

# Thermodynamics of IRFP\* and Technicolor theories from gauge/gravity duality

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with

Kajantie arXiv:0912.4128 and  
Kajantie, Tuominen work in progress

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\*Infrared fixed point

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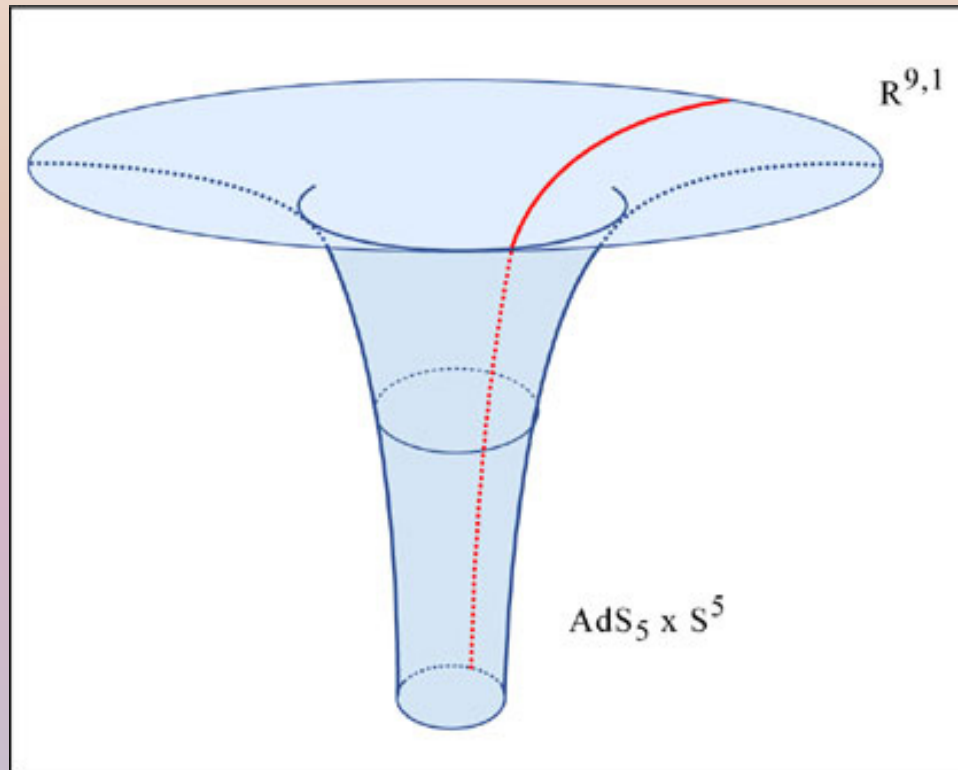
# 1. Motivation

- Study thermodynamics of field theories which have a strong coupling regime: QCD, IRFP, Technicolor...
- Strong coupling  $\rightarrow$  perturbation theory does not work
- Methods: Lattice, gauge/gravity duality...

## Aim of this talk

Introduce the 5D Einstein-Dilaton model and use it to calculate thermodynamics for various field theories.

## 2. Gauge/Gravity duality



# Gauge/gravity duality or Holography

- Idea: pure gauge theory is dual to higher dimensional theory including gravity
- First realization by Maldacena, -97:

- String theory on  $AdS_5 \times S^5$  dual to  $d=4$   $SU(N)$   $N=4$  supersymmetric theory

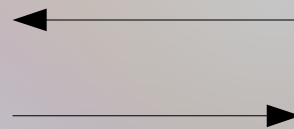
Gravity

Gauge theory

- 't Hooft limit

$$\lambda = g^2 N_c \rightarrow \infty, N_c \rightarrow \infty$$

**Classical gravity**



**Strongly coupled quantum theory**

# IHQCD: Improved Holographic QCD/SU(N). *Gursoy, Kiritsis, Mazzanti, Nitti...*

- Pure gravity in AdS is dual to a strongly coupled conformal field theory but realistic field theories have also a weak coupling regime and are non-conformal.
- In IHQCD these two aspects are introduced by extra scalar field

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left( R - \frac{12}{\mathcal{L}^2} \right) \longrightarrow S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left( R - \frac{4}{3} (\partial\Phi)^2 - V(\Phi) \right)$$

