluation of Str

- Str prescription only correct up to fourth order
- We use 2 different approaches:

  1) expand action to fourth order
  +: include maximal number of terms we can trust
  -: approximation breaks down in superfluid phase

  2) adapt Str prescription: set $\left(\tau^i\right)^2 = \mathbf{1}$ inside Str
  +: can handle **all** terms
  -: strictly valid only if $N_f \to \infty$ contradict probe approximation

# String Theory Embedding
# Evaluation of Str

- Str prescription only correct up to fourth order
- We use 2 different approaches:

  1) expand action to fourth order
  +: include maximal number of terms we can trust
  -: approximation breaks down in superfluid phase

  2) adapt Str prescription: set $\left(\tau^i\right)^2 = \mathbf{1}$ inside Str
  +: can handle all terms
  -: strictly valid only if $N_f \to \infty$ contradict probe approximation

give the same physics!!!

# Embedding in String Theory Comparison to Gravity Model

- D7-branes are probes in D3-brane background. Background determined by Type IIB SUGRA

$$S_{\mathrm{IIB}} \supset N_c^2 \int \mathrm{d}^5 x \sqrt{-g} R$$

# Embedding in String Theory Comparison to Gravity Model

- D7-branes are probes in D3-brane background. Background determined by Type IIB SUGRA

$$S_{\mathrm{IIB}} \supset N_c^2 \int \mathrm{d}^5 x \sqrt{-g} R$$

- DBI action determines embedding of the D7-branes and the gauge fields on these branes

$$S_{\mathrm{DBI}} = -T_{D7} \int \mathrm{d}^8 \xi \sqrt{\det\left(P[g] + 2\pi\alpha' F\right)}$$
$$\supset N_c N_f \int \mathrm{d}^5 x \sqrt{-g} F^2$$

# Embedding in String Theory Comparison to Gravity Model

- D7-branes are probes in D3-brane background. Background determined by Type IIB SUGRA

$$S_{\mathrm{IIB}} \supset N_c^2 \int \mathrm{d}^5 x \sqrt{-g} R$$

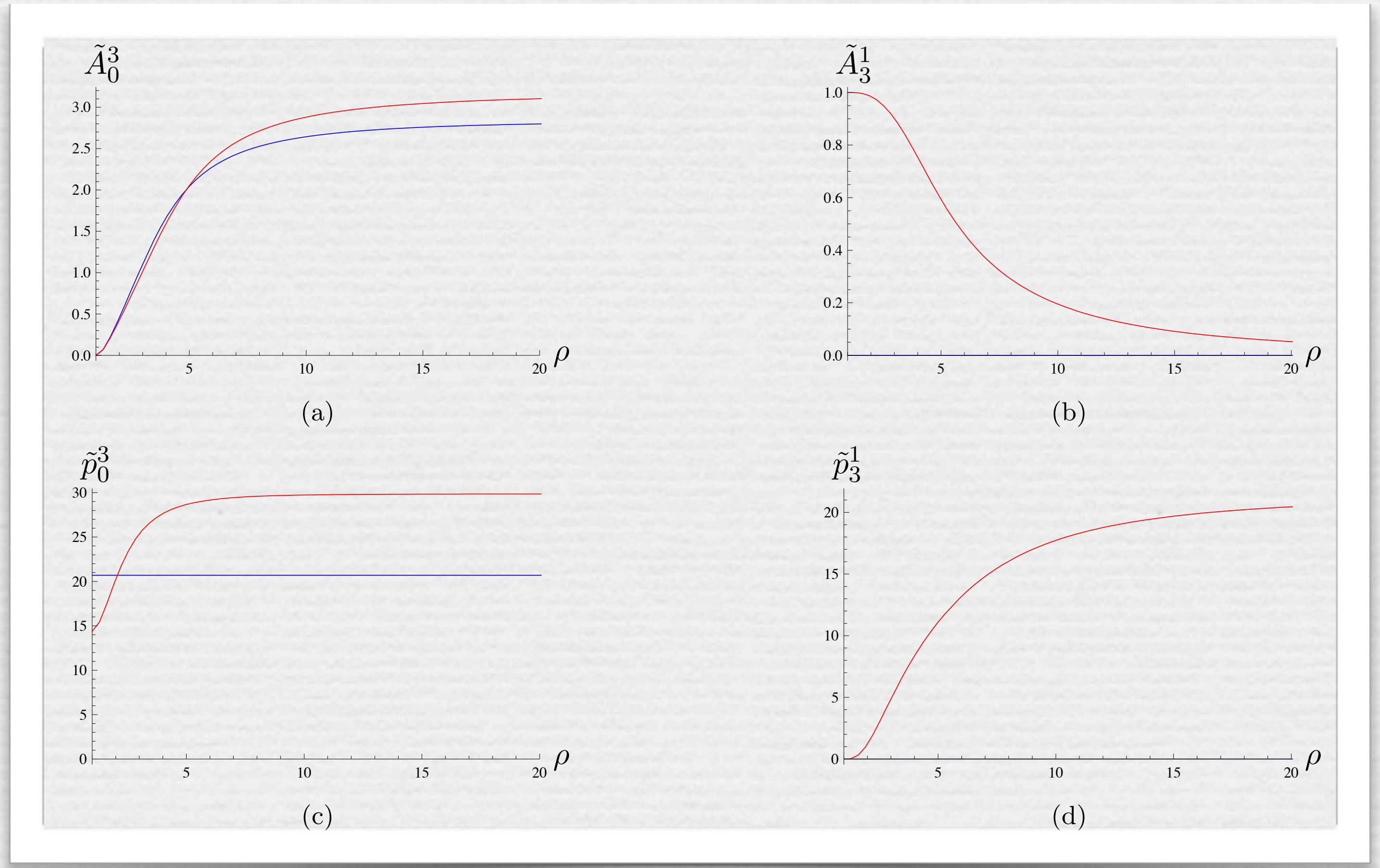
- DBI action determines embedding of the D7-branes and the gauge fields on these branes

