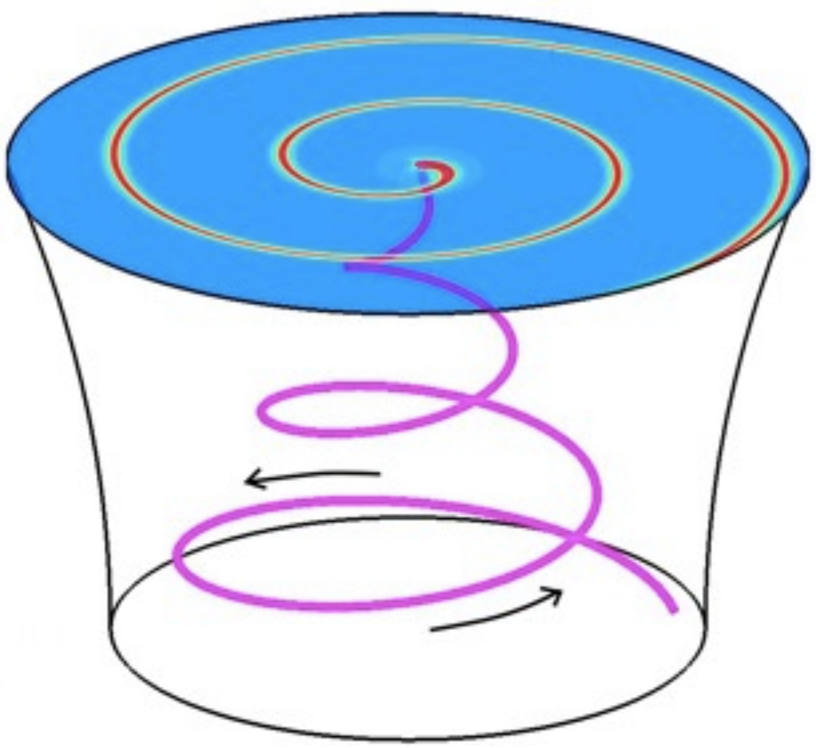
SYNCHROTRON RADIATION IN STRONGLY COUPLED CONFORMAL FIELD THEORIES

Dominik Nickel
Institute of Nuclear Theory, Seattle



C. Athanasiou, P. Chesler, H. Liu, DN, K. Rajagopal

arXiv:1001.3880

UNIVERSITY of WASHINGTON

OUTLINE

- motivation: patterns of radiation
- gauge/gravity duality
- synchrotron radiation
- outlook

RADIATION IN CLASSICAL ELECTRODYNAMICS

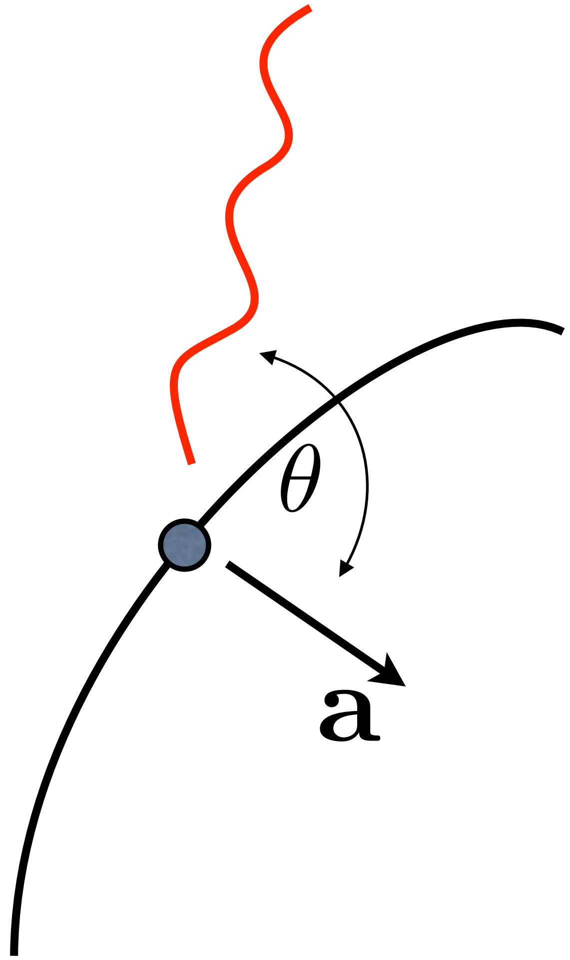
- accelerated charge radiates

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi} a^2 \sin^2\theta$$

➡ total power radiated (energy loss)

$$P = \frac{2}{3}\frac{e^2}{4\pi} a^2$$

➡ free propagation of radiation (distribution of power)



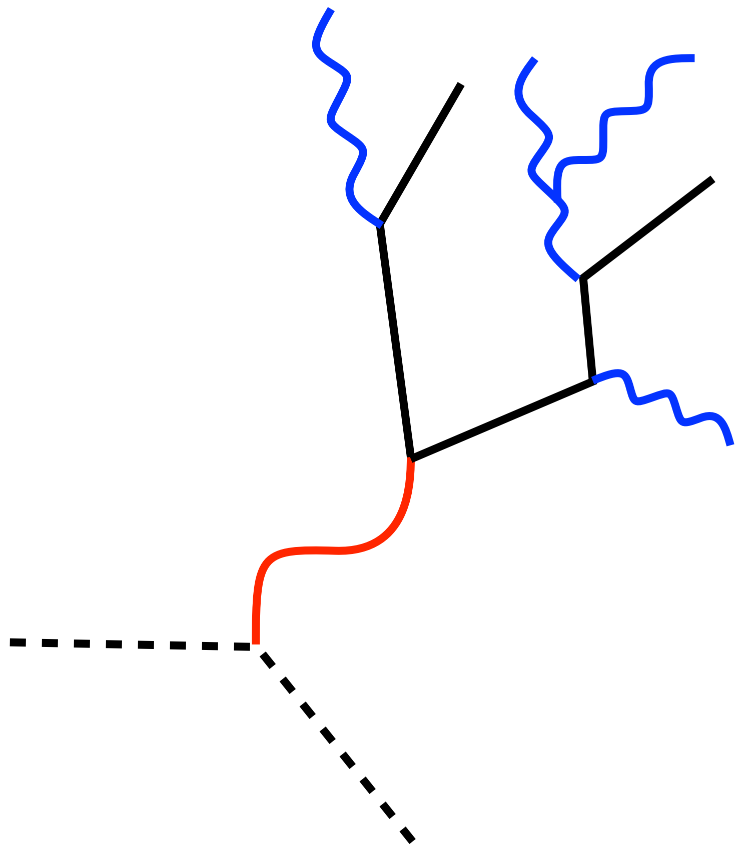
analogy in strongly coupled gauge theories?

RADIATION IN GAUGE THEORIES? - JETS

factorization in jet production

$$e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}$$

- hard production process
- showering
- hadronization



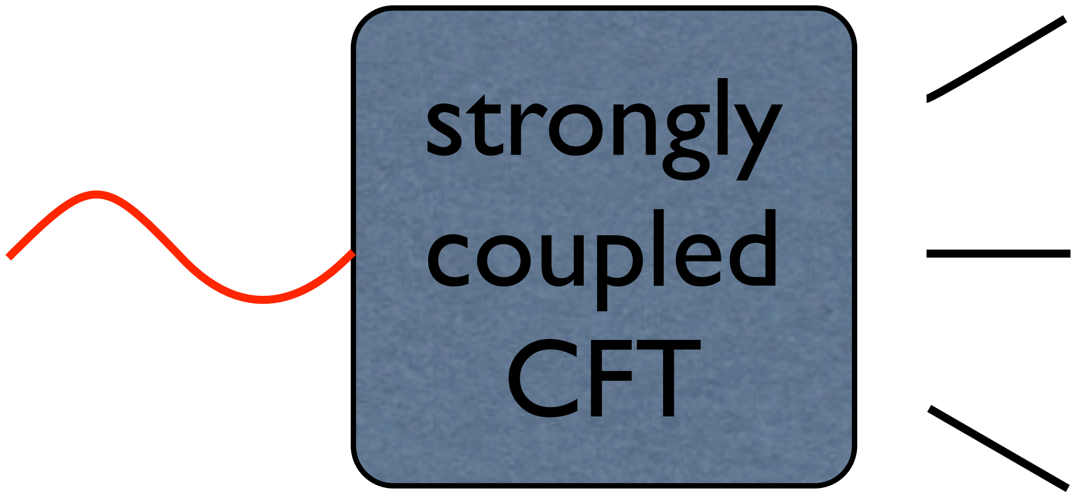
understanding from first principles challenging?

➡ guidance from other approaches?

CONFORMAL COLLIDER PHYSICS

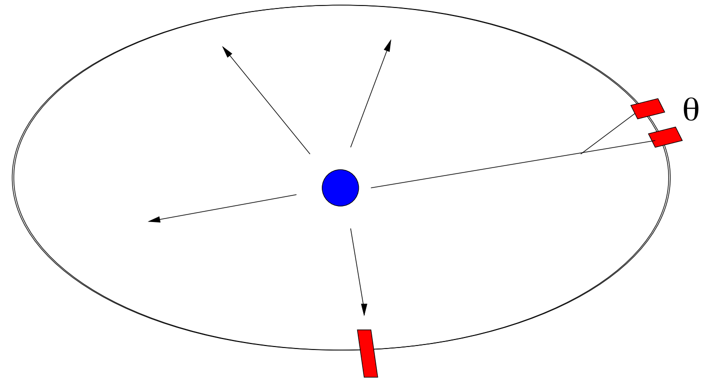
D. Hofman, J. Maldacena [arXiv:0803.1467]

study decay of off-shell bosons in a strongly coupled CFT



quantities of interest:

$$\langle \mathcal{E}(\hat{r}) \rangle = \lim_{r\to\infty} r^2 \int dt \langle \psi(t) | \hat{r}\cdot \mathbf{S} | \psi(t) \rangle$$
$$\langle \mathcal{E}(\hat{r}) \mathcal{E}(\hat{r}') \rangle = \ldots$$



key result: all N-point functions isotropic!