Arbitrary. Contains AdS constant

- Ansatz and identifications

$$ds^2 = b(r)^2 \left[ \frac{dr^2}{f(r)} - f(r) dt^2 + dx_m dx^m \right]$$

$$\Phi(r)$$

**Energy scale**

$$b(r)$$

**Coupling constant**

$$\lambda(r) = e^{\Phi(r)}$$

**Vacuum**

$$f(r) = 1$$

**Thermal state**

$$f(r) \neq 1$$

# Equations/Potential fixing

- By the choice of the coupling constant and energy scale we have fixed the scheme we are working in
- Beta function:

$$\frac{d\lambda}{d \ln E} = \frac{d\lambda}{d \ln b} = \beta(\lambda)$$

- Einstein equations:

$$6 \frac{b'^2}{b^2} - 3 \frac{b''}{b} = \frac{4}{3} \frac{\lambda'^2}{\lambda^2}$$

$$\frac{f''}{f'} + 3 \frac{b'}{b} = 0$$

$$6 \frac{b'^2}{b^2} + 3 \frac{b''}{b} + 3 \frac{b'}{b} \frac{f'}{f} = \frac{b^2}{f} V(\lambda)$$

Dilaton potential is chosen so that it produces expected dynamics for dual theory.

For example: QCD must be asymptotically free at UV and confining at IR

# QCD/SU(N) thermodynamics from IHQCD

*Gursoy, Kiritsis, Mazzanti,  
Nitti (0903.2859), Panero (0907.3719)...*

• Potential: 
$$V(\lambda) = \frac{12}{\ell^2} \left\{ 1 + V_0\lambda + V_1\lambda^{4/3} \left[ \log \left( 1 + V_2\lambda^{4/3} + V_3\lambda^2 \right) \right]^{1/2} \right\}$$

Correct UV asymptotics

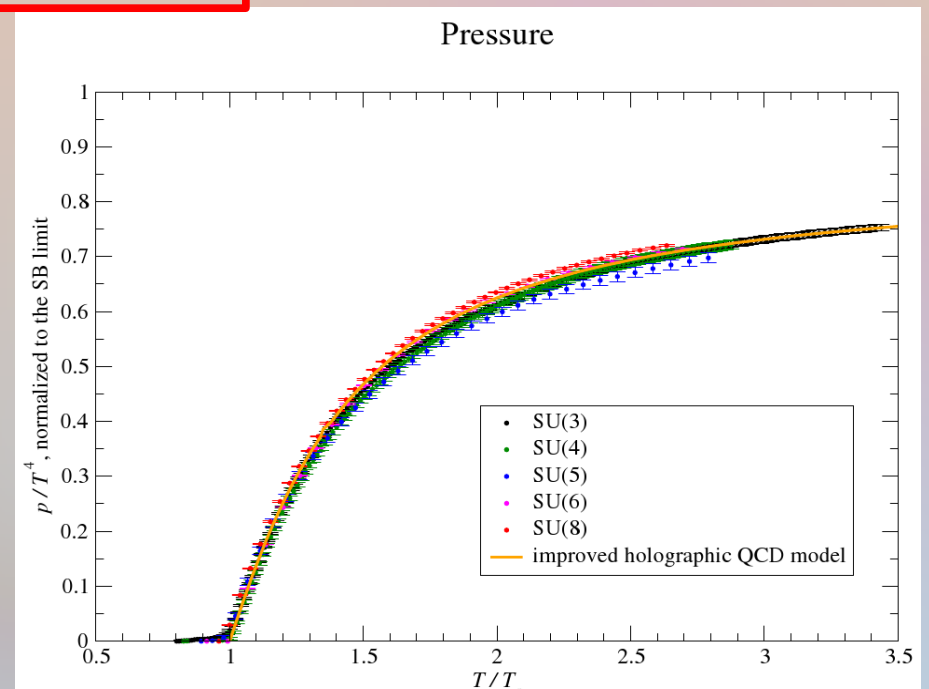
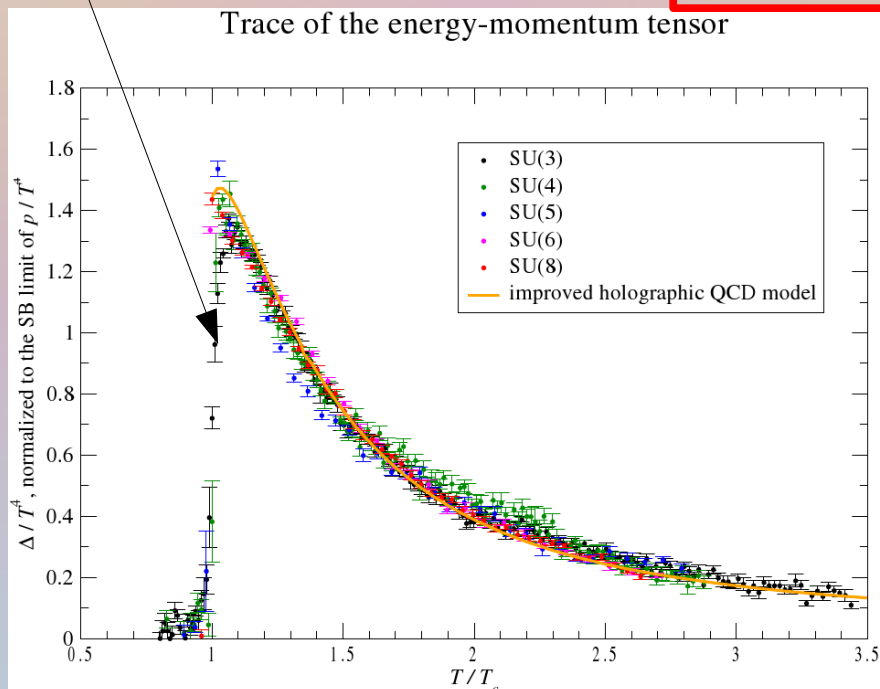
Confining + linear glue ball spectra