$$S_{\mathrm{DBI}} = -T_{D7} \int \mathrm{d}^8 \xi \sqrt{\det\left(P[g] + 2\pi\alpha' F\right)}$$
$$\supset N_c N_f \int \mathrm{d}^5 x \sqrt{-g} F^2$$

- Action is similar to EYM action $\alpha \propto \sqrt{N_f / N_c} \overset{\text{here}}{=} 0$ with back-reaction dilaton have to considered, too

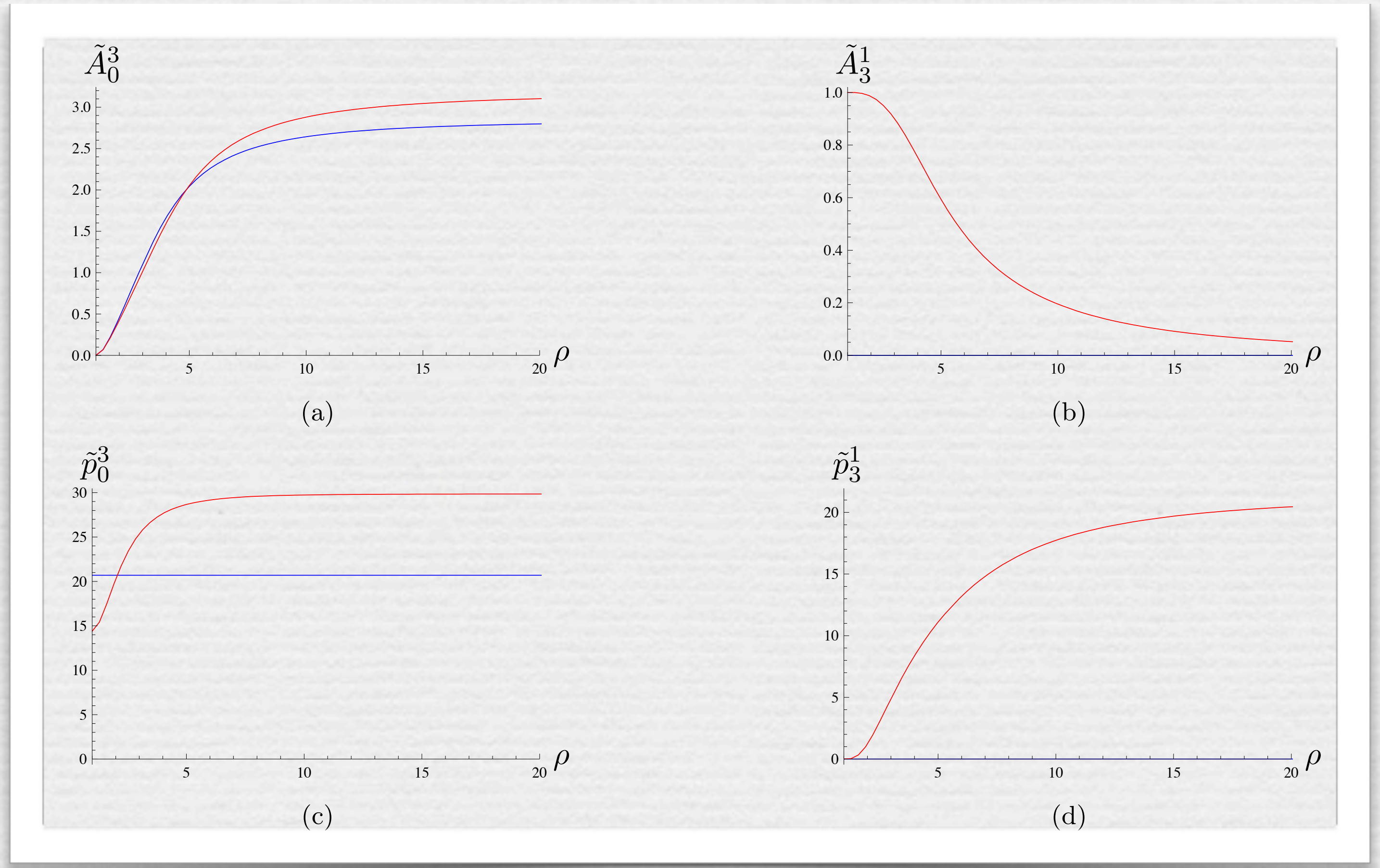
