Strings and numeric strings

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Outline

Physical motivation

Lattice QCD

Results of this work

Conclusions and outlook

Based on:

- M.P., *Thermodynamics of the QCD plasma and the large-N limit*, Phys. Rev. Lett. **103** 232001 (2009), `[arXiv:0907.3719 [hep-lat]]`

Outline

The physical problem

- Due to asymptotic freedom in non-Abelian gauge theories **[Gross and Wilczek, 1973; Politzer, 1973]**, hadronic matter is expected to undergo a change of state to a deconfined phase at sufficiently high temperatures or densities **[Cabibbo and Parisi, 1975; Collins and Perry, 1974]**.
- Extensive experimental investigation through heavy ion collisions since the Eighties: first at AGS (BNL) and SPS (CERN), then at RHIC (BNL)
- Present experimental evidence from SPS and RHIC: a ‘A new state of matter’ has been created **[Heinz and Jacob, 2000, Arsene *et al.*, 2004; Back *et al.*, 2004; Adcox *et al.*, 2004; Adams *et al.*, 2005]** . . .
- . . . which behaves as an almost ideal fluid **[Kolb and Heinz, 2003]** (‘The most perfect liquid observed in Nature’)
- Program to be continued with forthcoming experiments at LHC (CERN) and FAIR (GSI)
- However, the theoretical understanding of the QCD plasma **[Rischke, 2003]** is still far from complete . . .

Theoretical approaches - I

- Relativistic fluidodynamics is a successful phenomenological description **[Romatschke, 2009]**, but is not derived from QCD first principles
- The perturbative approach in thermal gauge theory has a non-trivial mathematical structure, involving odd powers of the coupling **[Kapusta, 1979]**, as well as contributions from diagrams involving arbitrarily large numbers of loops **[Linde, 1980; Gross, Pisarski and Yaffe, 1980]** . . .
- . . . and shows poor convergence at the temperatures probed in experiments **[Kajantie, Laine, Rummukainen and Schröder, 2002]**
- Dimensional reduction **[Ginsparg, 1980; Appelquist and Pisarski, 1981]** to EQCD and MQCD **[Braaten and Nieto, 1995]**, hard-thermal loop resummations **[Blaizot and Iancu, 2002]**, and other effective theory approaches **[Kraemmer and Rebhan, 2004]**
- Analytical progress in strongly interacting gauge theories: the AdS/CFT conjecture **[Maldacena, 1997]** and related theories as possible models for the non-perturbative features of QCD, including spectral **[Erdmenger, Evans, Kirsch and Threlfall, 2007]** and thermal properties **[Gubser and Karch, 2009]**
- In the large-N limit, the Maldacena conjecture relates a strongly interacting gauge theory to the classical limit of a gravity model

Theoretical approaches - II

- Numerical approach: Computer simulations of QCD regularized on a lattice allow first-principle, non-perturbative studies of the finite-temperature plasma
- The lattice determination of equilibrium thermodynamic properties in $\mathrm{SU}(3)$ gauge theory is regarded as a solved problem **[Boyd *et al.*, 1996]**
- In recent years, finite-temperature lattice QCD has steadily progressed towards parameters corresponding to the physical point **[Karsch *et al.*, 2000; Ali Khan *et al.*, 2001; Aoki *et al.*, 2005; Bernard *et al.*, 2006; Cheng *et al.*, 2007; Bazavov *et al.*, 2009]**—see also **[DeTar and Heller, 2009]** for a review of recent results
- Goals of this work: High-precision determination of the equilibrium thermodynamic properties in $\mathrm{SU}(N \geq 3)$ Yang-Mills theories, comparison with holographic models, investigation of possible non-perturbative contributions to the trace anomaly—see also **[Bringoltz and Teper, 2005]** and **[Datta and Gupta, 2009]** for related works

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Motivation, definition and scope - I

- QCD is the regnant theory of strong subnuclear interactions
- Very strong experimental evidence from processes at high energies, where, thanks to asymptotic freedom, theorists can rely on perturbative calculations
- On the contrary, the main qualitative features of the low-energy domain of hadron physics (confinement and chiral symmetry breaking) are non-perturbative in nature
- Lattice QCD **[Wilson, 1974]** is *the* non-perturbative regularization of QCD
- Continuum fields replaced by a discrete set of variables
- Divergent integrals regularized through a finite cutoff, inversely proportional to the lattice spacing a
- Amenable to numerical simulations
- Retains invariance under gauge transformations and a discrete subgroup of rotations and translations
- Continuum physics recovered for $\lim_{a\to 0} \lim_{V\to\infty}$

