Hadrons as Holograms

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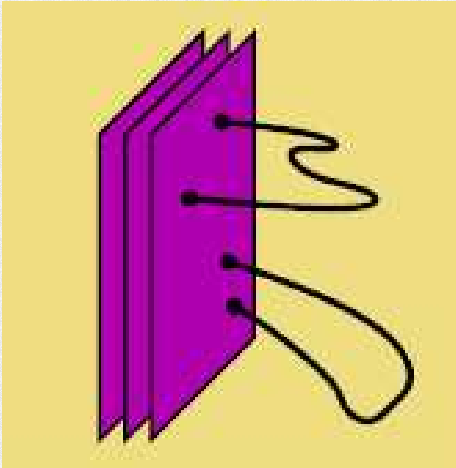
HF, T. Frederico, M. Beyer, JHEP 07 (2007) 077, IJMPE 16 (2007) 2794,
… + W. de Paula, PRD 79 (2009) 075019;
HF, PRD 78 (2008) 025071;
HF, E. Klempt, PLB 679 (2009) 77

- Gauge/string correspondence and `AdS/QCD`
- Meson & baryon spectra, lin. Regge trajs.
- Diquark correlations in baryons
- Dynamical AdS/QCD
- Hadron correlators, espec. glueballs
- Wrapping up...

Gauge/string correspondence in a (tiny) nutshell

2 equivalent descriptions of $E \ll 1/l_s$ excitations of $N_c \gg 1$ D3-branes:

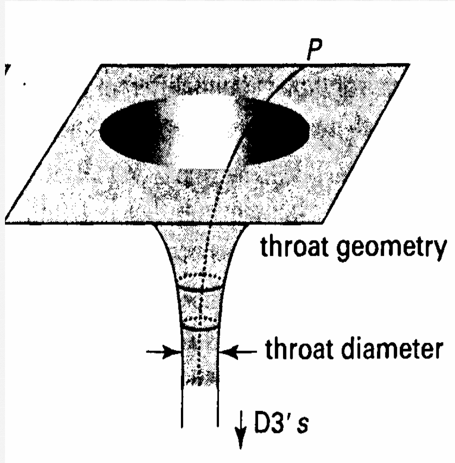
1) Open $m=0$ string modes on D3s :



$\lambda = gN_c \rightarrow 0$: $G_{(10)} \sim g^2 l_s^8 \rightarrow 0$:
little distortion of metric: (almost) flat
closed strings & bulk fields decouple,
+ open string modes ($m \neq 0$ inaccessible) →

(S)U(N_c) (S)Yang-Mills fields on D3s, strongly coupled (+...)

2) SUGRA soln. sourced by branes:



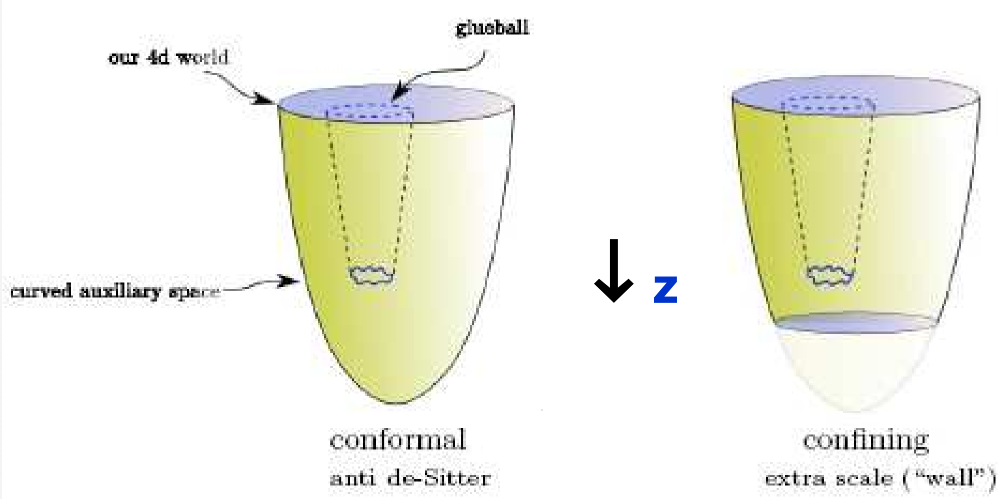
$\lambda \gg 1$: metric \`throat´, ends at horizon,
in far region: flat, gravity decoupled,
+ near horizon: fin. E excits. redshifted
at ∞, cannot escape from horizon

Graviton- & other excits. in $AAdS_5 \times X^5$, weakly coupled (+...)

$AAdS_5$ bulk backgrounds expected for QCD

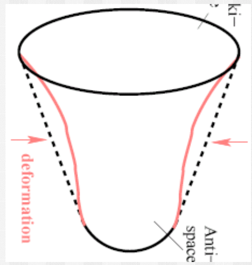
Minimal requirements on bulk geometry:

- Conf. UV, broken at Λ_{YM}: $AAdS_5$
- Poincaré invariant boundary
- Confinement: `IR wall´ in z
$\rightarrow$ IR-deformed AdS_5 :



$$ds^2 = g_{MN}(x)\, dx^M dx^N = e^{2A(z)} \frac{R^2}{z^2} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right) + R^2 ds^2_{X_5}$$

IR deform. $A(z)$ with $A(z \to 0) \to 0 \Rightarrow$ in UV
(ie. for $z \to 0$) metric remains conformal



+ dilaton $\Phi(z)$ in `soft wall´ + axions, tachyons, gauge fields + ...