First order transition

**Fitted to data**

$$V_1 = 14 \quad V_3 = 170$$

Very good agreement





### 3. Theories with Infrared Fixed Point

# Banks-Zaks

- Definition:

$$\beta(\lambda_{fp}) = 0$$

-Scheme independent:  $\lambda(\lambda_{fp}^*) = \lambda_{fp}$

$$\frac{d\lambda}{d \ln E} = \beta(\lambda) \Rightarrow \beta^*(\lambda^*) = \left( \frac{d\lambda(\lambda^*)}{d\lambda^*} \right)^{-1} \beta(\lambda(\lambda^*))$$

Beta function slope still depends on scheme!

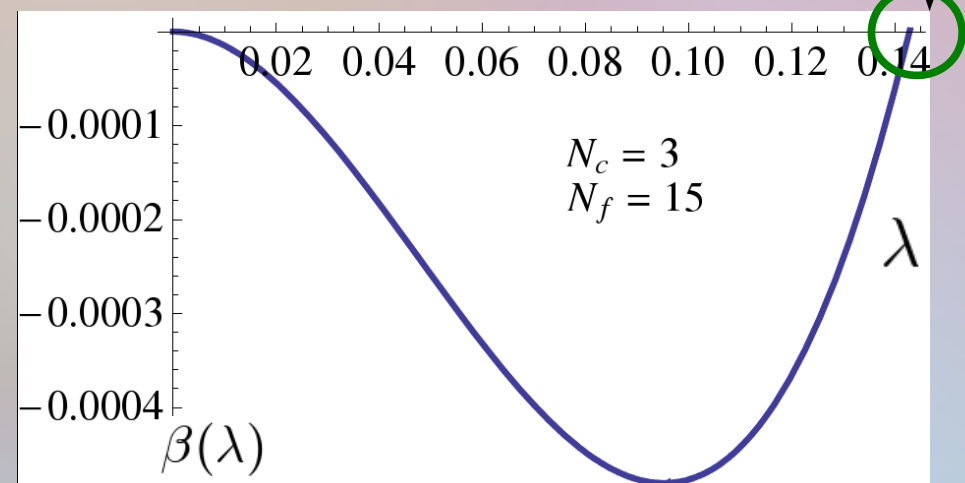
Perturbative FP  $\rightarrow$  in the weak coupling regime

- QCD with large number of flavors: *Banks-Zaks('82)* fixed point

$$\beta(\lambda) = -b_0(N_c, N_f)\lambda^2 - b_1(N_c, N_f)\lambda^3$$

$$\lambda_{fp} = -\frac{b_1(N_c, N_f)}{b_0(N_c, N_f)} > 0$$

$$N_c = 3, N_f \geq 9$$



# Non-perturbative fixed point

- In supersymmetric QCD the beta function can be calculated exactly. *Novikov-Shifman-Vainshtein-Zakharov*
- There has been a proposal for the exact beta function for non-supersymmetric theories. *Ryttov, Sannino...*

Generally these take a form

$$\beta(\lambda) = -2\beta_0\lambda^2 \frac{(1 - v\lambda)}{1 - j\lambda}$$

- Search for theories with IRFP is a hot topic in lattice field theory. *Catterall-Giedt-Sannino-Schneible 0807.0792, Hietanen-Rummukainen-Tuominen 0904.0864, Del Debbio-Lucini-Patella-Pica-Rago 0907.3896*