# Solutions in normal phase

- Conjugate momenta $p = \dfrac{\partial S}{\partial(\partial A)}$ approach $\langle J \rangle$ at boundary
- Normal phase:
  $p_t^3$ constant $\Rightarrow$ isospin density only produced at horizon



# Solutions in superfluid phase

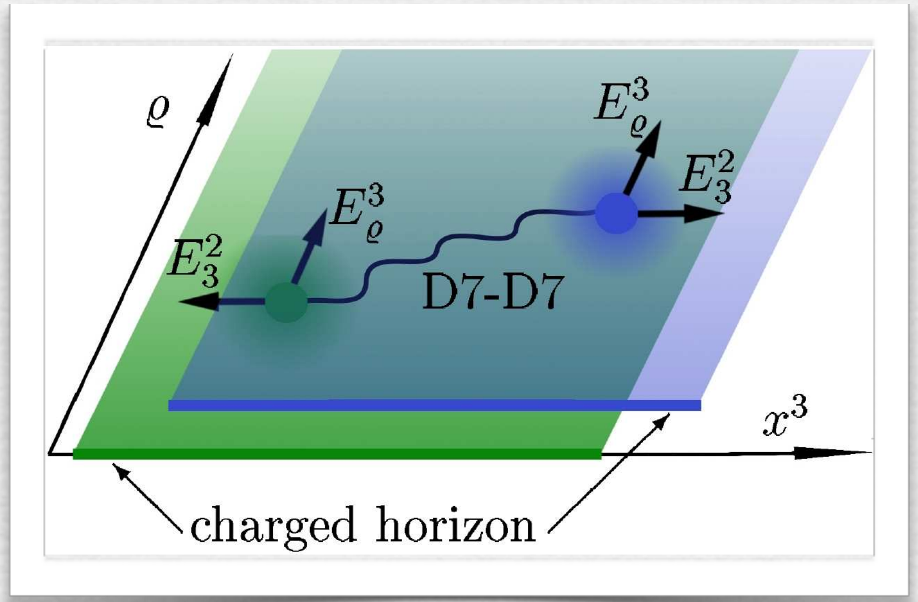
- Broken phase:
  $p_t^3$ not constant ⇒ isospin density generated in bulk
- $p_x^1$ increases towards boundary ⇒ condensate only created in bulk