Hadron dual-modes: rad. & orbital excitations in AAdS$_5$

Example: vector bulk modes:

$$V_z(x,z) = f_V(z)\,\varepsilon_z e^{-iPx}$$

$\Leftrightarrow$

dual to QCD meson interpolator:

$$\boxed{\mathcal{O}_{M,\bar\tau=L+2} = \bar q \gamma^\mu D_{\{\ell_1} \dots D_{\ell_m\}} q}$$

→ **bulk vector equation:**

with (Polchinski, Strassler) bc:

$$f_i(z) \overset{z\to 0}{\longrightarrow} z^{\tau_i}, \qquad \tau_i = \Delta_i - \sigma_i$$

$$\left[\partial_z^2 + 3\left(A' - \frac{1}{z}\right)\partial_z + 3\left(A'' + \frac{1}{z^2}\right) - \left(m_{5,V} R \frac{e^A}{z}\right)^2 + M^2\right] f_V(z) = 0.$$

⇒ **Sturm-Liouville problem:**

$$\boxed{\left[-\partial_z^2 + V_M(z)\right]\varphi(z) = M_M^2 \varphi(z)}$$

$$V_V(z) = \frac{3}{2}\left[-A'' + \frac{3}{2}A'^2 - 3\frac{A'}{z} + \frac{1}{2}\frac{1}{z^2}\right] + m_{5,V}^2 R^2 \frac{e^{2A}}{z^2}$$

Analog. for spin-0 mesons, baryons:

$$V_S(z) = \frac{3}{2}\left[A'' + \frac{3}{2}A'^2 - 3\frac{A'}{z} + \frac{5}{2}\frac{1}{z^2}\right] + m_{5,S}^2 R^2 \frac{e^{2A}}{z^2},$$

$$V_{B,\pm}(z) = m_{5,B} R \frac{e^A}{z}\left[\pm\left(A' - \frac{1}{z}\right) + m_{5,B} R \frac{e^A}{z}\right]$$

AdS/CFT dictionary sets bcs. ⇒

$$m_{5,S}^2 R^2 = \bar\tau_M(\bar\tau_M - 4) = L^2 - 4,$$

$$m_{5,V}^2 R^2 = \bar\tau_M(\bar\tau_M - 4) + 3 = L^2 - 1,$$

$$m_{5,B} R = \bar\tau_B - 2 = L + 1,$$

Trajectory structure in the light hadron spectrum

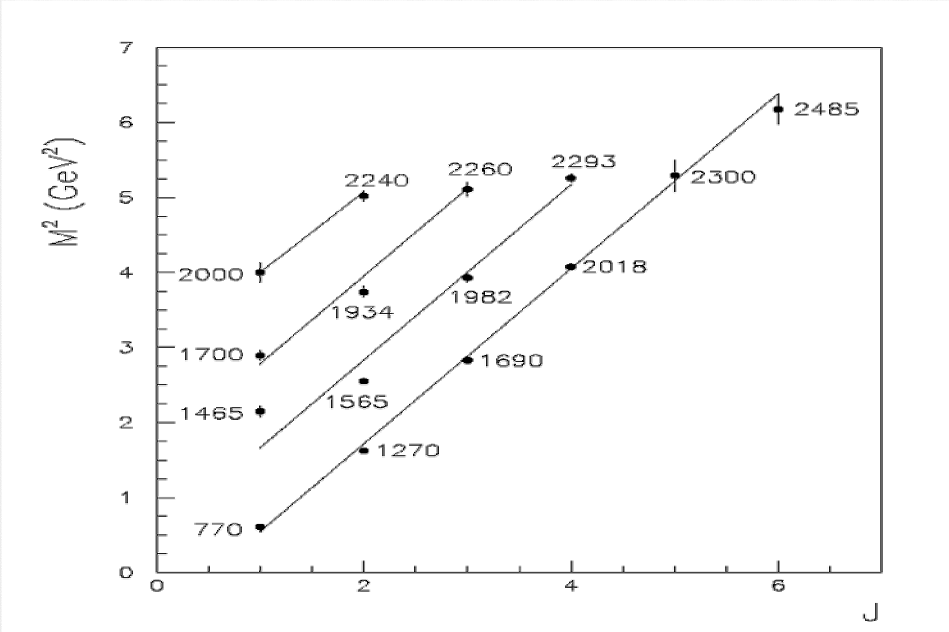
Simplest dual models, based on Polchinski-Strassler's 'hard wall':

$$\boxed{e^{2A^{(\mathrm{hw})}(z)} = \theta\left(z_m - z\right), \qquad z_m \simeq \Lambda_{\mathrm{QCD}}^{-1},}$$