# WARNING!

- Fermions are crucial for IRFP and TC theories
- Adding effects of the flavor to gauge/gravity duality is non-trivial. *Karch, Katz, Erdmenger, Meyer, O'Bannon, Ammon, Kiritsis, Kerner*
- We assume that 5D Einstein-Dilaton model can be used also with theories where flavor is important and that flavor affects only the form of the dilaton potential

$V(\lambda)$



QCD, infrared  
fixed point,  
Technicolor

# WARNING!

## Note

- For IRFP and TC duals the underlying field theory is unspecified and thermodynamic properties are calculated using the dilaton potential which produces typical beta functions.
- Dilaton potential contains free parameters which have to be restricted by the choice of underlying theory

# Holographic IRFP: thermodynamics

*Alanen, Kajantie, Tuominen...*

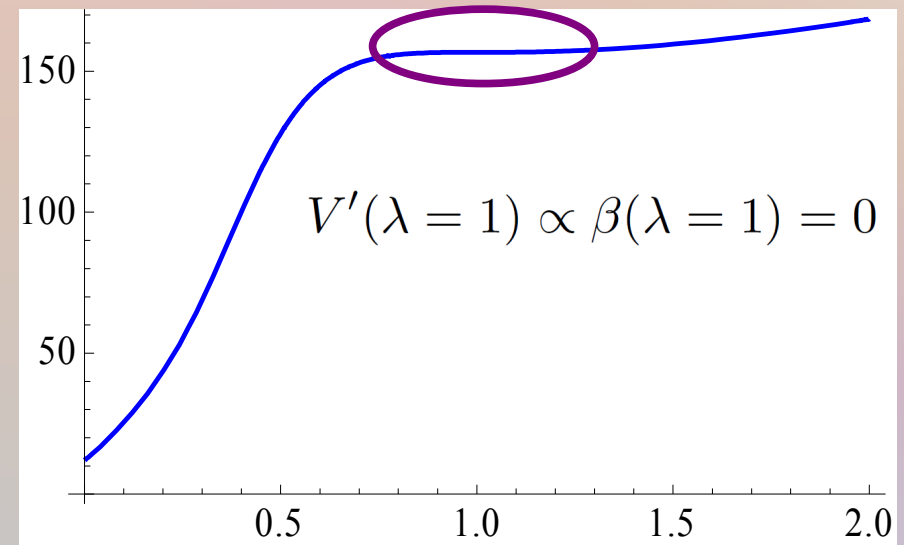
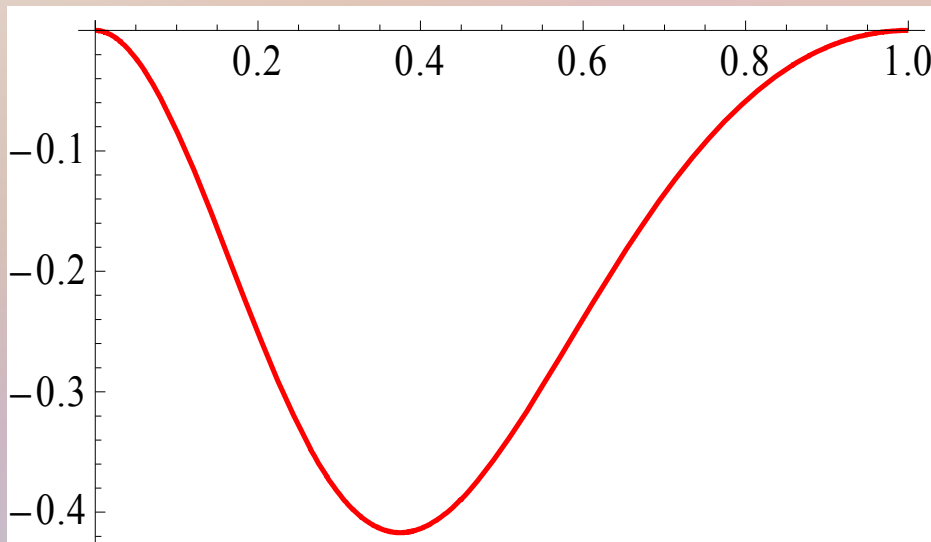
- Calculate dilaton potential from a given beta function