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Motivation, definition and scope - II

- A systematically improvable approach—no uncontrolled approximation involved
- Intrinsically non-perturbative, allows one to extract *first principle* QCD predictions at strong coupling (*e.g.* hadron spectrum, running coupling α_S, quark masses, hadronic matrix elements relevant for the CKM matrix, deconfinement at high temperature . . .)
- Based on the Feynman path integral formulation in Euclidean spacetime—real-time processes, non-equilibrium thermal quantities, *et c.* are typically dealt with indirectly
- Technically challenging ('sign problem') for systems at finite density

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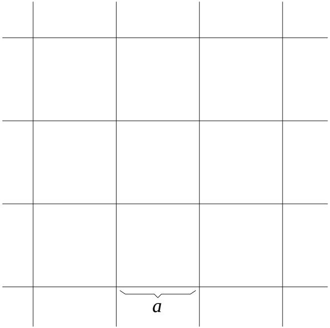
Lattice formulation for Yang-Mills theories

- Discretize a finite hypervolume in Euclidean spacetime by a regular grid with finite spacing a



- Transcribe gauge d.o.f. to lattice elements, build lattice observables
- Isotropic lattice action for the Yang-Mills theory:

$$S = \beta \sum_{\Box} \left(1 - \frac{1}{N} \mathrm{Re\,Tr}\, U_{\Box} \right), \quad \text{with: } \beta = \frac{2N}{g_0^2} a^{d-4}$$

- Invariant under gauge transformations $U_\mu(x) \to g(x) U_\mu(x) g^\dagger(x + a\hat{\mu})$
- Naïve continuum limit: The lattice action is equivalent to the continuum action, up to $O(a^2)$ corrections
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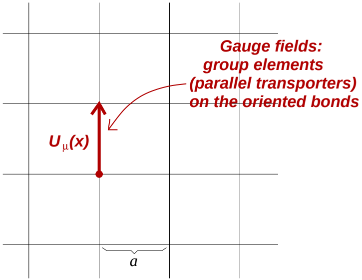

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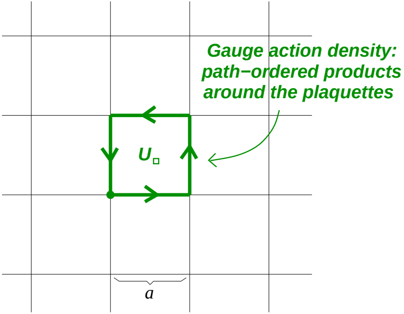

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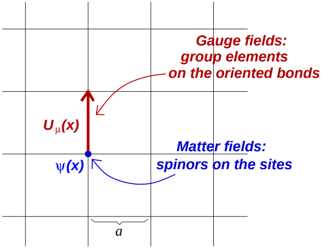
Including fermions

- Fermionic matter fields defined on the lattice sizes



- Fermionic contribution to the lattice action:

$$\sum_{q=1}^{N_f} \sum_x \left\{ m_q \bar{\psi}(x)\psi(x) + \frac{1}{2a} \sum_\mu \bar{\psi}(x)\gamma_\mu \left[U_\mu(x)\psi(x+a\hat{\mu}) - U^\dagger_\mu(x-a\hat{\mu})\psi(x-a\hat{\mu}) \right] \right\}$$

- Exact analytical integration of the Grassmann variables leads to the determinant of the Dirac matrix: large computational overhead
- Naïve lattice discretization yields 2^d *doublers*:

$$m_q + (i/a) \sum_\mu \gamma_\mu \sin(p_\mu a)$$

- Wilson's fix: Include a higher-dimensional operator to remove 2^d doublers (but chiral symmetry for $m_q = 0$ is lost)
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- Lattice QCD provides an *effective continuum theory* for the low-energy physics

$$S_{\text{eff}} = \int d^4x \left[\mathcal{L}_0(x) + a\mathcal{L}_1(x) + a^2\mathcal{L}_2(x) + \dots \right]$$