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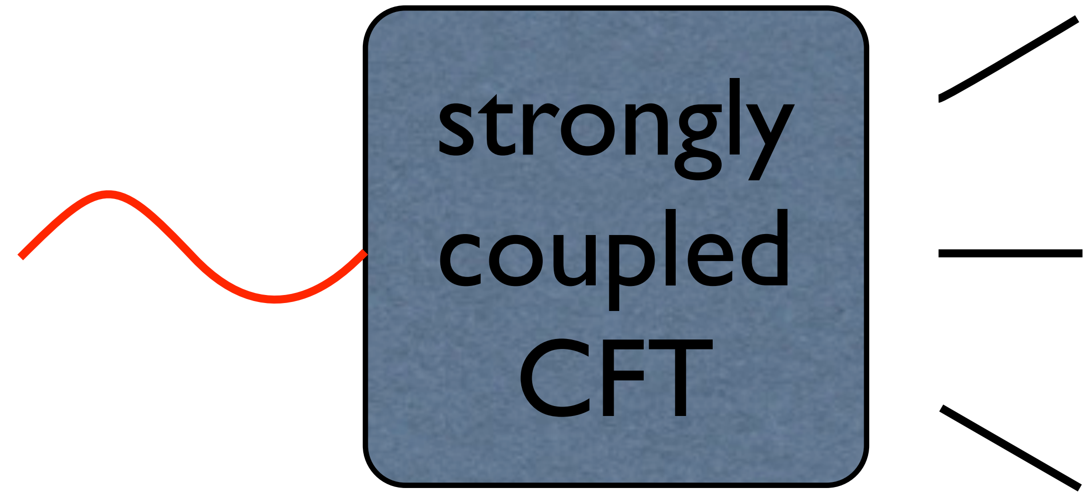

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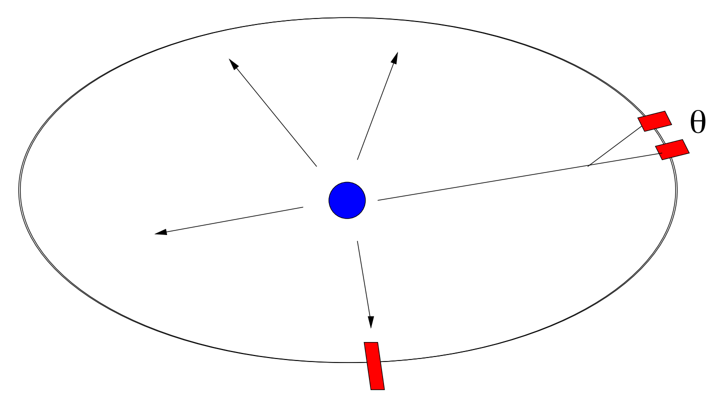


key result: all N-point functions isotropic!

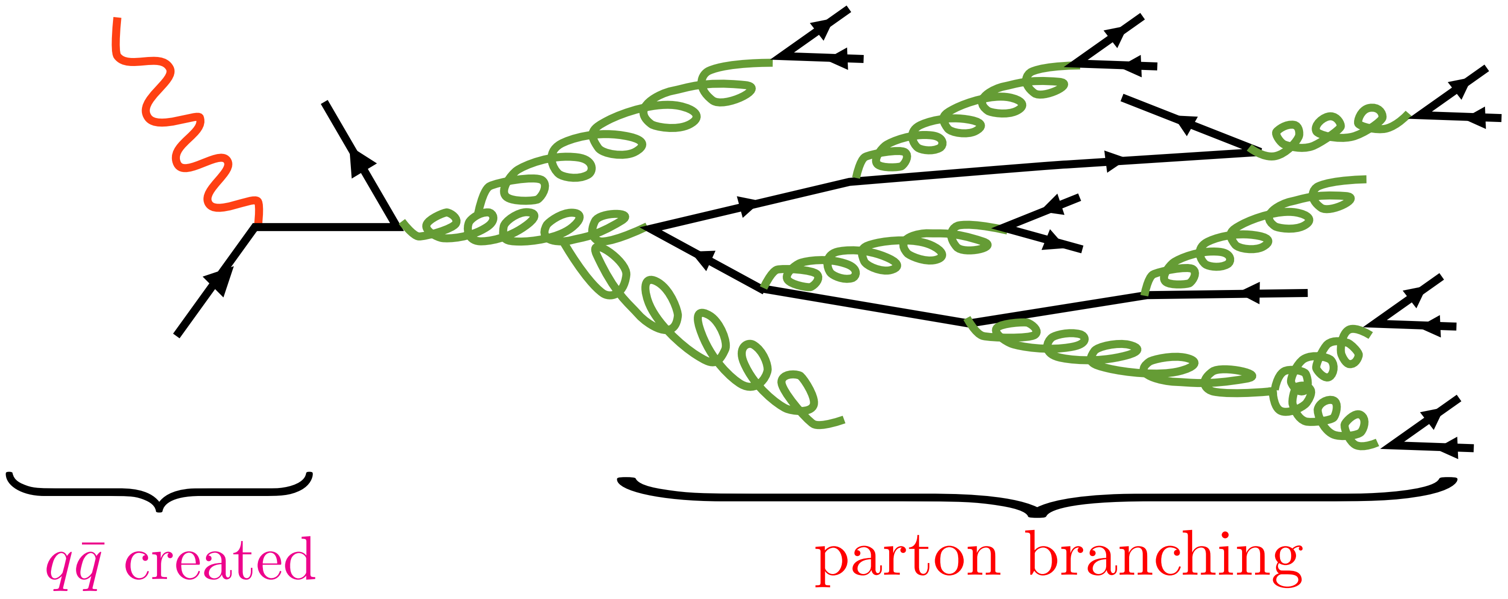
no jets at strongly coupling?

WHERE DOES THE ISOTROPY COME FROM?

- choice of initial conditions?
- propagation through strongly coupled vacuum?

parton branching picture:

Y. Hatta, E. Iancu, A. Mueller [arXiv:0803.2481]



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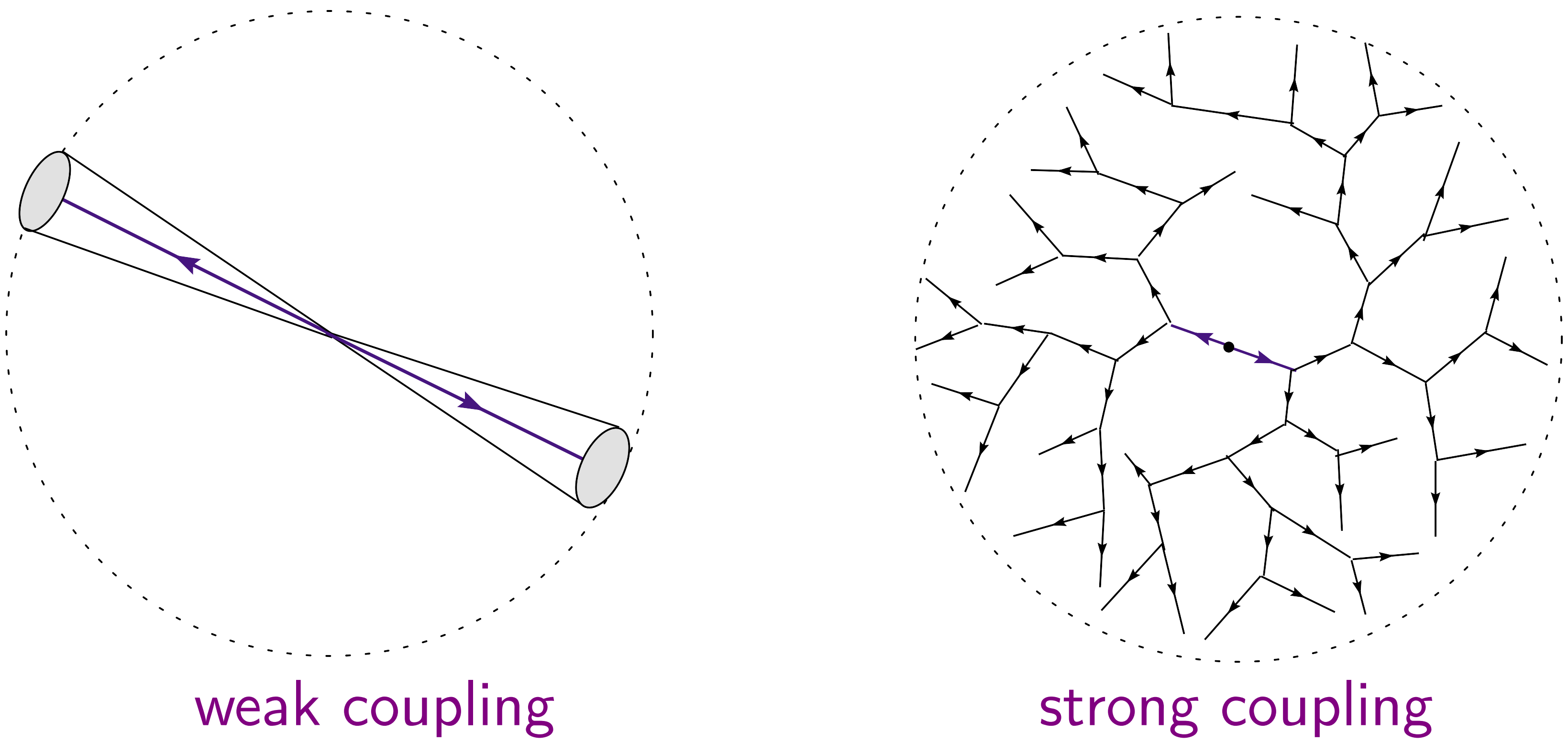

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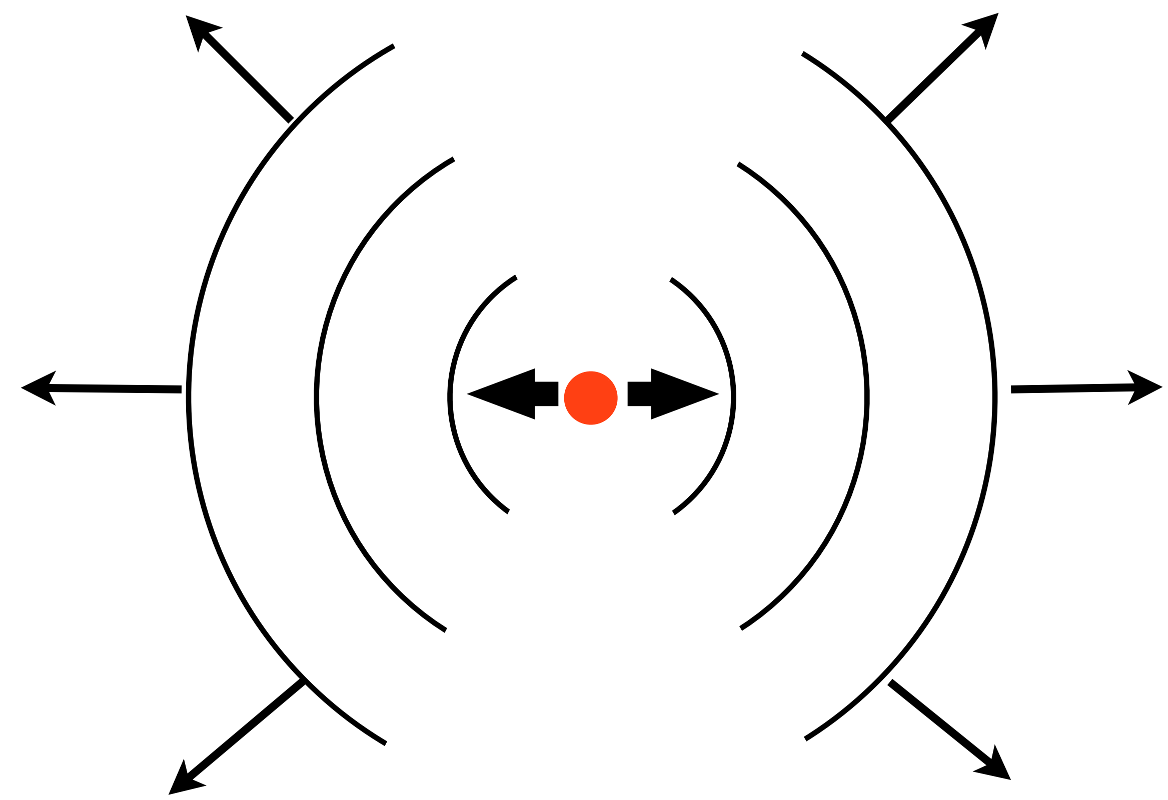
A SIMPLE EXPERIMENT

radiating source in compact domain



- real-time propagation
- power spectrum at infinity
- depletion of short-wavelength modes?