predict: **$M^2 \sim N^2, L^2, J^2$**

but experiment finds:

$$\boxed{M^2 = M_0^2 + W\left(N + L\right)}$$



$W = 1.14 \pm 0.013\ \mathrm{GeV}^2$ mesons

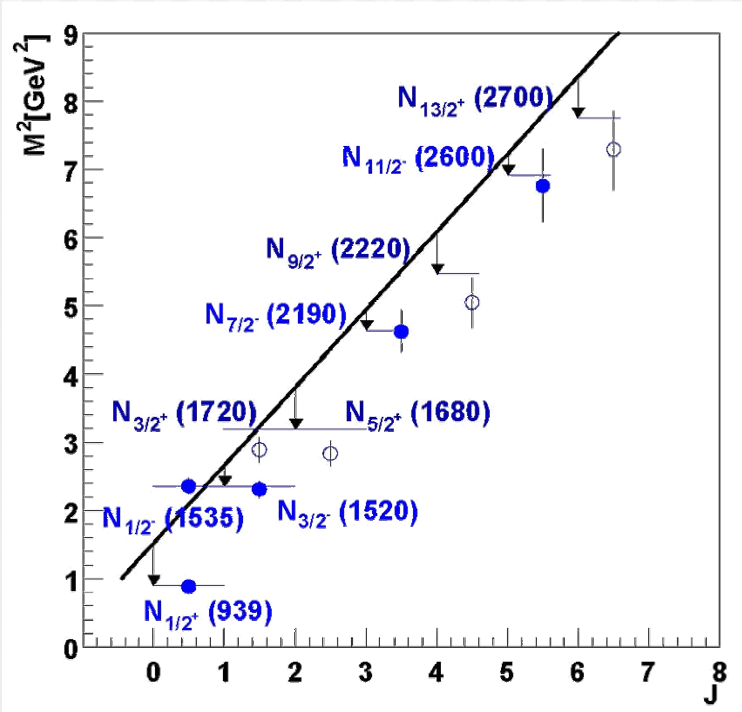
$W = 1.081 \pm 0.035\ \mathrm{GeV}^2$ baryons

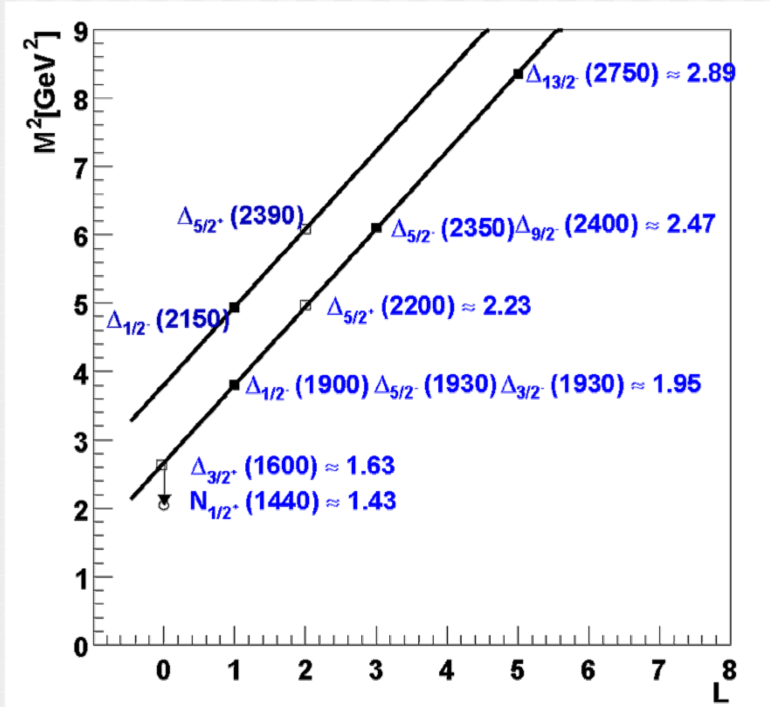
~ universal slopes!

Karch et al.: dilaton 'soft wall':
⇒ linear trajs. but only for mesons!

$$\boxed{A^{(\mathrm{sw})}\left(z\right) \equiv 0, \qquad \Phi^{(\mathrm{sw})}\left(z\right) = \lambda^2 z^2.}$$

Linear trajectories in the baryon spectra





How to obtain observed meson & baryon trajs. with universal slope?
Addtl. bulk fields necessary, or 5d metric background sufficient?

(Affirmative) answer constructively: assume only background metric, check whether A can be constructed such that desired spectra arise!