- The physical value of the spacing a is set using a low-energy observable (*e.g.*, the asymptotic slope σ of the potential between infinitely heavy external sources)
- Lattice renormalization: hadronic renormalization schemes, mean-field improved perturbation theory **[Parisi, 1980; Lepage and Mackenzie, 1993]**, recursive finite-size technique **[Lüscher, Weisz and Wolff, 1991]**
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Thermodynamics on the lattice

- Thermal averages from simulations on a lattice with compactified Euclidean time direction, with $T = 1/(aN_\tau)$
- Pressure $p(T)$ via the 'integral method' **[Engels *et al.*, 1990]**:

$$
\begin{aligned}
p &= T\frac{\partial}{\partial V}\log \mathcal{Z} \simeq \frac{T}{V}\log \mathcal{Z} = \frac{1}{a^4 N_s^3 N_\tau}\int_{\beta_0}^{\beta} d\beta' \frac{\partial \log \mathcal{Z}}{\partial \beta'} \\
&= \frac{6}{a^4}\int_{\beta_0}^{\beta} d\beta' \left(\langle U_\Box\rangle_T - \langle U_\Box\rangle_0\right)
\end{aligned}
$$

Thermodynamics on the lattice

- Other thermodynamic observables obtained from indirect measurements
  - Trace of the stress tensor $\Delta = \epsilon - 3p$:

$$\Delta = T^5 \frac{\partial}{\partial T} \frac{p}{T^4} = \frac{6}{a^4} \frac{\partial \beta}{\partial \log a} \left(\langle U_\Box \rangle_0 - \langle U_\Box \rangle_T \right)$$

  - Energy density:

$$\epsilon = \frac{T^2}{V} \frac{\partial}{\partial T} \log \mathcal{Z} = \Delta + 3p$$

  - Entropy density:

$$s = \frac{S}{V} = \frac{\epsilon - f}{T} = \frac{\Delta + 4p}{T}$$

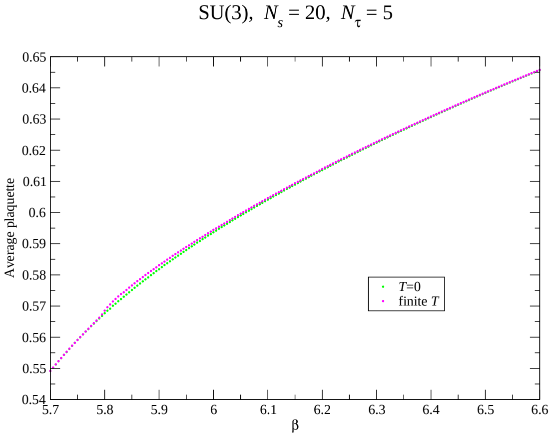
Outline

Simulation details

- Lattice sizes $N_s^3 \times N_\tau$, with $N_s = 20$ or 16, and $N_\tau = 5$
- Simulation algorithm: heat-bath **[Kennedy and Pendleton, 1985]** for $\mathrm{SU}(2)$ subgroups **[Cabibbo and Marinari, 1982]** and full-$\mathrm{SU}(N)$ overrelaxation **[Kiskis, Narayanan and Neuberger, 2003; Dürr, 2004; de Forcrand and Jahn, 2005]**
- Cross-check with $T = 0$ simulations run using the Chroma suite **[Edwards and Joó, 2004]**
- Physical scale for $\mathrm{SU}(3)$ set using r_0 **[Necco and Sommer, 2001]**
- Physical scale for $\mathrm{SU}(N > 3)$ set using known values for the string tension σ **[Lucini, Teper and Wenger, 2004; Lucini and Teper, 2001]** in combination with the 3-loop lattice β-function **[Allés, Feo and Panagopoulos, 1997; Allton, Teper and Trivini, 2008]** in the mean-field improved lattice scheme **[Parisi, 1980; Lepage and Mackenzie, 1993]**

Measurements of the plaquette

- High precision determination of $(\langle U_\Box \rangle_T - \langle U_\Box \rangle_0)$ required