concentrate on theories with classical gravity dual

>exact solution for synchrotron radiation<

ADS/CFT

Maldacena's conjecture:

string theory on aymptotic $AdS_5 \times X^5$	$\longleftrightarrow$	$4d$ (supersymmetric) CFT
classical gravity		large $N_c\,,\ \lambda$

J. Maldacena [hep-th/9711200]

geometrical realization / 'dictionary':

gravity in 5d		4d CFT
gravitational fields		source for stress-energy tensor
gauge fields		source for global currents
open strings	$\longleftrightarrow$	quarks
black hole		finite temperature/density

$$S_{\rm gravity}[\phi] = S_{\rm gravity}^{\rm boundary}[\phi_0] = S_{\rm CFT}[\phi_0]$$

➡ classical dynamics in 5d captures dynamics of 4d QFT

A PICTORIAL PRESENTATION OF HOLOGRAPHY

deformation of background near boundary encodes relevant information for CFT

(cartoon for illustrative purposes only; courtesy of P. Chesler)

background deformation

- similarity to induced surface charge on a conductor
- deformations from AdS_5 in bulk give different ensembles

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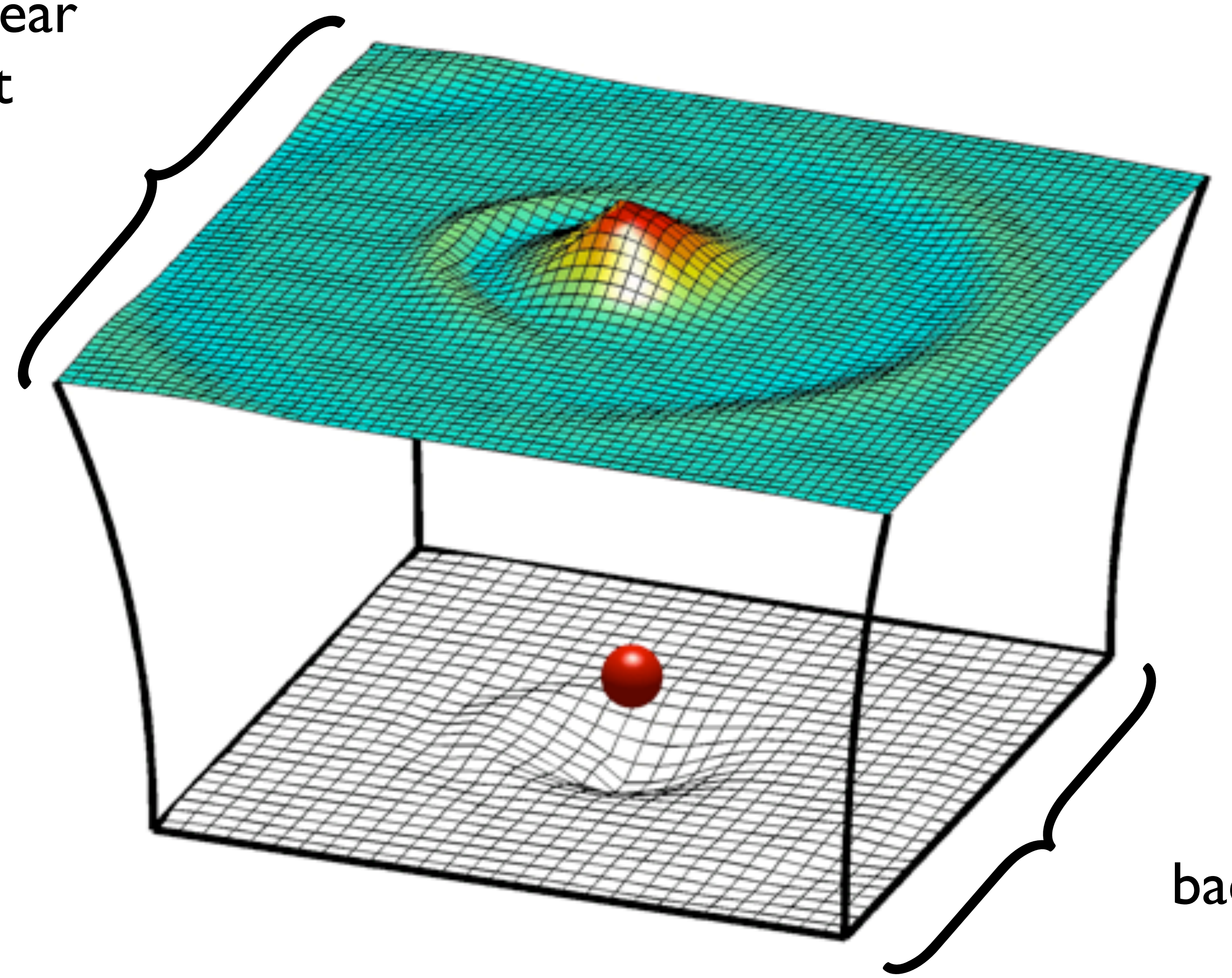

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THE GRAVITATIONAL SETUP

AdS background:

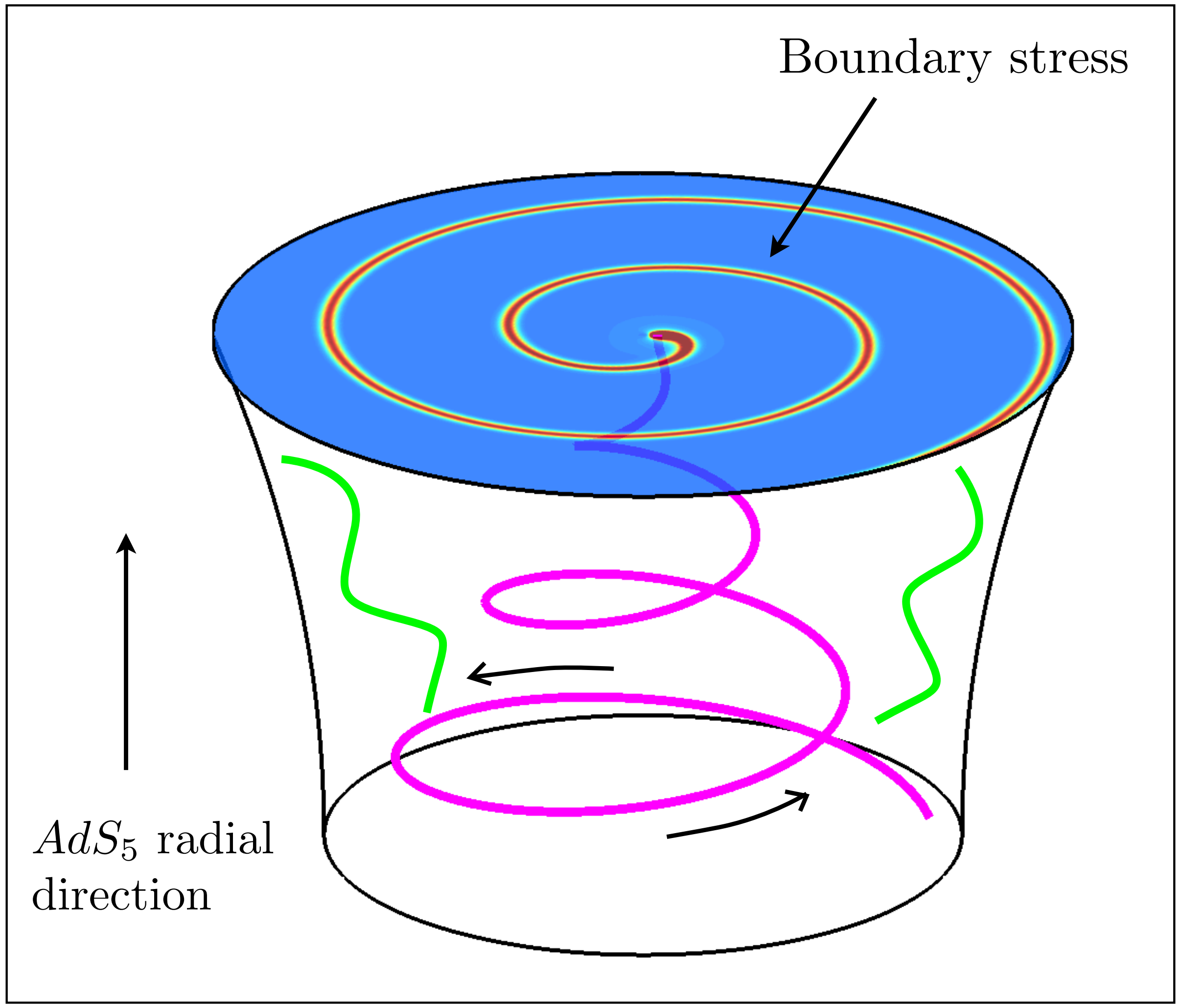
$$ds^2 = \frac{L^2}{u^2}\left(-dt^2 + d\mathbf{x}^2 + du^2\right)$$

➡ rotating string $X^\mu(\sigma,\tau)$

➡ gravitational waves $h_{\mu\nu}(x)$ towards boundary

➡ stress-energy tensor/energy density

$$T^{\mu\nu}(x) = \lim_{u\to 0} \frac{2}{\sqrt{-g}} \frac{\delta S_{\text{gravity}}}{\delta h_{\mu\nu}(x)}$$



simplification: $G_{\text{Newton}} \sim 1/N_c^2 \;\longrightarrow$ gravitational response linear