Construction of metric for univ. lin. trajectory spectra

Lin. Trajectories require oscillator-type Sturm-Liouville potentials

Univ. slopes can be obtained from AdS$_5$ 1/z^2 potentials via: $\bar{\tau}_i \to \bar{\tau}_i + \lambda^2 z^2.$

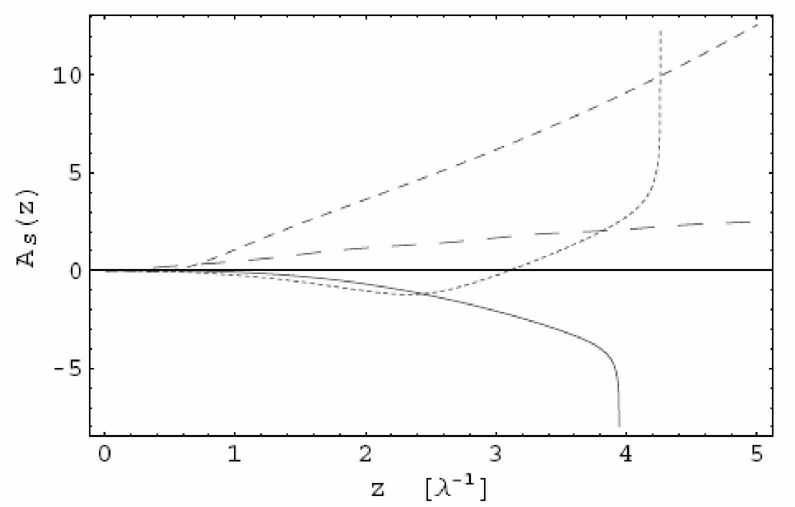
$$V_M^{(\mathrm{LT})}(z) = \left[\left(\lambda^2 z^2 + L\right)^2 - \frac{1}{4}\right]\frac{1}{z^2}$$

$$V_{B,\pm}^{(\mathrm{LT})}(z) = \left\{(L+1)(L+1\mp 1) + \left[2(L+1)\pm 1\right]\lambda^2 z^2 + \lambda^4 z^4\right\}\frac{1}{z^2}.$$

Derive dual geometry by equating to gen.-A potentials ⇒ ODEs for A(z), ∃ unique warp factor solutions A(z):

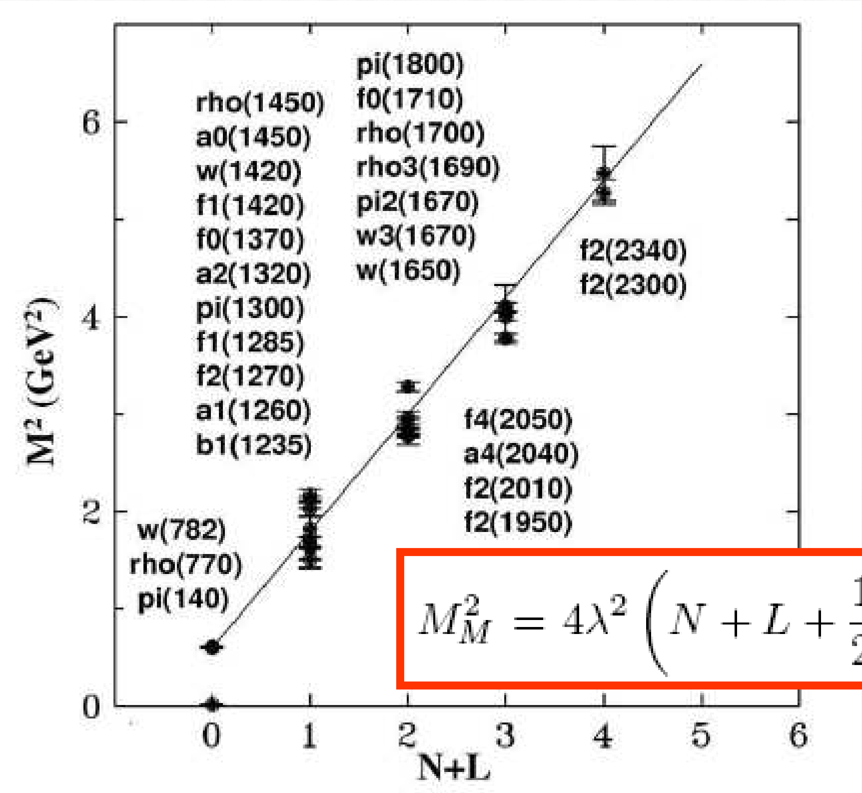
$$\boxed{A_S(z) = \frac{1}{2}\ln\left[1 + \sum_{n=1}^{\infty} A_n(L)\left(\frac{\lambda^2 z^2}{2}\right)^n\right]}$$