Can be calculated exactly

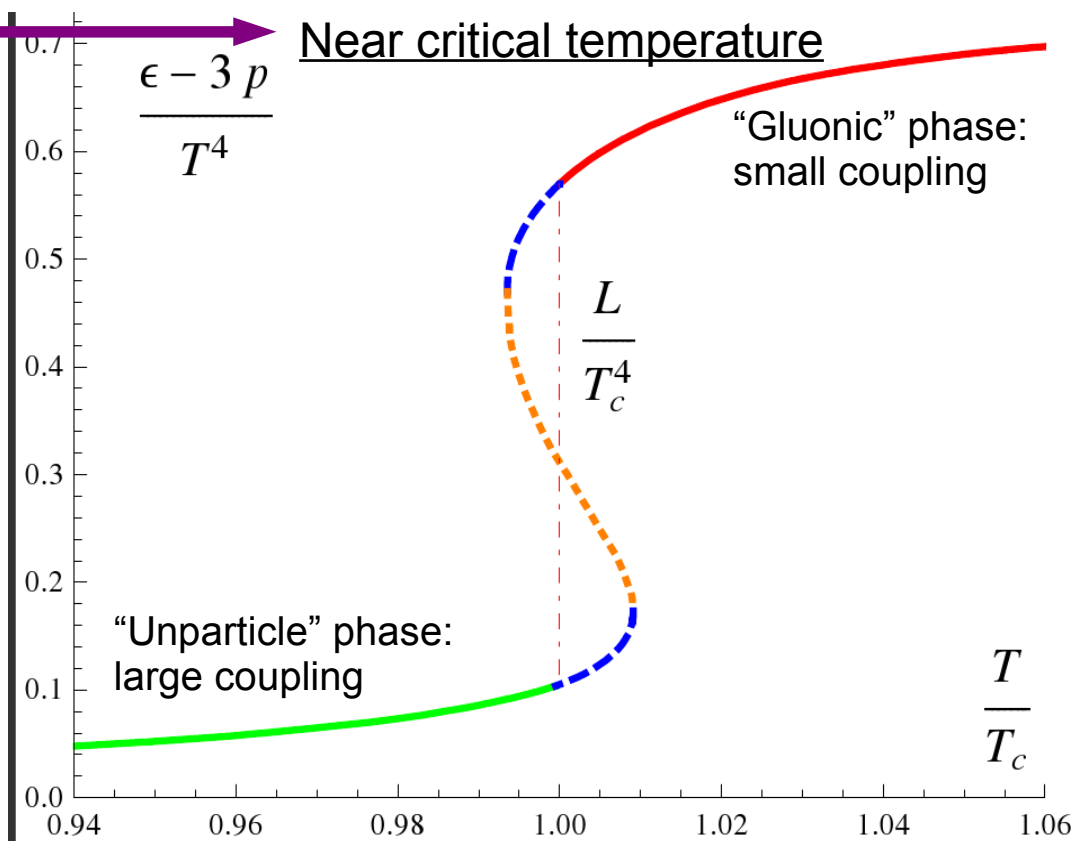
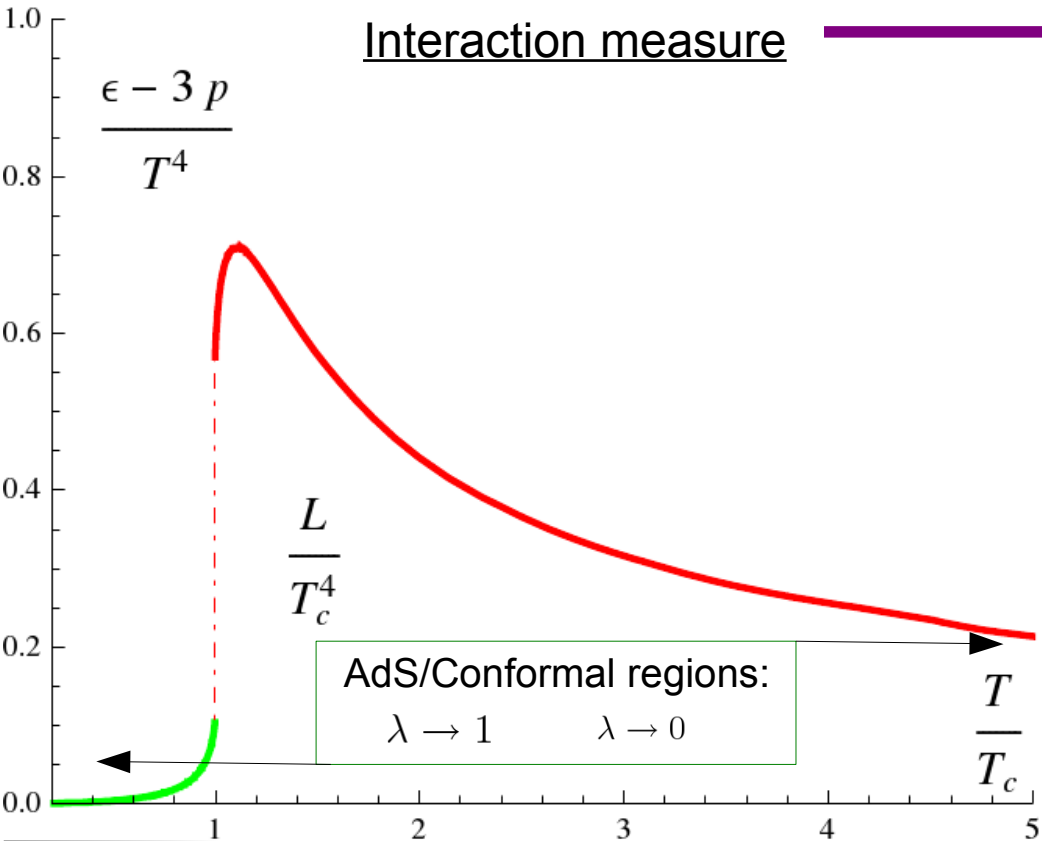
$$\beta(\lambda) = -c\lambda^2 \frac{(1-\lambda)^2}{1+a\lambda^3}$$