THE ROTATING STRING

- Nambu-Goto action in AdS background:

$$S_{\rm NG} = -T_0 \int dt du \sqrt{-\det(\gamma_{ab})}\,, \quad \gamma_{ab} = G^{\rm AdS}_{\mu\nu} X^\mu X^\nu$$

- parameterization of worldsheet: $X^\mu \;=\; (t, R(u), \pi/2, \phi(u) + \omega_0 t, u)^\mu$

- equations of motion:

$$R'' \;=\; -\frac{R(u + 2RR')(1 + R'^2)}{u(u^4\Pi^2 - R^2)} - \frac{1 + R'^2}{R(1 - \omega_0^2 R^2)}\,,$$

$$\phi'^2 \;=\; \frac{u^4\Pi^2(1 - \omega_0^2 R^2)(1 + R'^2)}{R^2(R^2 - u^4\Pi^2)}\,.$$

- additional constraint: $1 - \omega_0^2 R(u_\Lambda)^2 = 0 \quad \Leftrightarrow \quad R(u_\Lambda)^2 - u_\Lambda^2 \Pi^4 = 0$

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- solution (remarkable luck!):

$$R(u) = \sqrt{R_0^2 + \frac{v^2 u^2}{\sqrt{1 - v^2}}}\,, \quad \phi(u) = -\frac{u\omega_0}{\sqrt{1 - v^2}} + \arctan\left(\frac{u\omega_0}{\sqrt{1 - v^2}}\right)$$

- bonus: energy loss from flux on string A. Mikhailov [hep-th/0305196]

$$E = -\pi^u_t \equiv \frac{\delta S_{\rm NG}}{\delta \partial_u X^t} = \frac{\lambda}{2\pi}\left(\frac{R_0 \omega_0^2}{1 - v^2}\right)^2$$

GRAVITATIONAL WAVES AND GAUGE INVARIANTS

- linearized equations of motion for $G_{MN} = G^{\mathrm{AdS}}_{MN} + h_{MN}$ from

$$R_{MN} - \frac{1}{2} G_{MN}(R + 2\Lambda) \quad = \quad 8\pi G_{\mathrm{Newton}}\, t^{\text{rotating string}}_{MN}$$

- decomposition into helicities: $h_{MN} \quad = \quad h^{\mathrm{tensor}}_a \mathbb{T}^a_{MN} + h^{\mathrm{vector}}_a \mathbb{V}^a_{MN} + h^{\mathrm{scalar}}_a \mathbb{S}^a_{MN}$

- coordinate invariance/conservation laws: only one gauge invariant per helicity required

- scalar equation

$$\partial_u^2 Z(u) - \frac{5}{u}\,\partial_u Z(u) + \left(-\partial_t^2 + \nabla^2 + \frac{9}{u^2}\right) Z(u) \quad = \quad S(u)$$

$$S(u)/L^3 \quad = \quad -u^2 \partial_u \left(\frac{t_{00}}{u^2}\right) + \frac{u}{6}\nabla^2(2t_{00} - 2t_{55} + t_{ii}) - \frac{1}{2} u \nabla_i \nabla_j t_{ij} + \partial_t t_{05} + \nabla_i t_{i5}$$

- energy density:

$$\boxed{\mathcal{E}(t, \mathbf{r}) \quad = \quad \lim_{u \to 0} \frac{Z(t, \mathbf{r}, u)}{u^3}}$$

ANALYTIC SOLUTION FOR ENERGY DENSITY

- retarded Green's function for desired boundary conditions:

$$\mathcal{G}(t,\mathbf{r},u) \quad = \quad \frac{1}{\pi u^2}\delta'(-t^2+\mathbf{r}^2+u^2)$$

- energy density via convolution:

$$\begin{aligned}\mathcal{E}(t,\mathbf{r}) &= \int dt d\mathbf{r}' \int_0^\infty du\; \mathcal{G}(t-t',\mathbf{r}-\mathbf{r}',u)S(t',\mathbf{r}',u) + \text{regulators} \\ &= \frac{\sqrt{\lambda}}{24\pi^2\gamma^4 r^6 \Xi^6}\Big[-2r^2\Xi^2 + 4r\gamma^2\Xi(t_{\text{ret}}-t) + (2\gamma^2 - 4r^2v^2\gamma^2\omega_0^2\sin^2\theta + 3r^2\gamma^4\omega_0^2\Xi^2)(t_{\text{ret}}-t)^2 \\ &\quad + 7r\gamma^2\omega_0^2\Xi(t_{\text{ret}}-t)^3 + 4\gamma^2\omega_0^2(t_{\text{ret}}-t)^4 + 8v\gamma^2\omega_0 r(t_{\text{ret}}-t)(t_{\text{ret}}-t+r\,\Xi)\sin\theta\cos(\varphi-\omega_0 t_{\text{ret}})\Big]\end{aligned}$$

where

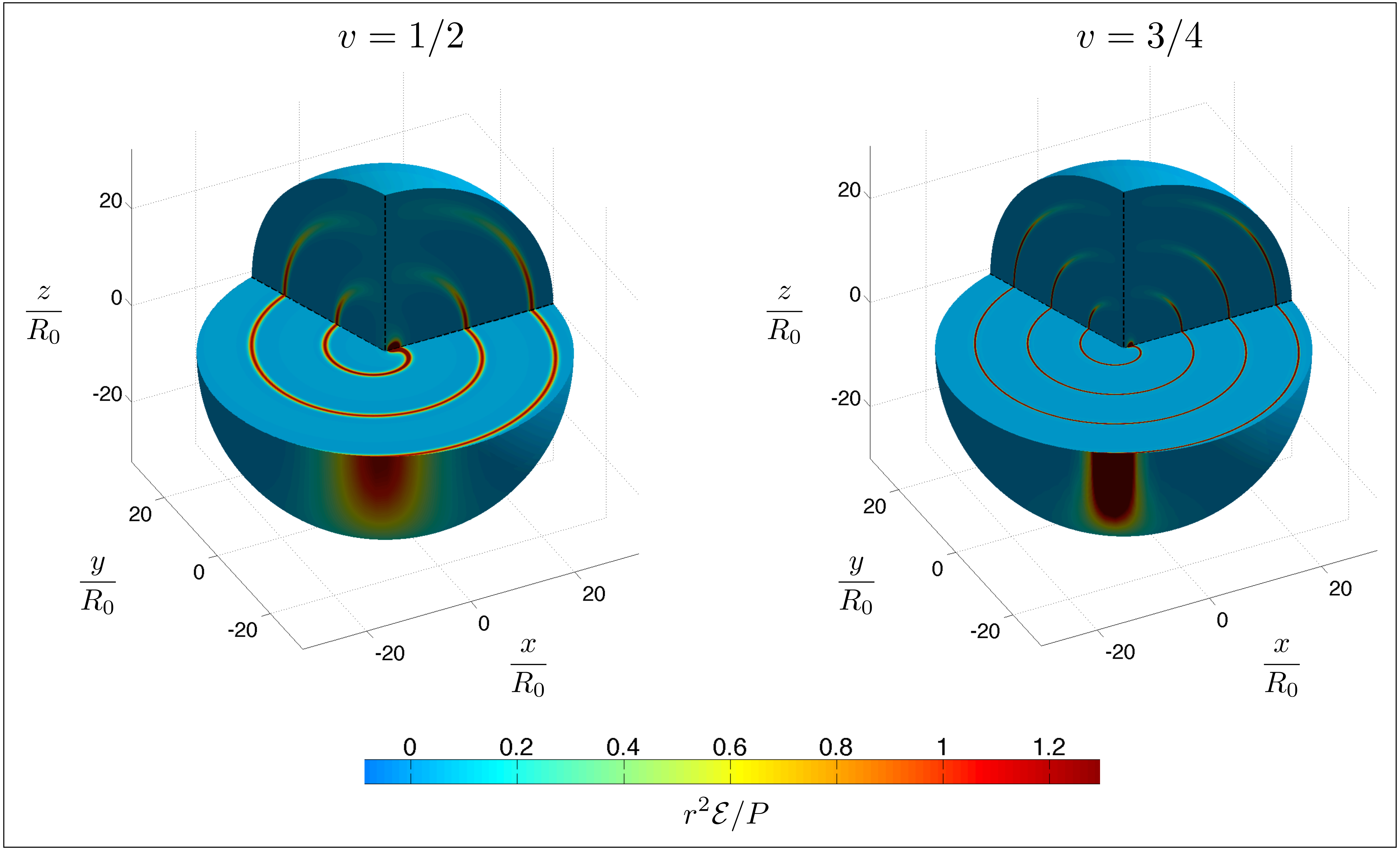
$$t - t_{\text{ret}} - |\mathbf{r} - \mathbf{r}_{\text{quark}}(t_{\text{ret}})| = 0$$

$$\Xi \equiv \frac{|\mathbf{r} - \mathbf{r}_{\text{quark}}(t_{\text{ret}})| - \mathbf{r}\cdot\dot{\mathbf{r}}_{\text{quark}}(t_{\text{ret}})}{r}$$

$$\mathbf{r}_{\text{quark}} = \text{quark's trajectory}\,, \quad \mathbf{r} \equiv \{r,\theta,\phi\} = \text{observer}\,, \quad \gamma = 1/\sqrt{1-v^2}$$