normal phase
superfluid phase

# Pairing Mechanism

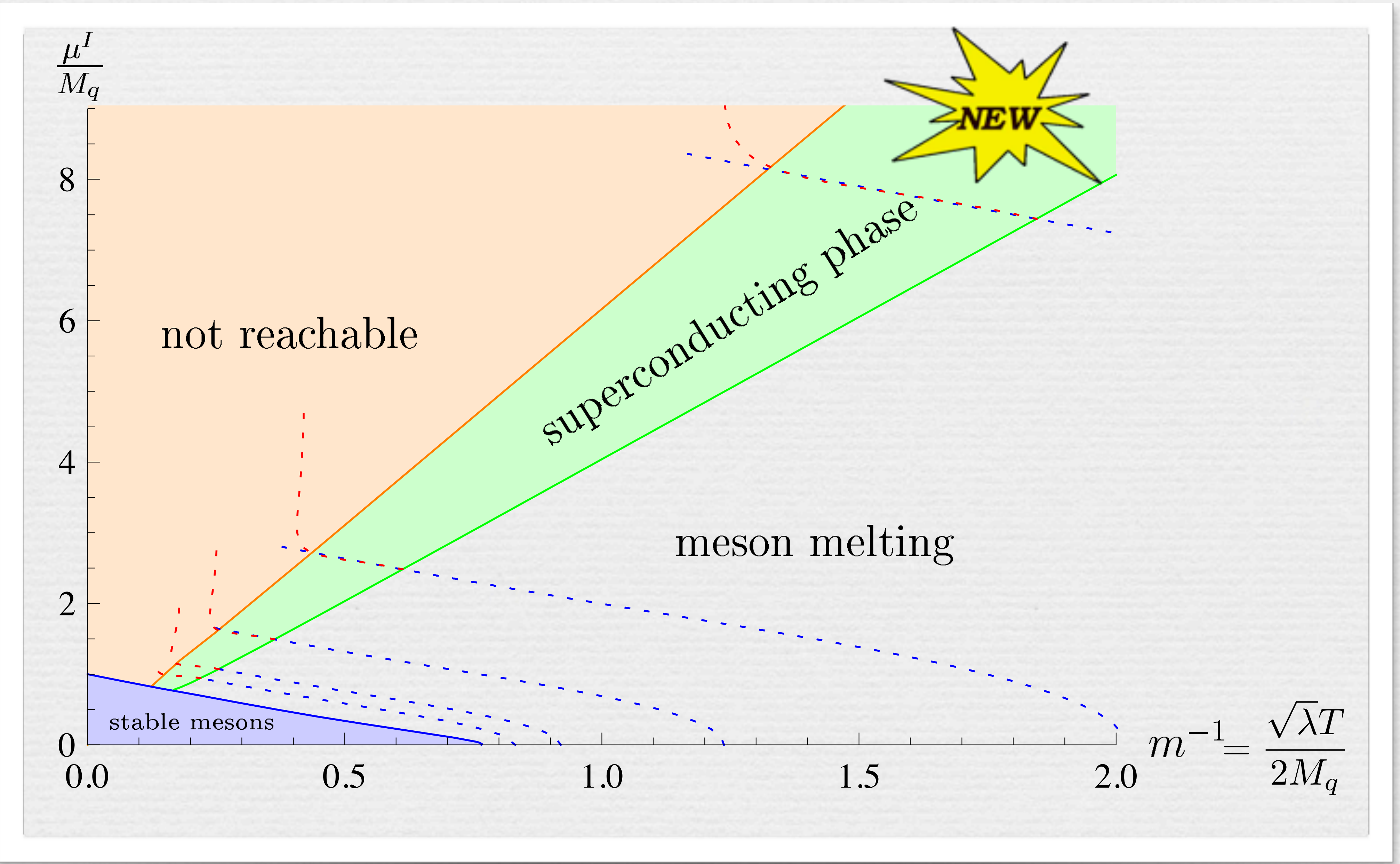
- D3-D7 strings at horizon produce finite isospin density $\Rightarrow$ branes have different charges.
- At large densities system unstable (cf. flashover)
- D3-D7 strings merge and form D7-D7 strings.
- D7-D7 strings propagate towards boundary $\Rightarrow$ energy minimized