$$\boxed{A_B(z) = \ln\left(1 + \frac{\lambda^2 z^2}{L+1}\right)}$$

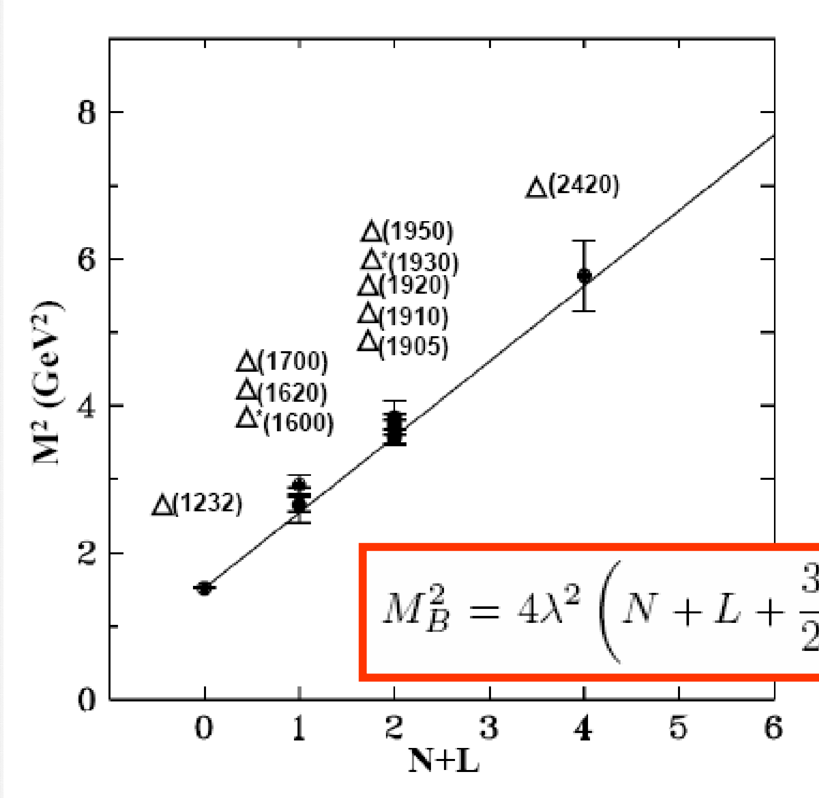


Resulting spectra and properties

Dual eigenmodes and mass spectra (obtained analytically):



$$M_M^2 = 4\lambda^2 \left(N + L + \frac{1}{2}\right)$$



$$M_B^2 = 4\lambda^2 \left(N + L + \frac{3}{2}\right)$$

- **Lin. trajectories of universal slope in baryon sector, too**
- **Encoded via AdS$_5$ IR deform. (orbital flucts. backreacted?)**
- **Dual confinement signatures (metric singularities)**
- **New relations between slopes and ground state masses**

Diquark correlations in light baryons holographically

Nucleon excitations improved by incl. diquark-content dependent anomalous dims of interpolators

$$\boxed{\eta_t(x) = 2\left[\eta_{\rm pd}(x) + t\,\eta_{\rm sd}(x)\right]}$$

$$\eta_{\rm pd} = \varepsilon_{abc}(u_a^T C d_b)\gamma^5 u_c$$

$$\eta_{\rm sd} = \varepsilon_{abc}(u_a^T C\gamma^5 d_b) u_c$$

⇒ string-mode mass corrections:

$$\Delta m_5^{(\kappa_{\rm gd})} = \gamma_t^{(\kappa_{\rm gd})} = \frac{\Delta M^2_{\kappa_{\rm gd}}}{4\lambda^2 R}$$

dep. on "good-diquark fraction" κ !

L, N	$\kappa_{\rm gd}$	Resonance					Pred.
0, 0	$\frac{1}{2}$	$N(940)$				input:	**0.94**
0, 0	0	$\Delta(1232)$					1.27
0, 1	$\frac{1}{2}$	$N(1440)$					1.40
1, 0	$\frac{1}{4}$	$N(1535)$	$N(1520)$				1.53
1, 0	0	$N(1650)$	$N(1700)$	$N(1675)$			1.64
1, 0	0	$\Delta(1620)$	$\Delta(1700)$		$L, N = 0, 1$:	$\Delta(1600)$	1.64
2, 0	$\frac{1}{2}$	$N(1720)$	$N(1680)$		$L, N = 0, 2$:	$N(1710)$	1.72
2, 0	0	$N(1900)$	$N(1990)$	$N(2000)$	$\Delta(1910)$	$\Delta(1920)$	1.92
2, 0	0	$\Delta(1905)$	$\Delta(1950)$	$\Delta(1900)^*$	$\Delta(1940)^*$	$\Delta(1930)^*$	1.92
0, 3	$\frac{1}{2}$	$N(2100)$					2.03
3, 0	$\frac{1}{4}$	$N(2070)$	$N(2190)$	$L, N = 1, 2$:	$N(2080)$	$N(2090)$	2.12
3, 0	0	$N(2200)$	$N(2250)$	$\Delta(2223)$	$\Delta(2200)$	$L, N = 1, 2$: $\Delta(2150)$	2.20
4, 0	$\frac{1}{2}$	$N(2220)$					2.27
4, 0	0	$\Delta(2390)$	$\Delta(2300)$	$\Delta(2420)$	$L, N = 3, 1$:	$\Delta(2400)$ $\Delta(2350)$	2.43
5, 0	$\frac{1}{4}$	$N(2600)$					2.57
6, 0	$\frac{1}{2}$	$N(2700)$					2.71
6, 0	0	$\Delta(2950)$			$L, N = 5, 1$:	$\Delta(2750)$	2.84

$$M^2_{N,L} = M^{(\rm ms)2}_{N,L+\Delta m_5 R} = 4\lambda^2\left(N + L + \frac{3}{2}\right) + \Delta M^2_{\kappa_{\rm gd}}$$

$$\boxed{\Delta M^2_{\kappa_{\rm gd}} = -2\left(M^2_\Delta - M^2_N\right)\kappa_{\rm gd}}$$

Manifestly simple spectrum, reprod. all 48 observed N res. better than quark models !