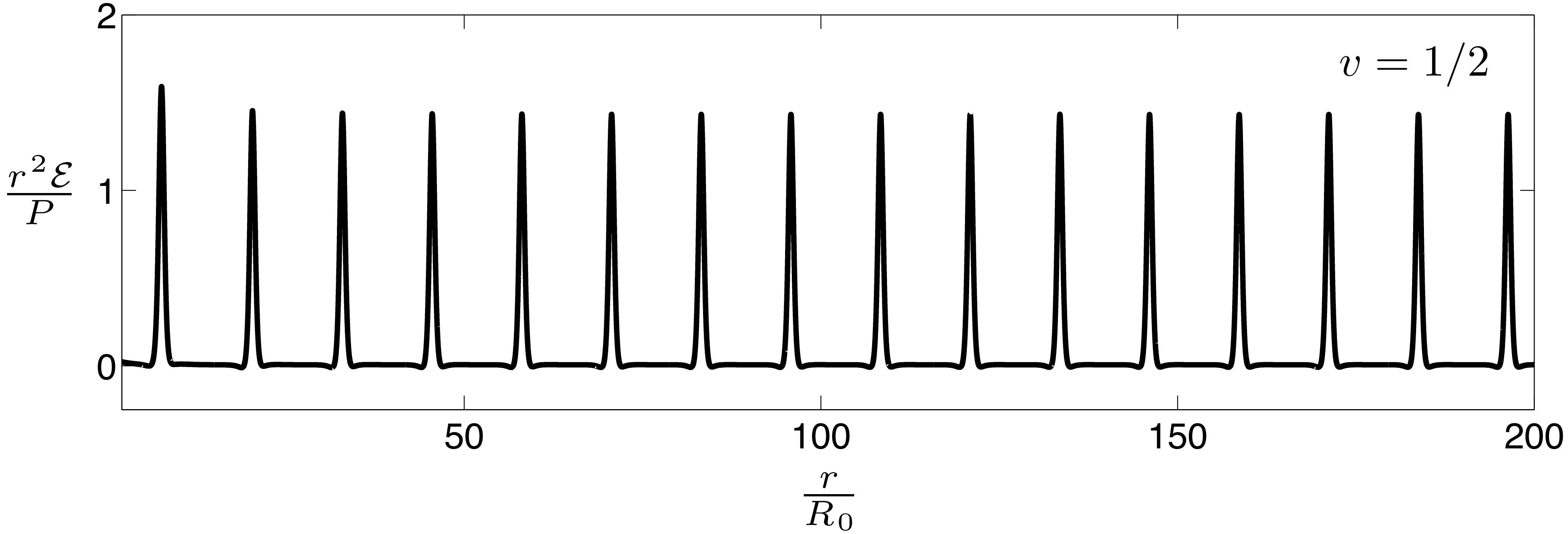
- analytic solution up to t_{ret} (as in electrodynamics)
- similar dependence on Ξ as in electrodynamics

ILLUSTRATION OF $\mathcal{E}(t, \mathbf{r})$

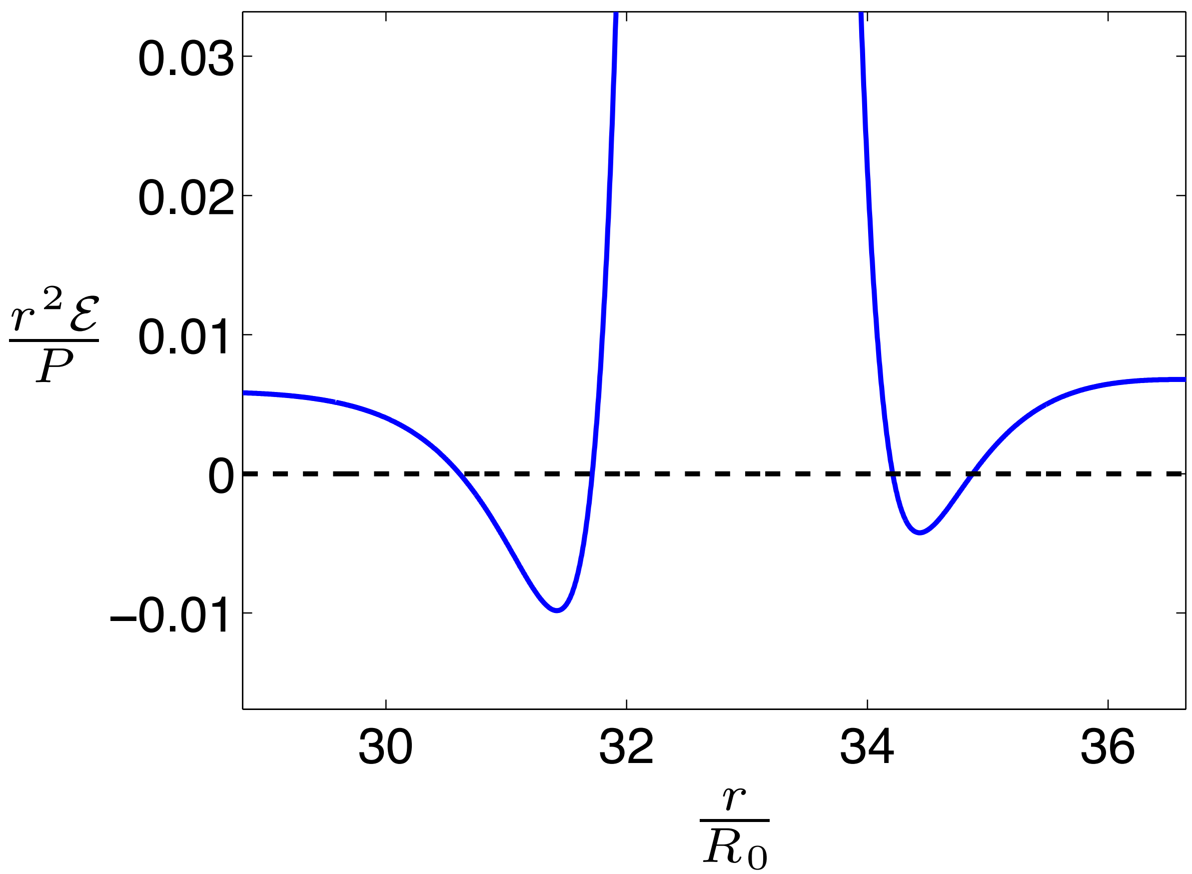


radial thickness: $\sim 1/\gamma^3$, azimuthal thickness: $\sim 1/\gamma$

BEHAVIOR IN FAR ZONE



- radiation does not broaden in far zone!
- similar struture as classical synchrotron radiation
- negative energy density ⟷ quantum effects



GEOMETRICAL INTERPRETATION

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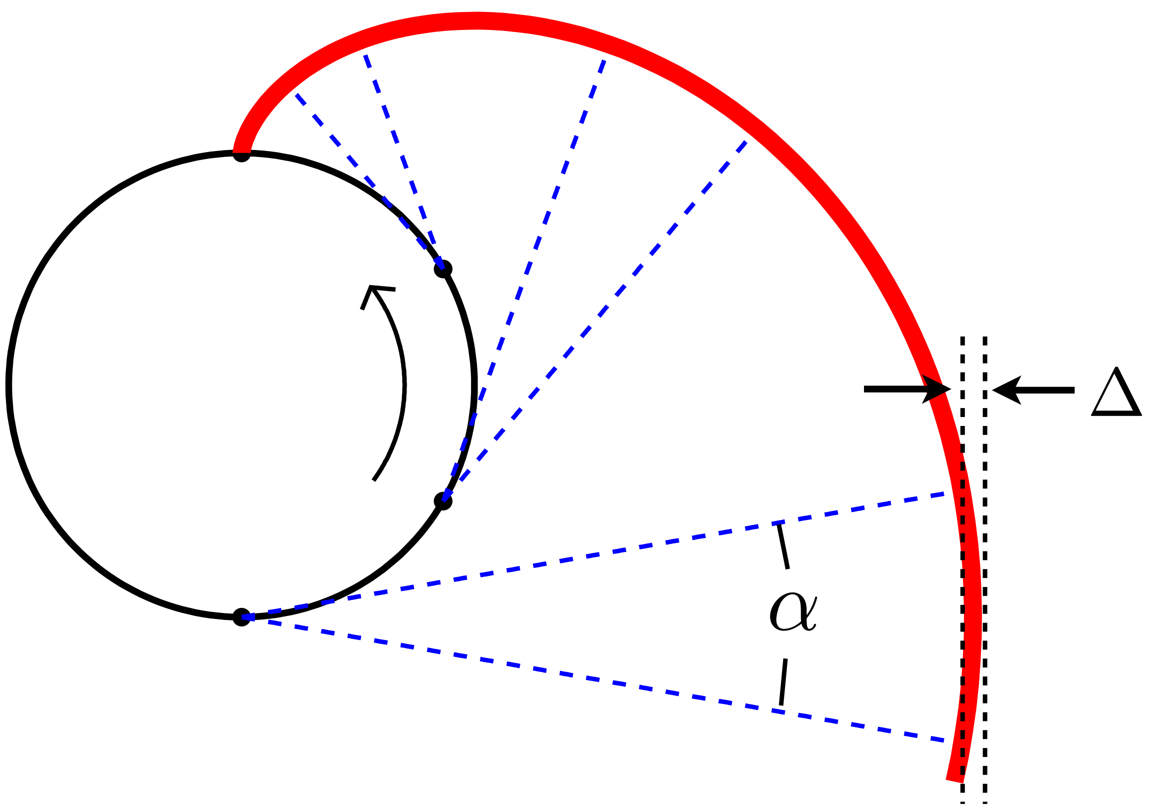

- radiation in fixed relative direction and free without broadening propagation: spiral structure

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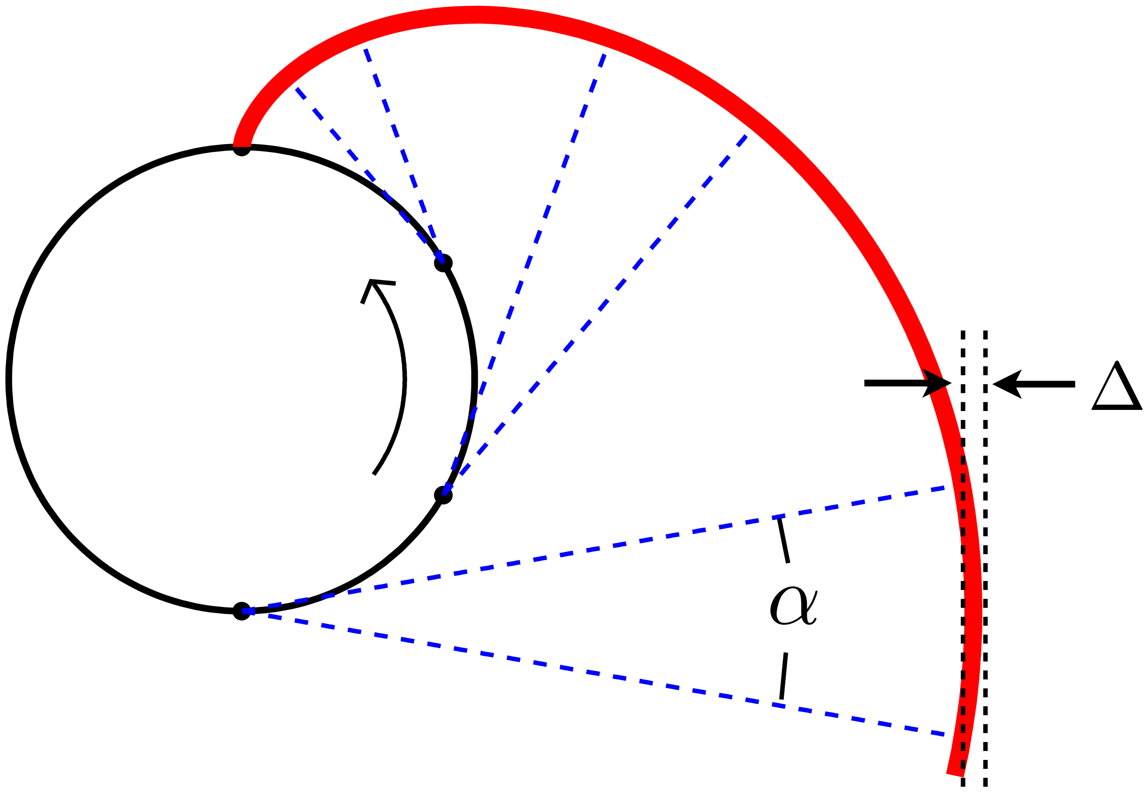