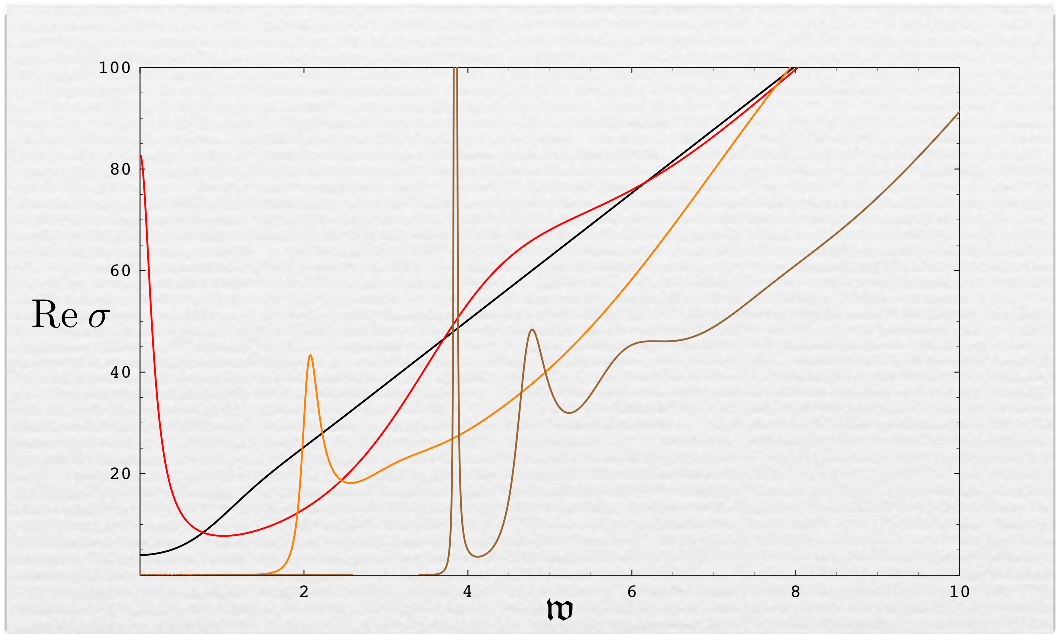
# Importance of D7-D7 strings

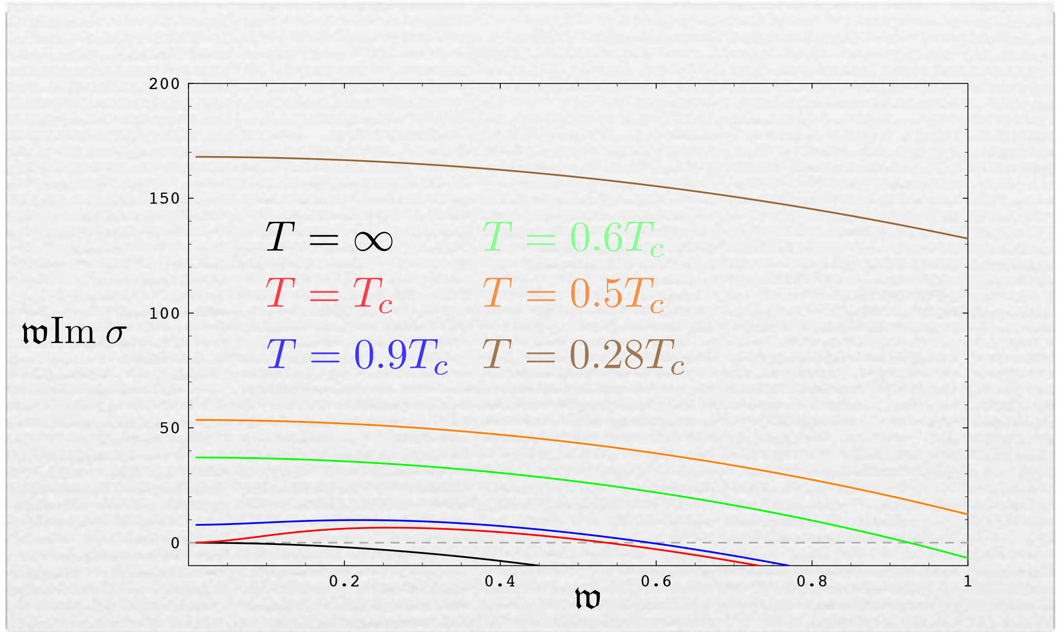
- They stabilize the system (minimal energy).
- They generate the condensate in the bulk.
  $\Rightarrow$ They break the $U(1)_3$ symmetry.
- They are the duals of the Cooper pairs.