$$f(r) = 1$$

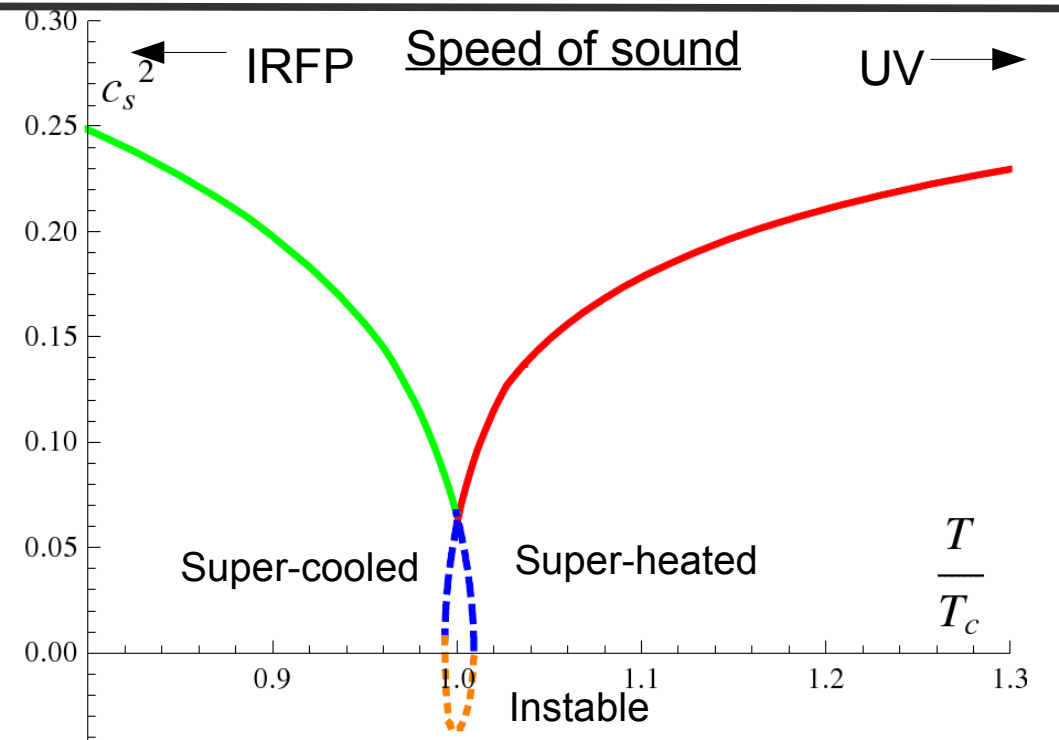
$$V(\lambda) = \frac{12}{\mathcal{L}^2} e^{-\frac{8}{9} \int d\lambda \frac{\beta(\lambda)}{\lambda^2}} \left[ 1 - \left( \frac{\beta(\lambda)}{3\lambda} \right)^2 \right]$$