Regge trajectories in a dynamical AdS/QCD dual

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{|g|}\left(-R + \frac{1}{2}g^{MN}\partial_M\Phi\partial_N\Phi - V(\Phi)\right),$$

$\phi = \phi(z)$ and g_{AAdS5} → Einstein-dilaton eqs.:

$$6A'^2 - \tfrac{1}{2}\Phi'^2 + e^{-2A}V(\Phi) = 0,$$

$$3A'' - 3A'^2 - \tfrac{1}{2}\Phi'^2 - e^{-2A}V(\Phi) = 0,$$

$$\Phi'' - 3A'\Phi' - e^{-2A}\frac{dV}{d\Phi} = 0$$

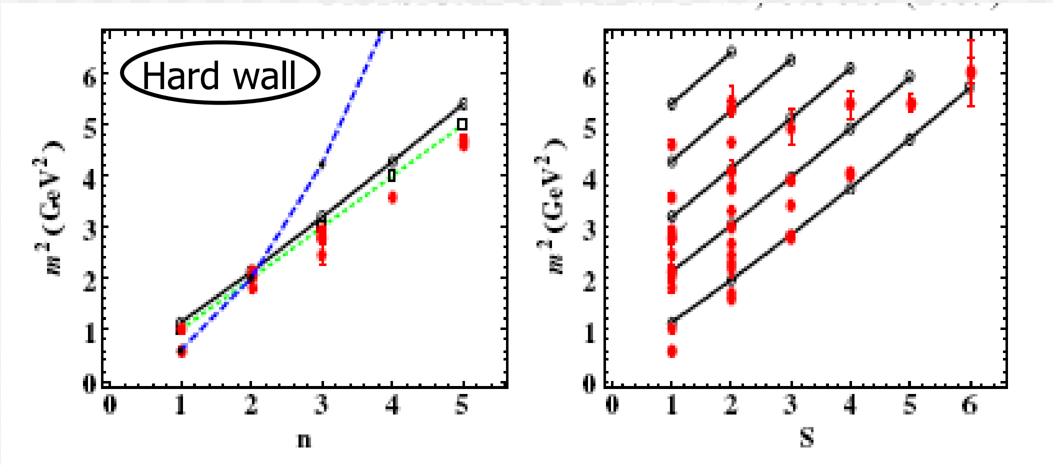
Find UV-conformal solution for ϕ, $V(\phi)$ with area-law gen. warp factor

$$A(z) = \ln z + C(z)$$

$$\boxed{C(z) = \frac{1+\sqrt{3}}{2S+\sqrt{3}-1}\,\frac{(z\Lambda_{QCD})^2}{1+e^{(1-z\Lambda_{QCD})}}}$$

→ Mass spectra

$$\boxed{m^2_{n,S} \simeq \frac{1}{10}(11n + 9S - 9),}$$



Pert. Log. Corrections can be included in C…

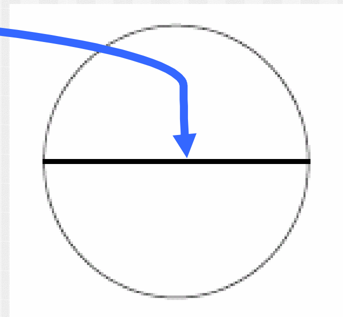
Glueball correlators: calculational strategy

On-shell action: insert solutions $\varphi(x,z)$ subject to boundary source $\varphi^{(s)}(x)$, i.e.

$$\varphi(x,z) = \int \frac{d^4q}{(2\pi)^4} e^{-iqx} \hat{K}(q,z) \int d^4x' e^{iqx'} \varphi^{(s)}(x')$$

where K is the bulk-to-boundary propagator

$$\hat{K}(q,z) = -\frac{R^3}{\varepsilon^3} \sum_n \frac{\psi_n'(\varepsilon)\psi_n(z)}{q^2 - m_n^2 + i\varepsilon'}$$



into the bulk action

$$S[\varphi; g, \Phi] = \frac{1}{2\kappa^2} \int d^{d+1}x \sqrt{|g|} e^{-\Phi} g^{MN} \partial_M \varphi \partial_N \varphi$$

2 funct. derivatives of exp. of on-shell bulk action wrto. $\varphi^{(s)}$ ⇒

$$\boxed{\hat{\Pi}(-q^2) = -\frac{R^3}{\kappa^2}\left[\frac{e^{-\Phi(z)}}{z^3}\hat{K}(q,z)\,\partial_z\hat{K}(q,z)\right]_{z=\varepsilon\to 0}}$$