# Phase diagram
## for $\mathcal{N} = 4$ $SU(N_c)$ SYM
## coupled to $\mathcal{N} = 2$ hypermultiplets



# Conductivity

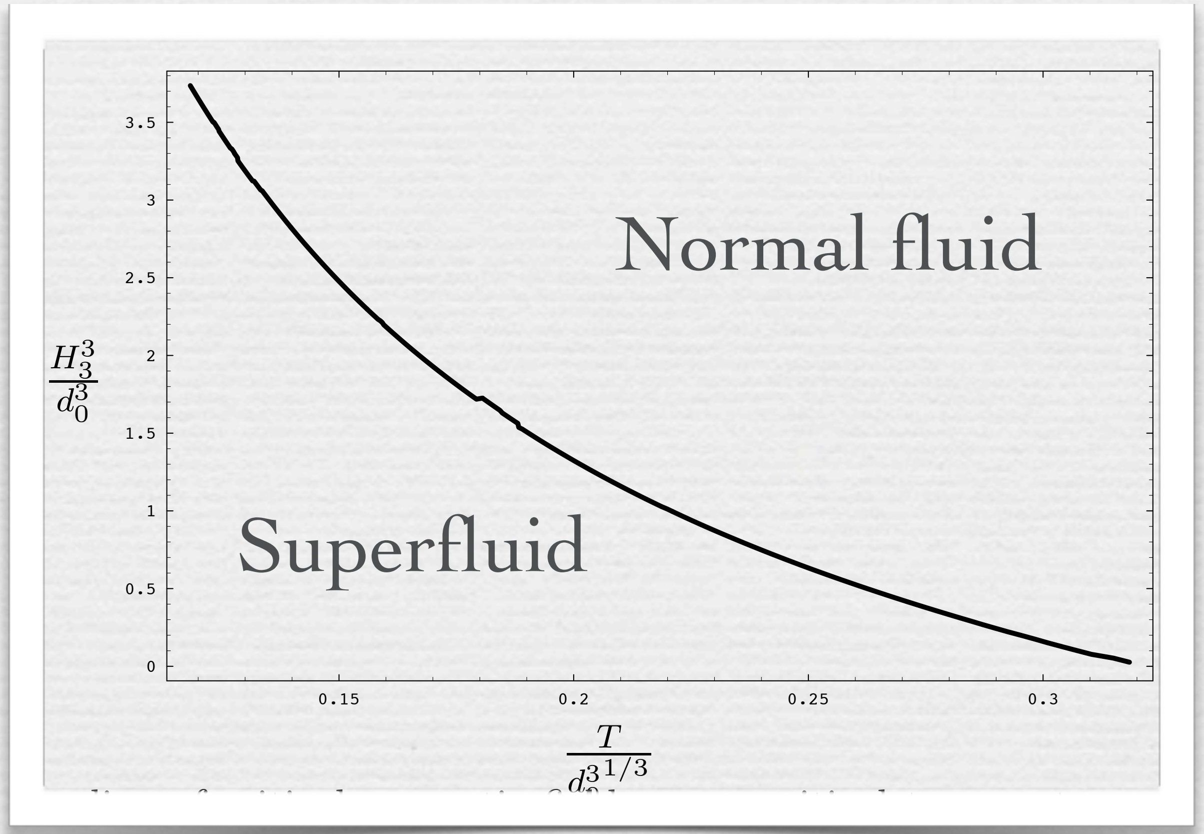



- Real part of conductivity develops a gap.
- Additional resonances (mesons) can appear in the gap.
- Imaginary part behaves like $n_s/\mathfrak{w} \Rightarrow \mathrm{Re}\,\sigma \sim \pi n_s \delta(\mathfrak{w})$
  Superfluid density behaves as in GL $n_s \propto (1 - T/T_c)$

# Remnant of Meissner-Ochsenfeld effect

- Magnetic fields can destroy the superfluid state.
- Only for small magnetic field, magnetic field and condensate can coexist.
- If symmetry was gauged, superconducting current would generate magnetic field $\Rightarrow$ magnetic field would be expelled.