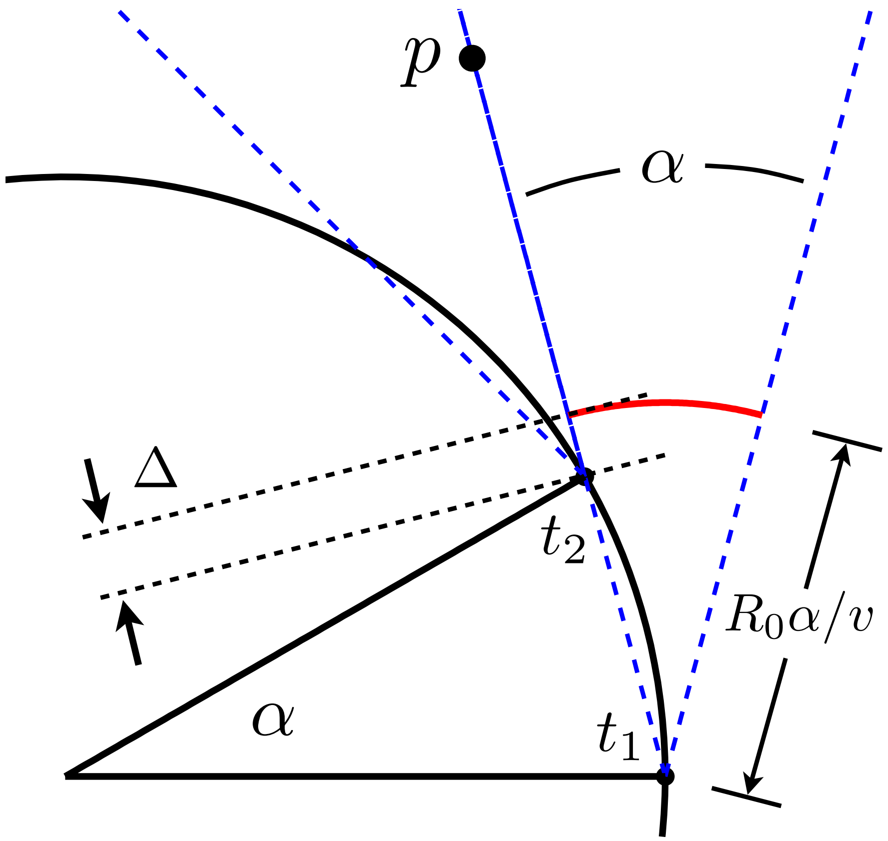

- radiation in fixed relative direction and free without broadening propagation: spiral structure
- radiation in cone with width α : - when radiated in moving direction $\Delta \sim \alpha R_0/v - \alpha R_0 \sim R_0/\gamma^3$
  - otherwise $\Delta \sim \alpha R_0$

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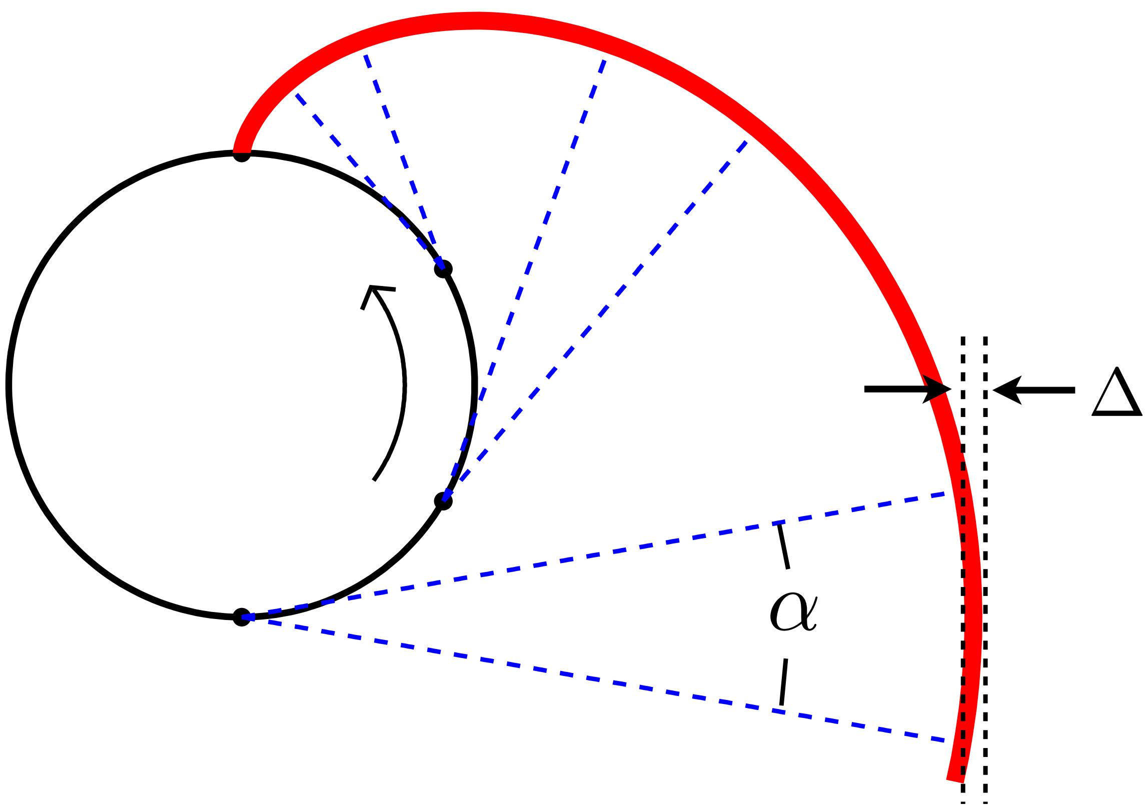

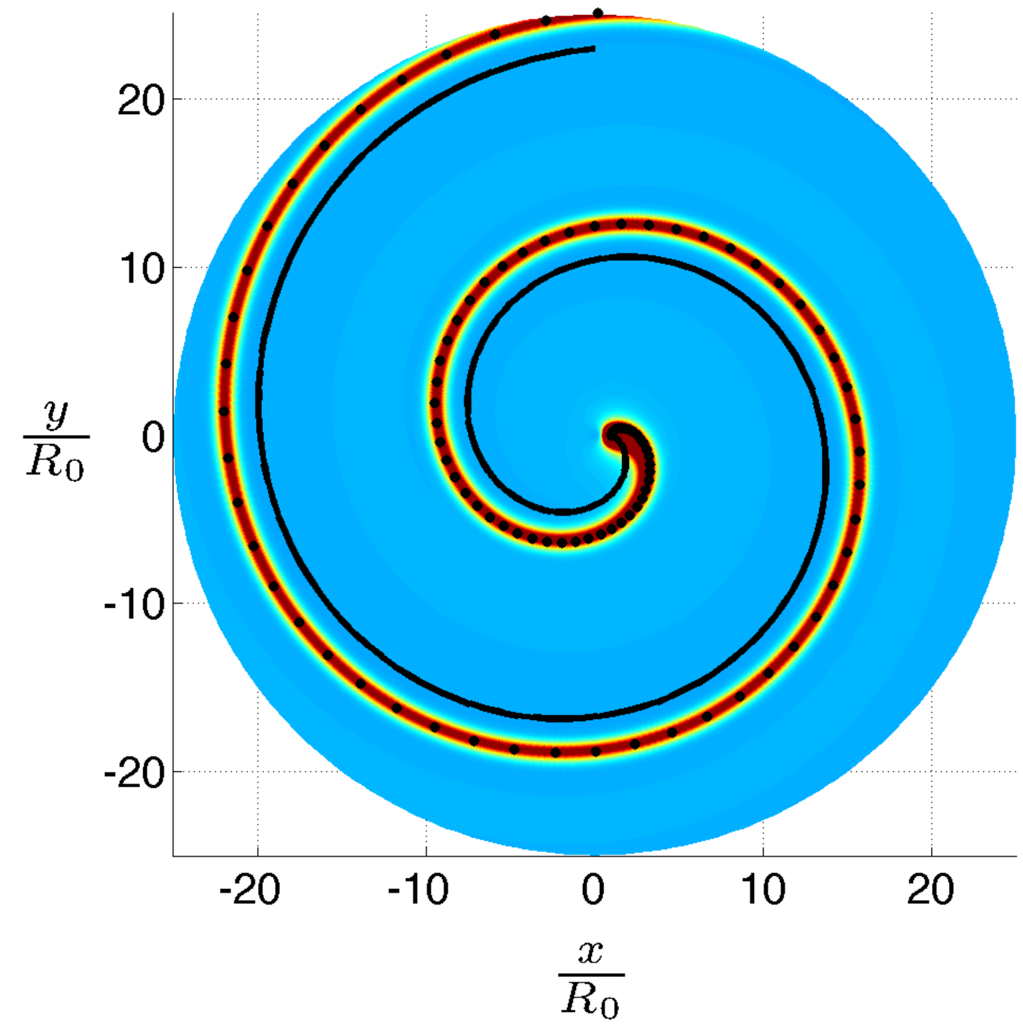

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➡ **Lesson:**
- radiation more or less isotropic in rest frame ($\alpha \sim 1/\gamma$)
- free propagation without broadening
- strong similarities to ED/weak coupling regime

COMPARISON TO WEAK COUPLING AND ED

- time-averaged power distributions:

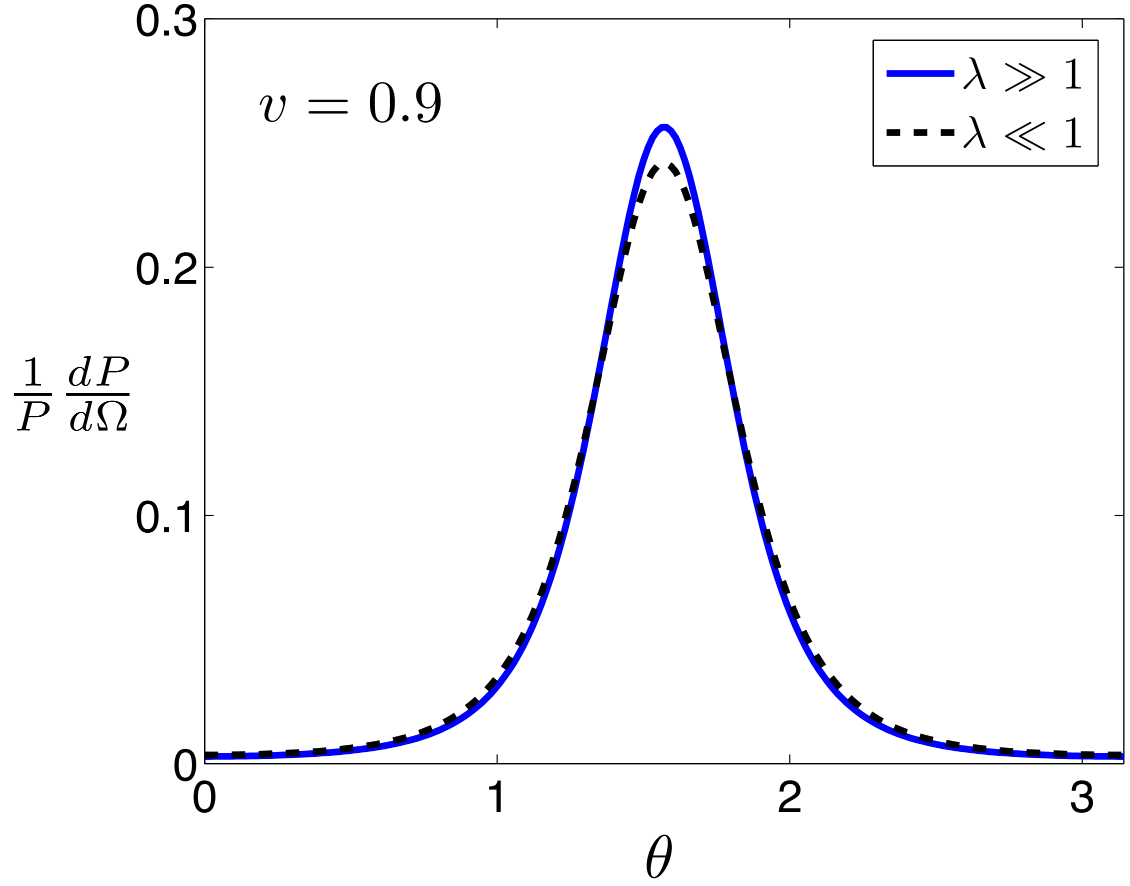
$$\frac{dP}{d\Omega} = \frac{1}{T}\int_t^{t+T} dt' \underbrace{\hat{\mathbf{n}}\cdot\mathbf{S}}_{\to r^2\mathcal{E}}$$

- weak coupling: one vector and one scalar field coupling to $e_{\rm eff}^2 = \lambda/2$

$$\frac{dP^{\lambda\ll1}}{d\Omega} \propto \frac{dP^{\rm vector}}{d\Omega} + \frac{dP^{\rm scalar}}{d\Omega}$$
$$\frac{dP^{\lambda\gg1}}{d\Omega} \propto \frac{dP^{\rm vector}}{d\Omega} + 2\frac{dP^{\rm scalar}}{d\Omega}$$

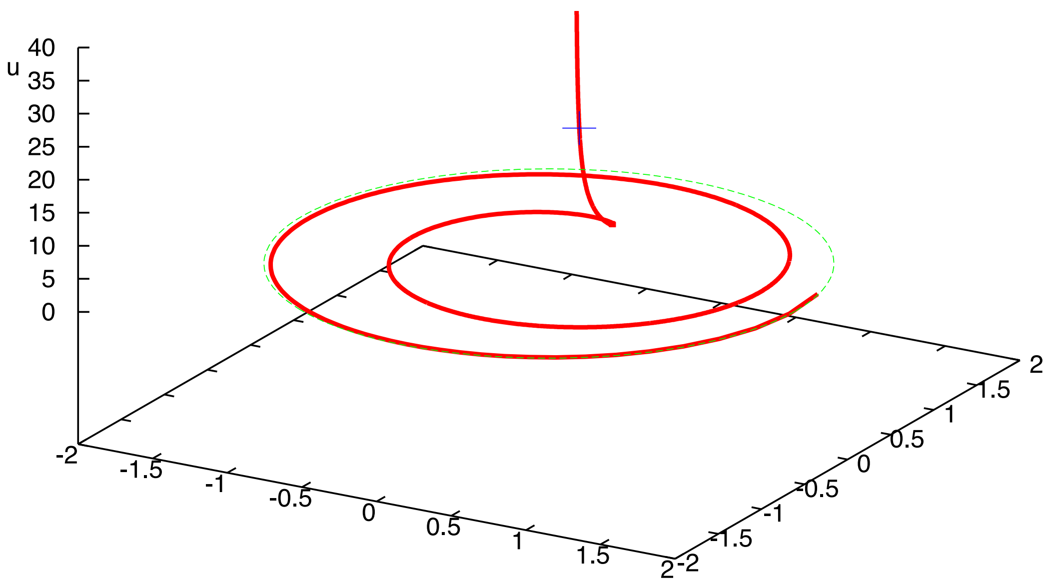
- overall power radiated:

$$P^{\lambda\ll1} = \frac{5\lambda}{12}\frac{a^2}{2\pi} \quad \longleftrightarrow \quad P^{\lambda\gg1} = \sqrt{\lambda}\,\frac{a^2}{2\pi}$$

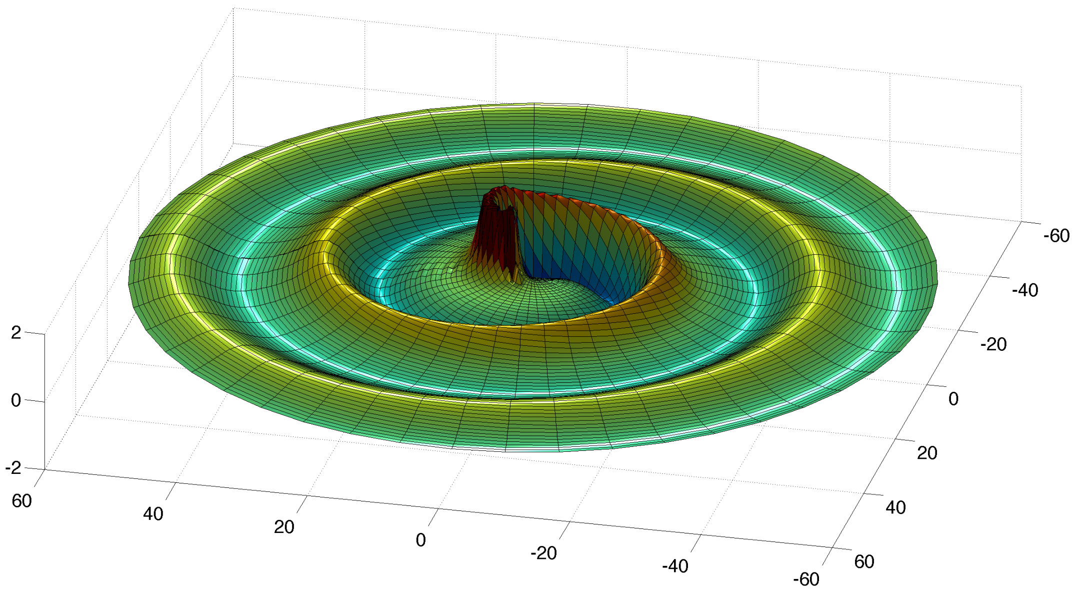


different parametric dependence on coupling, but similar profiles

POSSIBLE EXTENSIONS



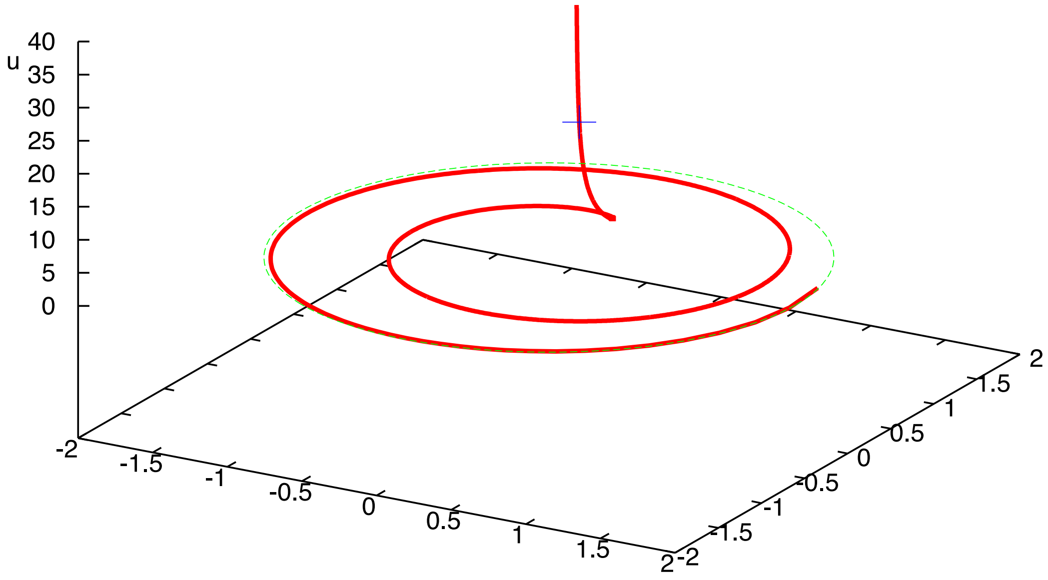
$$\omega_0^2 \gamma^3 \ll \pi^2 T^2$$



K. Fadafan, H. Liu, K. Rajagopal, U. Wiedemann [arXiv:0809.2869]

POSSIBLE EXTENSIONS

- finite temperature: competition between radiation and thermal diffusion - - laboratory for jet quenching?!