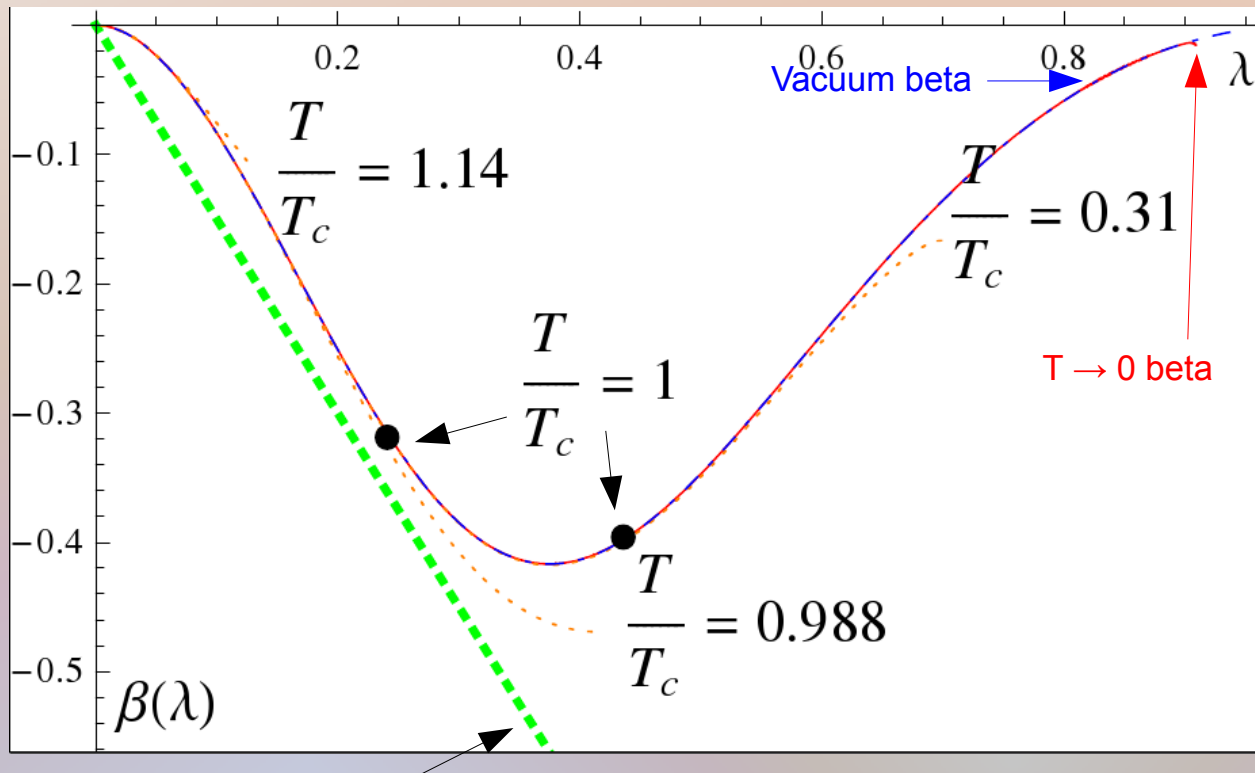
- Search black hole solutions with  $f(r) \neq 1 \rightarrow$  **Thermodynamics**



- QCD-like interaction measure
- First order phase transition between two asymptotically conformal phases: gluonic  $\rightarrow$  unparticle  
 $\epsilon - 3p \rightarrow 0$      $p = \frac{g_{eff}}{90} \pi^2 T^4$
- Super-heated/cooled phases: thermodynamically unfavored
- Unstable: speed of sound squared is negative
- Not confining/deconfining transition! BH phase always dominates. BH  $\rightarrow$  BH transition.



# Beta function



- Asymptotically free at UV
- First order phase transition
- Infrared fixed point
- "Thermal" beta function

"confining line"

$$\beta(\lambda) = -\frac{3}{2}\lambda$$



## 4. Walking Technicolor

# Walking Technicolor

Weinberg,  
Susskind, Eichten,  
Lane, Sannino...

- TC idea: add new strongly coupled theory to EW scale which produces masses for weakly interacting bosons by chiral condensate

VEV for Higgs  $250\text{GeV}$   
 $\langle \phi \rangle^3 = v^3 \sim \Lambda_{EWSB}^3$



Techniquark condensate  
 $\langle \bar{Q}_i Q_i \rangle \propto \Lambda_{TC}^3 \sim \Lambda_{EWSB}^3$

Similar to  
QCD chiral  
condensate  
 $\langle \bar{\psi}_i \psi_i \rangle \propto \Lambda_{QCD}^3$