# Summary

- Realization of holographic p-wave superfluids by black holes with vector hair

- Order of transition depends on number of charged degrees of freedom

- Embedding into string theory:
  $\Rightarrow$ Superfluidity in explicit field theory
  $\Rightarrow$ Pairing Mechanism in string terms

# Outlook

- May shed light on:

  meson superfluids and non-conventual superconductors due to QCP

- Can study additional properties:

  hydrodynamic transport, response to non-zero superfluid velocities, ...

# Outlook

- May shed light on:

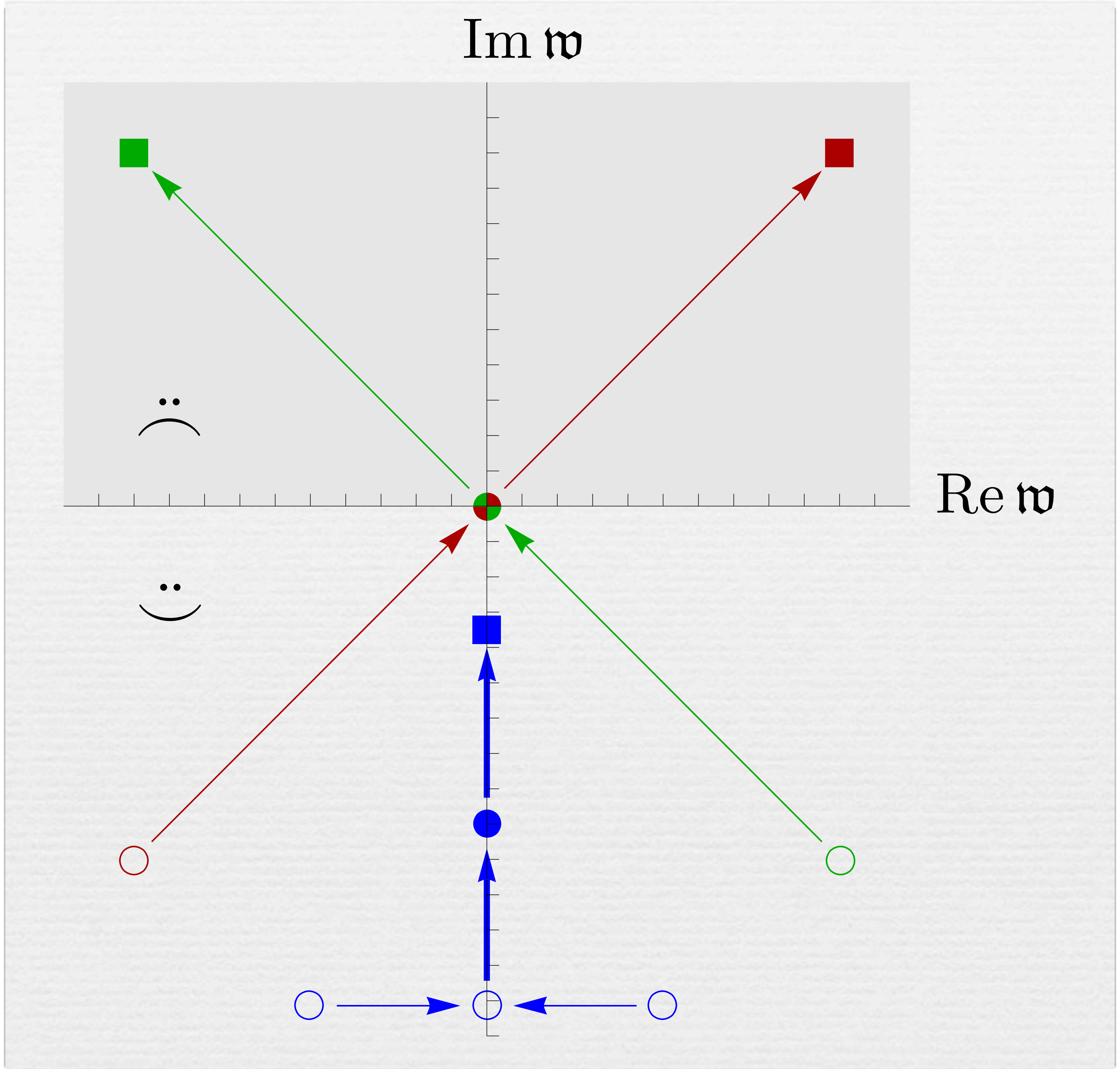
  meson superfluids and non-conventual superconductors due to QCP

- Can study additional properties:

  hydrodynamic transport, response to non-zero superfluid velocities, ...