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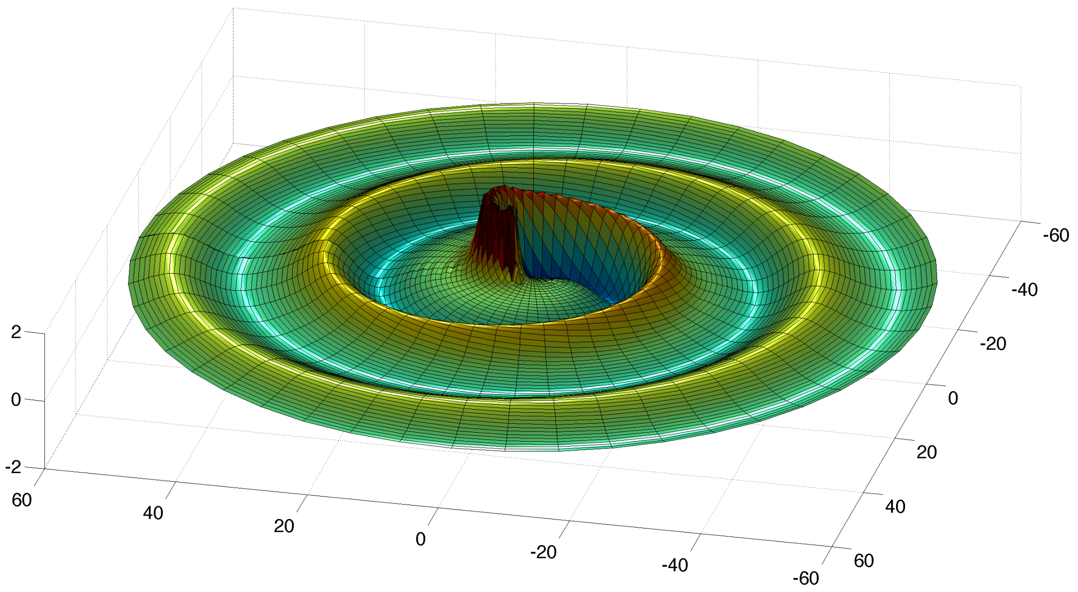

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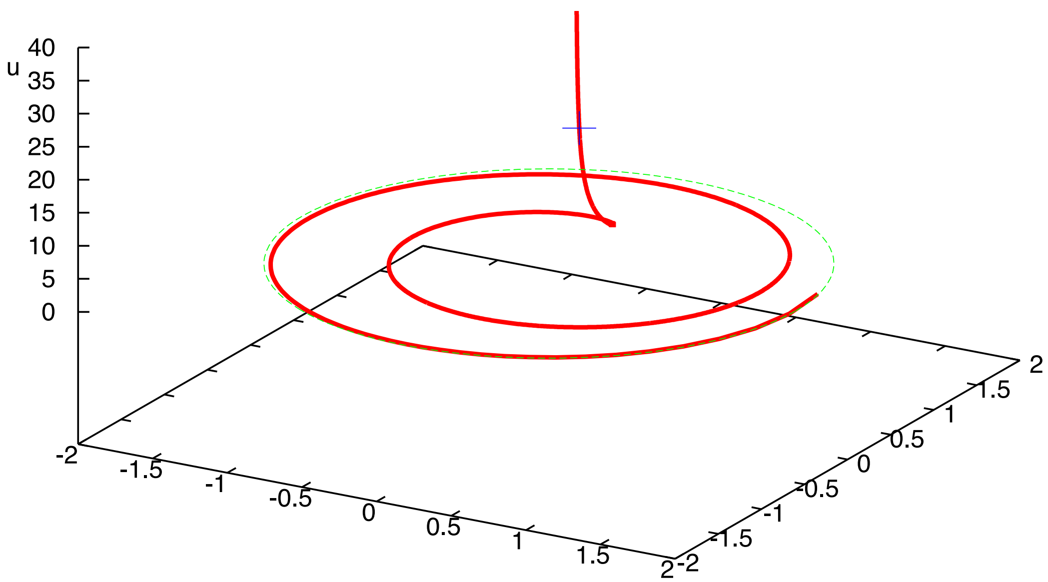

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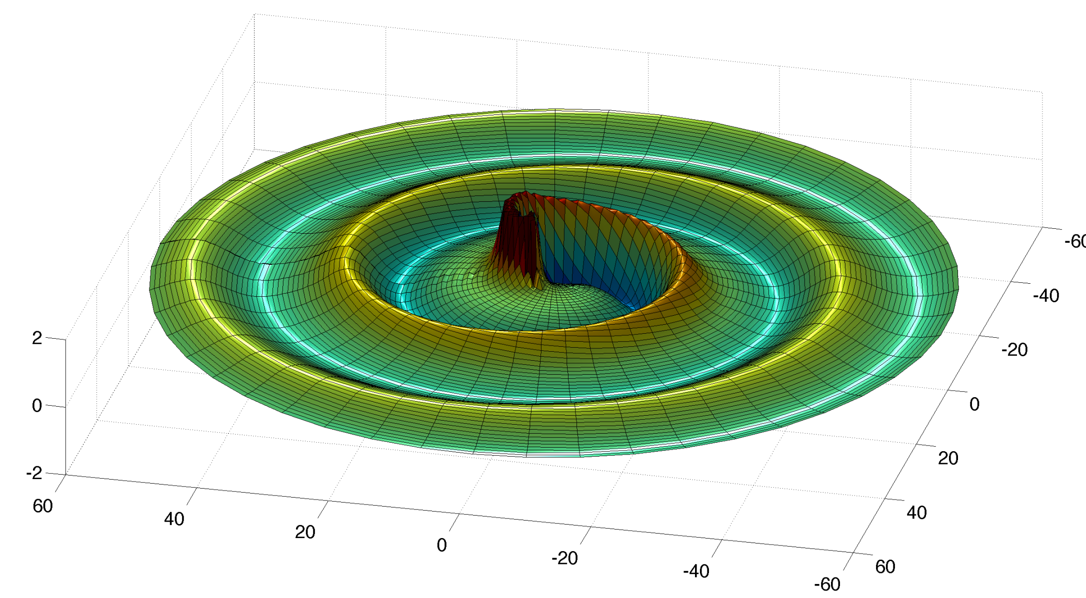

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- solution for general acceleration at T=0 - unlikely (due to memory effects)

$$\begin{aligned}\mathcal{E}(t,\mathbf{r}) \;=\; & \frac{L^3}{\pi}\int d^4r' \int_0^\infty du\;\theta(t-t')\Big[\,(4ut_{00}-t_{M5}\nabla'_M\mathcal{W})\,\frac{\delta''(\mathcal{W})}{u^2} \\ & +|\mathbf{r}-\mathbf{r}'|^2\,(4t_{00}-4t_{55}+2t_{ii})\,\frac{\delta'''(\mathcal{W})}{3u} - (t_{ij}\nabla'_i\mathcal{W}\nabla'_j\mathcal{W})\,\frac{\delta'''(\mathcal{W})}{2u}\Big]\end{aligned}$$

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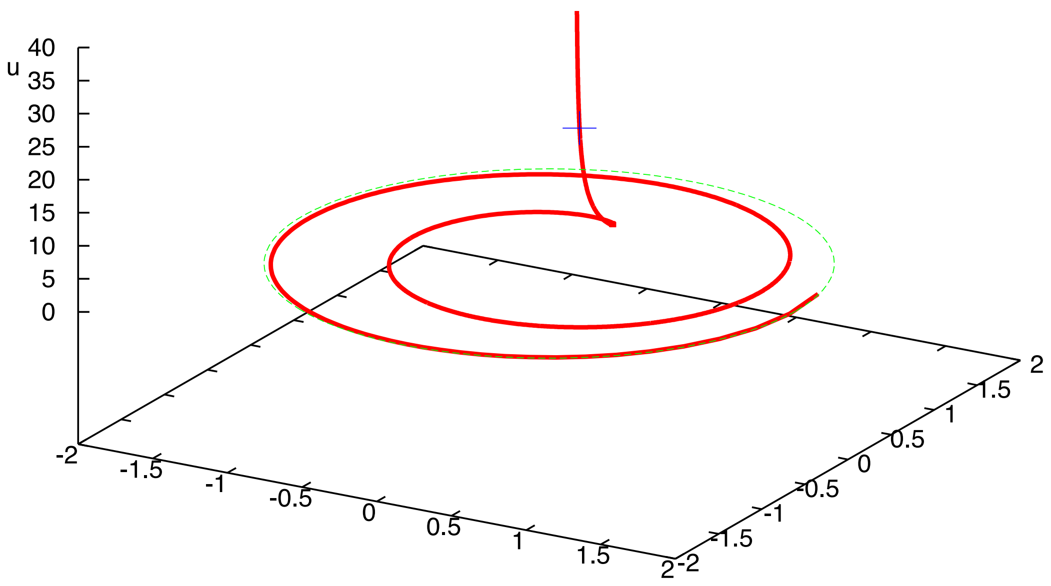

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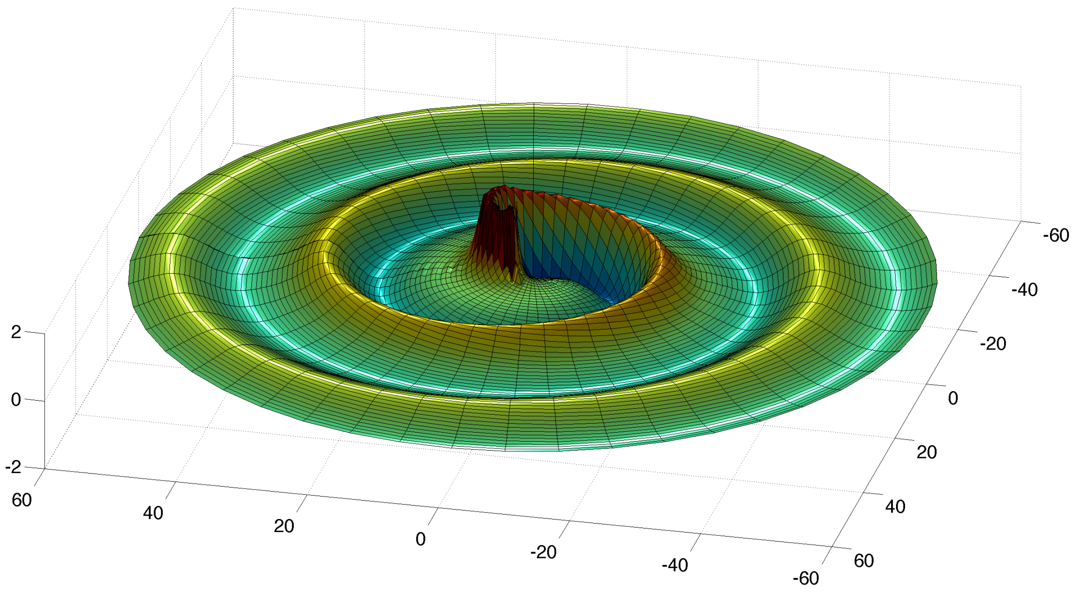

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- (late-time) model for single jets - study $\mathcal{E}(t,\mathbf{x})$ for classes of string profiles

SUMMARY

- gauge/gravity duality nice tool to study real-time evolution
- synchrotron radiation solved analytically

➡ no isotropization observed

- hope/ideas for modeling jets

Thank you for your attention!