Glueball correlators in "hard- and soft-wall" duals

Hard-wall correlator (IR brane at $z=z_m$):

$$\hat{\Pi}\left(Q^2\right) = \frac{R^3}{8\kappa^2} Q^4 \left[2\frac{K_1\left(Qz_m\right)}{I_1\left(Qz_m\right)} - \ln\left(\frac{Q^2}{\mu^2}\right)\right]$$

$$\frac{R^3}{\kappa^2} = \frac{2\left(N_c^2 - 1\right)}{\pi^2}.$$

Dilaton soft-wall correlator ($\phi = \lambda^2 z^2$):

$$\hat{\Pi}\left(Q^2\right) = -\frac{2R^3}{\kappa^2}\lambda^4 \left[1 + \frac{Q^2}{4\lambda^2}\left(1 + \frac{Q^2}{4\lambda^2}\right)\psi\left(\frac{Q^2}{4\lambda^2}\right)\right]$$

Spectral representation: $\hat{\Pi}\left(Q^2\right) = \int_{m_1^2}^{\infty} ds \frac{\rho(s)}{s + Q^2}$ **with spect. density ρ:**

$\Rightarrow$ **both correlators have $\rho \geq 0$, mass gap, zero-width Gb poles $\leftrightarrow$ large N_c:**

Hard wall: $\rho(s) = \frac{R^3}{2\kappa^2 z_m^2} s^2 \sum_{n=1}^{\infty} \frac{\delta\left(s - m_n^2\right)}{J_0^2\left(j_{1,n}\right)}$ $\Rightarrow$ $m_n^{(\mathrm{N})} = \frac{j_{1,n}}{z_m},$

lin. "pomeron" trajectory!

Soft wall: $\rho(s) = \frac{\lambda^2 R^3}{2\kappa^2} s \left(s - m_0^2/2\right) \sum_{n=0}^{\infty} \delta\left(s - m_n^2\right).$ $\Rightarrow$ $m_n^2 = 4(n+2)\lambda^2$

Hard-wall content: small-instanton contributions

At large $Q \gg z_m^{-1}$ npert. Q dependence becomes exponential:

$$\hat{\Pi}^{(np)}\left(Q^2\right) \equiv \frac{R^3}{4\kappa^2}\frac{K_1\left(Qz_m\right)}{I_1\left(Qz_m\right)}Q^4$$
$$\xrightarrow{Qz_m \gg 1} \frac{4}{\pi}\left[1+\frac{3}{4}\frac{1}{Qz_m}+O\left(\frac{1}{\left(Qz_m\right)^2}\right)\right]Q^4 e^{-2Qz_m}$$

→ **No power corrections** (expected to be weak from QCD-OPE!)!
→ **LET** trivially satisfied (no anomaly, no gluon condensate...)

Compare with small-scale QCD instanton contribution to OPE:

$$\hat{\Pi}^{(I+\bar{I})}\left(Q^2\right) \xrightarrow{Q\bar{\rho} \gg 1} 2^4 5^2 \pi \zeta \bar{n}\left(Q\bar{\rho}\right)^3 e^{-2Q\bar{\rho}}$$

⇒ $$\bar{\rho} \simeq z_m, \qquad \bar{n} \simeq \frac{3}{2^4 5^2 \pi^2 \zeta}\frac{1}{z_m^4},$$ $$\rho^{(\mathrm{hw})} \leq z_m \sim \mu^{-1}$$

Crucial **small-instanton contribs**. (semi-quant.) reproduced!

Soft-wall dynamcis: power corrections, UV gluon mass

Asymptotic expansion for $Q^2 \gg \lambda^2 \cong \Lambda^2$:

$$\hat{\Pi}\left(Q^2\right) = -\frac{2R^3}{\kappa^2}\lambda^4 \left[1 + \frac{Q^2}{4\lambda^2}\left(1 + \frac{Q^2}{4\lambda^2}\right)\left(\ln\frac{Q^2}{4\lambda^2} - \frac{2\lambda^2}{Q^2} - \sum_{n=1}^{\infty}\frac{B_{2n}}{2n}\left(\frac{4\lambda^2}{Q^2}\right)^{2n}\right)\right]$$
$$= -\frac{2}{\pi^2}Q^4\left[\ln\frac{Q^2}{\mu^2} + \frac{4\lambda^2}{Q^2}\ln\frac{Q^2}{\mu^2} + \frac{2^2 5}{3}\frac{\lambda^4}{Q^4} - \frac{2^4}{3}\frac{\lambda^6}{Q^6} + \frac{2^5}{15}\frac{\lambda^8}{Q^8} + \ldots\right].$$

Contains all OPE power corrections:

$$\left\langle G^2\right\rangle \simeq -\frac{10}{3\pi^2}\lambda^4,$$
$$\left\langle gG^3\right\rangle \simeq \frac{4}{3\pi^2}\lambda^6,$$
$$\left\langle G^4\right\rangle \simeq -\frac{8}{15\pi^3\alpha_s}\lambda^8.$$

Even addtl. term expected in QCD from UV-ren., gluon mass, 2d-nonlocal cond. :

$$\hat{\Pi}^{(\mathrm{CNZ})}\left(Q^2\right) = -\frac{2}{\pi^2}Q^4\ln\frac{Q^2}{\mu^2}\left(1 + 6\frac{\bar{\lambda}^2}{Q^2} + \ldots\right)$$

$$\boxed{\bar{\lambda}^2 \simeq \frac{2}{3}\lambda^2}$$

Leading coeff. has sign opposite to QCD, LET → artefact of strong-coupl. UV!