Thank you!

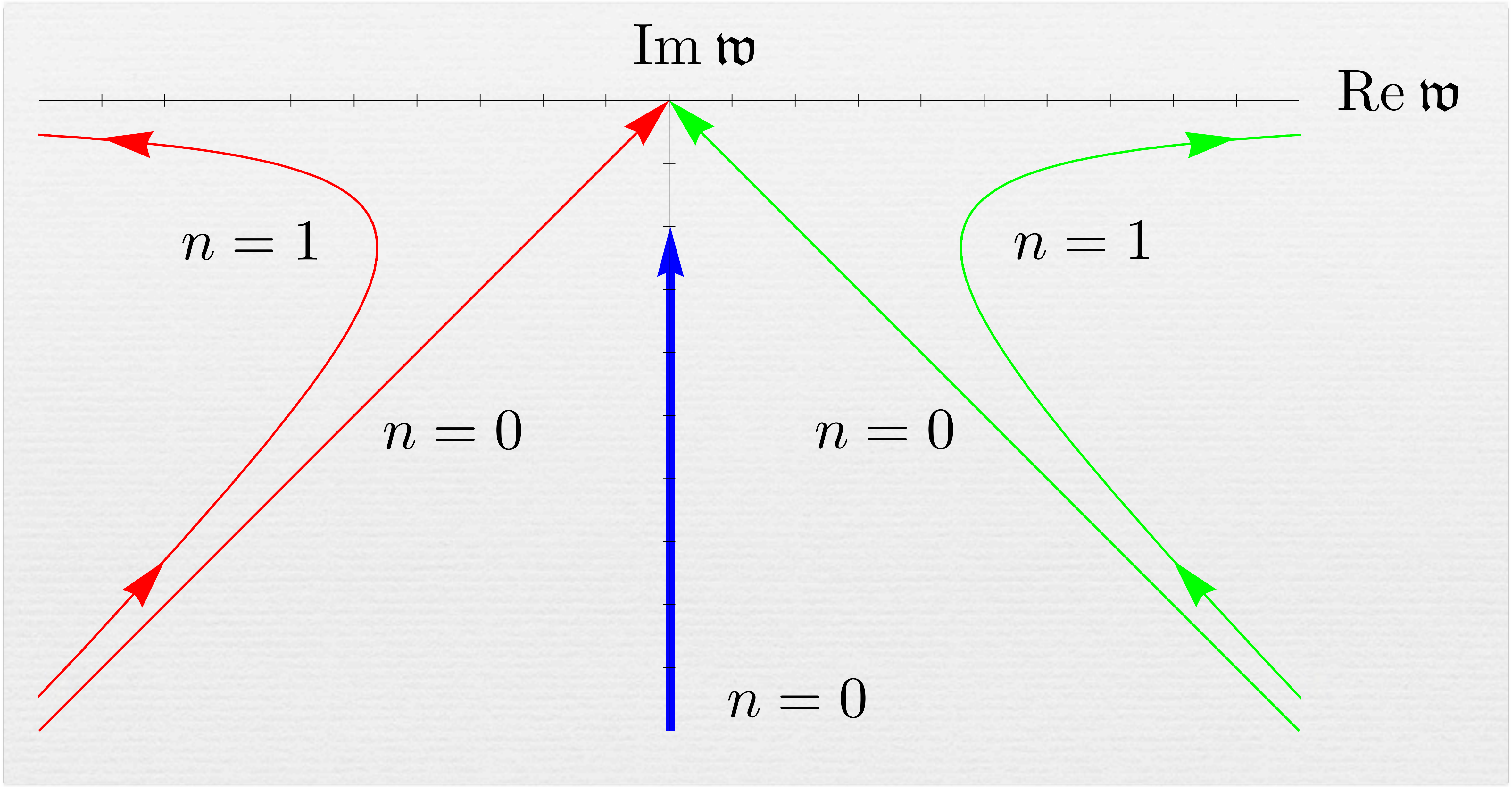
# Backup slides




Fourier decomposition:

$$e^{-i\mathfrak{w}} = e^{+\mathrm{Im}\,\mathfrak{w}} e^{-i\mathrm{Re}\,\mathfrak{w}}$$

# Quasinormal modes in normal phase




# Quasinormal modes in superfluid phase