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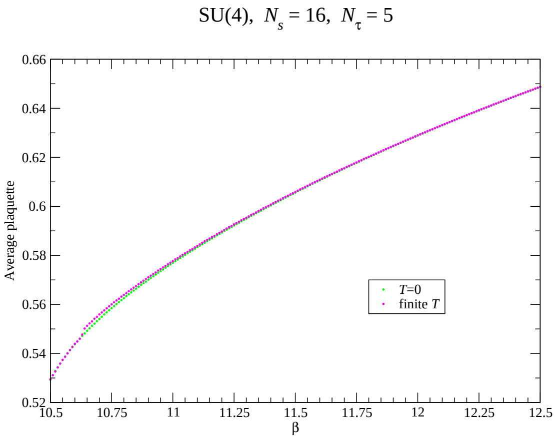
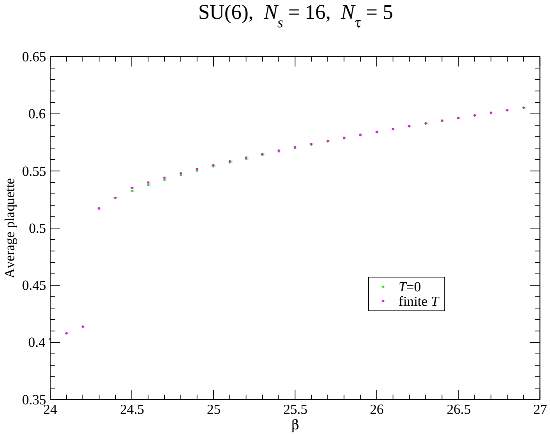


Measurements of the plaquette

- High precision determination of $(\langle U_\Box \rangle_T - \langle U_\Box \rangle_0)$ required
- Data reveal a strong first order bulk transition for $\mathrm{SU}(N \geq 4)$



Improved holographic QCD model *vs.* lattice data - I

- Kiritsis and collaborators **[Gürsoy, Kiritsis, Mazzanti and Nitti, 2008]** proposed an AdS/QCD model based on a 5D Einstein-dilaton gravity theory, with the fifth direction dual to the energy scale of the $\mathrm{SU}(N)$ gauge theory
- Field content on the gravity side: metric (dual to the $\mathrm{SU}(N)$ energy-momentum tensor), the dilaton (dual to the trace of F^2) and the axion (dual to the trace of $F\tilde{F}$)
- Gravity action:

$$S_{IHQCD} = -M_P^3 N^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} (\partial \Phi)^2 + V(\lambda) \right] + 2M_P^3 N^2 \int_{\partial M} d^4x \sqrt{h}\, K$$

- Φ is the dilaton field, $\lambda = \exp(\Phi)$ is identified with the running 't Hooft coupling of the dual $\mathrm{SU}(N)$ YM theory
- The effective five-dimensional Newton constant $G_5 = 1/\left(16\pi M_P^3 N^2\right)$ becomes small in the large-N limit

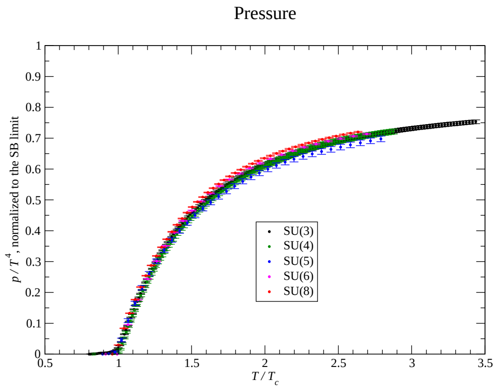
Improved holographic QCD model *vs.* lattice data - II

- Dilaton potential $V(\lambda)$ defined by requiring asymptotic freedom with a logarithmically running coupling in the UV and linear confinement in the IR of the gauge theory; a possible *Ansatz* is:

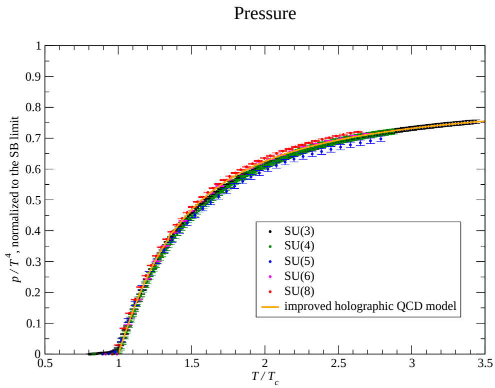
$$V(\lambda) = \frac{12}{\ell^2}\left[1 + V_0\lambda + V_1\lambda^{4/3}\sqrt{\log\left(1 + V_2\lambda^{4/3} + V_3\lambda^2\right)}\right], \tag{1}$$

  where ℓ is the AdS scale (overall normalization)
- V_0, V_1, V_2 and V_3 are free parameters: two of them can be fixed by imposing that the dual model reproduces the first two (scheme-independent) perturbative coefficients of the $\mathrm{SU}(N)$ β-function, and one is left with two independent parameters

Improved holographic QCD model *vs.* lattice data - III



Improved holographic QCD model *vs.* lattice data - III



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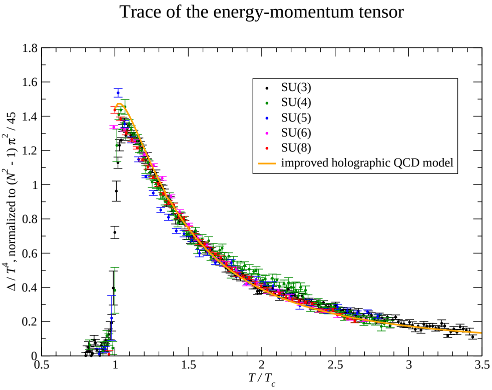

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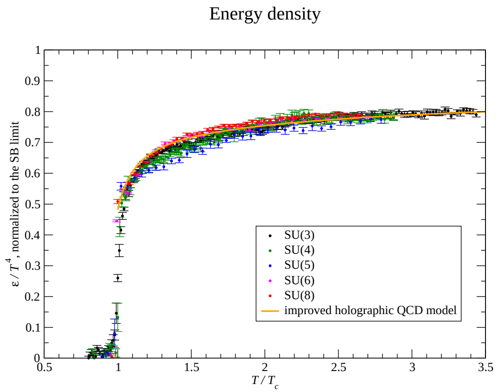

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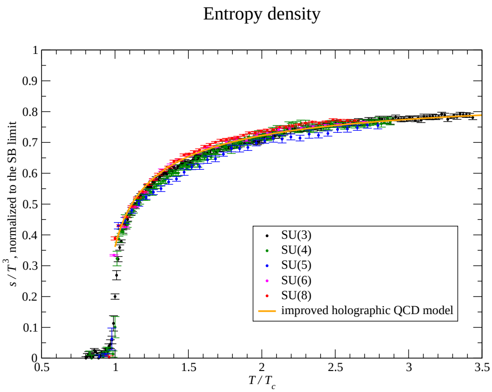
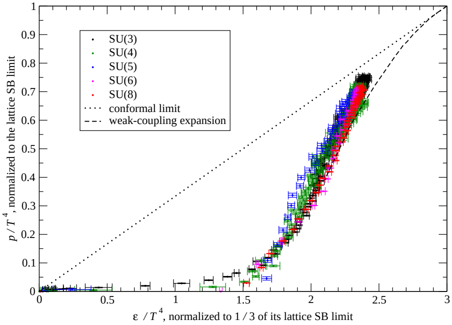


AdS/CFT *vs.* lattice data in a 'quasi-conformal' regime

For $T \simeq 3T_c$, the lattice results reveal that the deconfined plasma, while still strongly interacting and far from the Stefan-Boltzmann limit, approaches a scale-invariant regime . . .

$p(\varepsilon)$ equation of state and approach to conformality

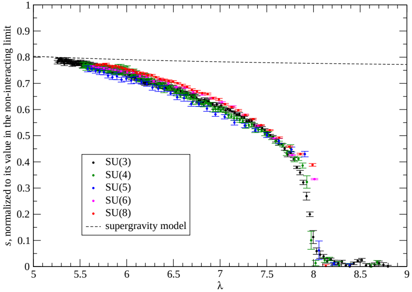


AdS/CFT *vs.* lattice data in a 'quasi-conformal' regime

...in which the entropy density is comparable with the supergravity prediction for $\mathcal{N} = 4$ SYM **[Gubser, Klebanov and Tseytlin, 1998]**