Glueball decay constants

Glueball observables beyond spectrum (expt. glueball signatures,...):

$$\boxed{f_n := \frac{1}{m_n^2} \left\langle 0 \left| \mathcal{O}_S(0) \right| 0_n^{++} \right\rangle}$$

where

$$\mathcal{O}_S(x) = G_{\mu\nu}^a(x) \, G^{a,\mu\nu}(x),$$

Contain crucial information on Gb structure (size!): `WF at origin´ x→0:

$$\chi_n(x) = \left\langle 0 \left| 2\mathrm{tr} \left\{ G_{\mu\nu}\left(-\frac{x}{2}\right) U\left(-\frac{x}{2}, \frac{x}{2}\right) G^{\mu\nu}\left(\frac{x}{2}\right) \right\} \right| 0_n^{++} \right\rangle$$

Existing QCD information:

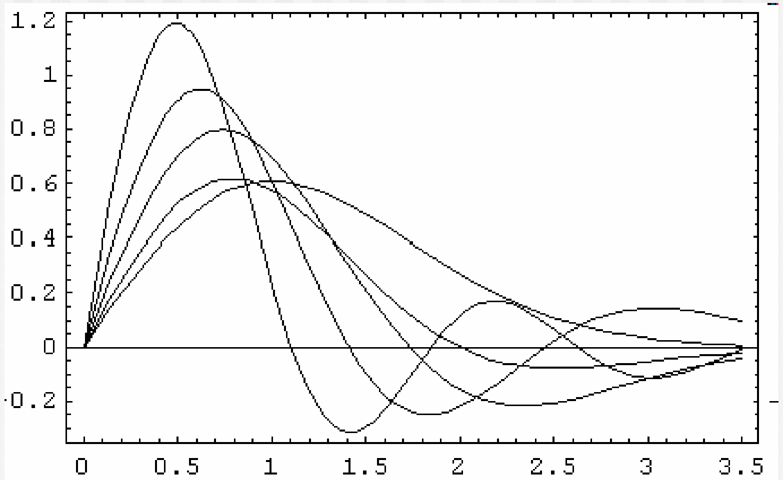
- older lattice evidence: except. small 0^{++} size (via BS ampl.)
- conflicting predictions of QCD sum rule analyses:
  - neglect npert. Wilson coeffs. (Narison): $f_S = 0.390 \pm 0.15$ GeV
  - include npert. (small inst.) Wilson coeffs. (HF): $f_S = 1.050 \pm 0.10$ GeV
- instanton liquid model (Schäfer, Shuryak) $f_S \cong 0.8$ GeV
- (quenched) lattice (Chen et al. 2006): $f_S = 0.86 \pm 0.18$ GeV

Decay constants holographically

Solve bulk field eq. for norm. eigenmodes:

$$\psi_n(z) = N_n \hat{\varphi}(m_n, z)$$

(for plot mult. by sqrt of int. measure)



For dilaton soft wall:

$$f_n^{(\mathrm{sw})} = 4 I_n^{-1/2} \frac{\lambda^3 R^{3/2}}{m_n^2 \kappa} = \frac{1}{\sqrt{2}} \sqrt{\frac{n+1}{n+2}} \frac{\lambda R^{3/2}}{\kappa} \quad \Rightarrow \quad \boxed{f_S^{(\mathrm{sw})} \simeq 0.3 \text{ GeV}.}$$

Hard wall (expected to represent more relevant Gb physics):

$$f_n = \lim_{\varepsilon \to 0} \frac{R^3}{\kappa m_n^2} \frac{\psi_n'(\varepsilon)}{\varepsilon^3} = \frac{N_n}{2} \frac{R^3}{\kappa} m_n^2 \quad \Rightarrow \quad \boxed{f_S^{(\mathrm{hw})} \simeq 0.8 - 0.9 \text{ GeV},}$$

Consistent with ILM, IOPE sum rules & lattice results

Summary and conclusions

Hadron spectra:

- Linear Regge-type trajectories of universal slope for mesons AND baryons
- Minimal 'metric soft wall': encoded solely via IR deformed AdS5 metric
- New relations between trajectory slopes and ground-state masses

- Diquark effects can enter via anom. dims. of baryon interpolators
- Decisively improve predictions for light baryon spectra!

- Backreacted 5d Einstein-dilaton solution: generates area law,
- Vector meson excitations with linear trajectories dynamically!

Hadron correlators:

Detailed, quantitative testing ground → generates new insights:

→ In comparison with QCD-OPE: map impact of strongly-coupled UV
→ Hard/soft wall duals contain complementary gauge/QCD physics
→ (beyond spectra) Predictions of glueball decay constants, coupls.
→ Suggests systematic bottom-up improvement strategies