$$\frac{s}{s_0} = \frac{3}{4} + \frac{45}{32}\zeta(3)(2\lambda)^{-3/2} + \dots$$

Entropy density *vs.* 't Hooft coupling



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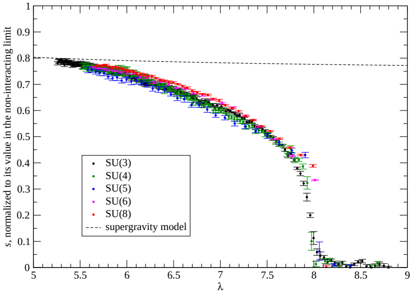
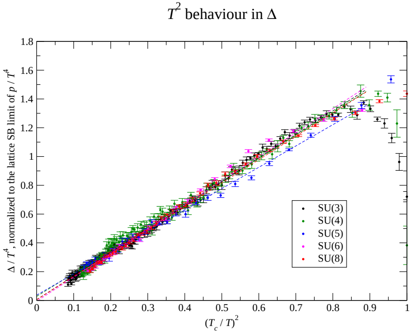


Note that a comparison of $\mathcal{N} = 4$ SYM and full-QCD lattice results for the drag force on heavy quarks also yields $\lambda \simeq 5.5$ **[Gubser, 2006]**

T^2 contributions to the trace anomaly?

The trace anomaly reveals a characteristic T^2-behavior, possibly of non-perturbative origin **[Megías, Ruiz Arriola and Salcedo, 2003; Pisarski, 2006; Andreev, 2007]**

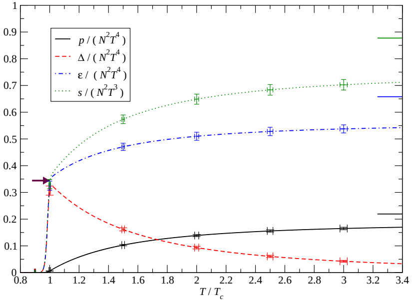


Extrapolation to $N \to \infty$

Based on the parametrization **[Bazavov *et al.*, 2009]**:

$$\frac{\Delta}{T^4} = \frac{\pi^2}{45}(N^2-1)\cdot\left(1-\left\{1+\exp\left[\frac{(T/T_c)-f_1}{f_2}\right]\right\}^{-2}\right)\left(f_3\frac{T_c^2}{T^2}+f_4\frac{T_c^4}{T^4}\right)$$

Extrapolation to the large-N limit



Outline

Conclusions

- Equilibrium thermodynamic observables in $\mathrm{SU}(N)$ YM theories at temperatures $0.8T_c \leq T \leq 3.4T_c$ show a mild dependence on N
- Successful comparison with the IHQCD model
- Quasi-conformal regime of YM and $\mathcal{N} = 4$ SYM predictions—Can lattice data help to pin down realistic parameters for AdS/CFT models of the sQGP? **[Noronha, Gyulassy and Torrieri, 2009]**
- Δ seems to have a T^2 dependence also at large N
- Extrapolation to the $N \to \infty$ limit

Projects for the future - I

- $\mathrm{SU}(N)$ screening masses and spatial string tensions, comparisons with AdS/CFT **[Bak, Karch and Yaffe, 2007]** and with IHQCD **[Alanen, Kajantie and Suur-Uski, 2009]**
- $\mathrm{Tr}F^2$ correlators and dilaton potential **[Noronha, 2009]**
- Observables related to thermodynamic fluctuations: specific heat, speed of sound *et c.* **[Gavai, Gupta and Mukherjee, 2005]**—relevant for the elliptic flow **[Ollitrault, 1992; Teaney, Lauret and Shuryak, 2001]**
- Renormalized Polyakov loops in various representations **[Damgaard, 1987; Damgaard and Hasenbusch, 1994; Dumitru, Hatta, Lenaghan, Orginos and Pisarski, 2004; Gupta, Hübner and Kaczmarek, 2008]**
- Transport coefficients **[Meyer, 2007]**

Projects for the future - II

- High-precision thermodynamics for $\mathrm{SU}(N)$ theories in 3D (work in progress with Caselle, Castagnini, Feo and Gliozzi; see also **[Bialas, Daniel, Morel and Petersson, 2008]**)