- ETC idea: masses for SM-fermions

Yukawa interaction  
 $\phi \bar{\psi} \psi$

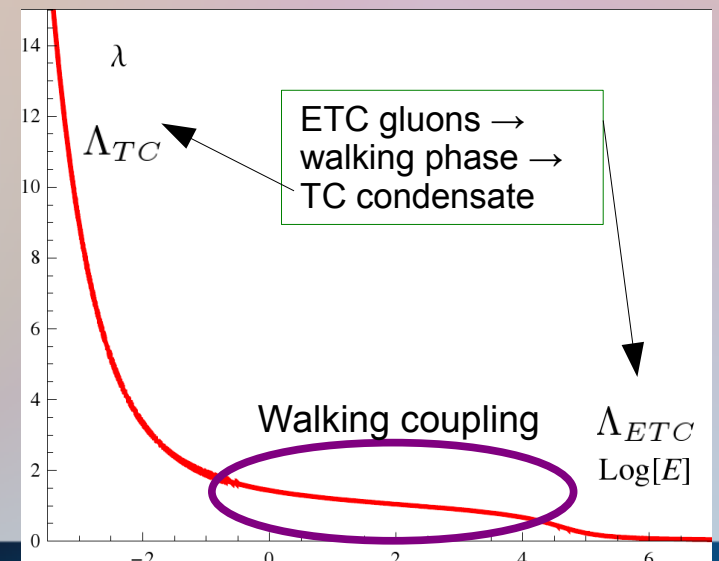
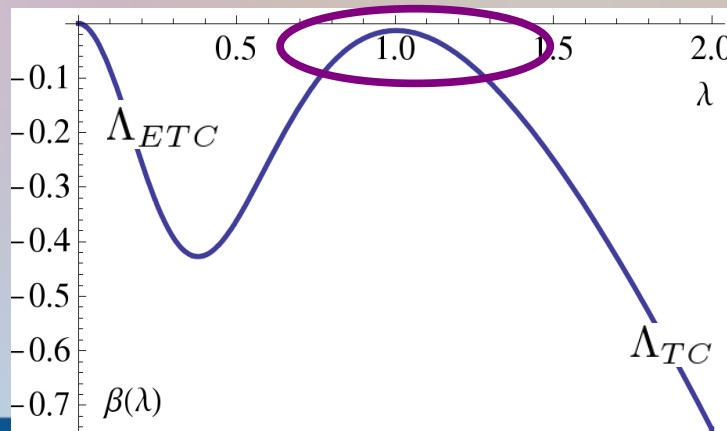


Interactions with techniquarks  
 $\frac{\bar{Q} Q \bar{\psi} \psi}{\Lambda_{ETC}^2}$        $\frac{\Lambda_{TC}}{\Lambda_{ETC}} \lesssim 10^{-3}$

- Large top-quark mass  $\rightarrow$  TC must “walk”

Coupling almost independent of the scale E

$$\frac{d\lambda}{d \ln E} = \beta(\lambda) \approx 0$$



# Holographic Walking TC: thermodynamics

*Alanen, Kajantie, Tuominen...*

Walking beta

$$\beta_0(\lambda) = -c\lambda^2 \frac{[(1-\lambda)^2 + e]}{1+a\lambda^3}$$

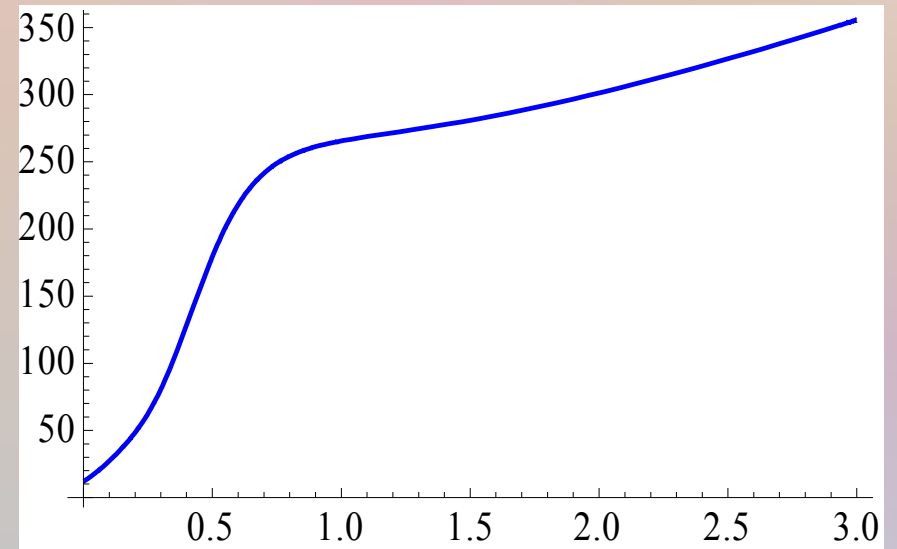
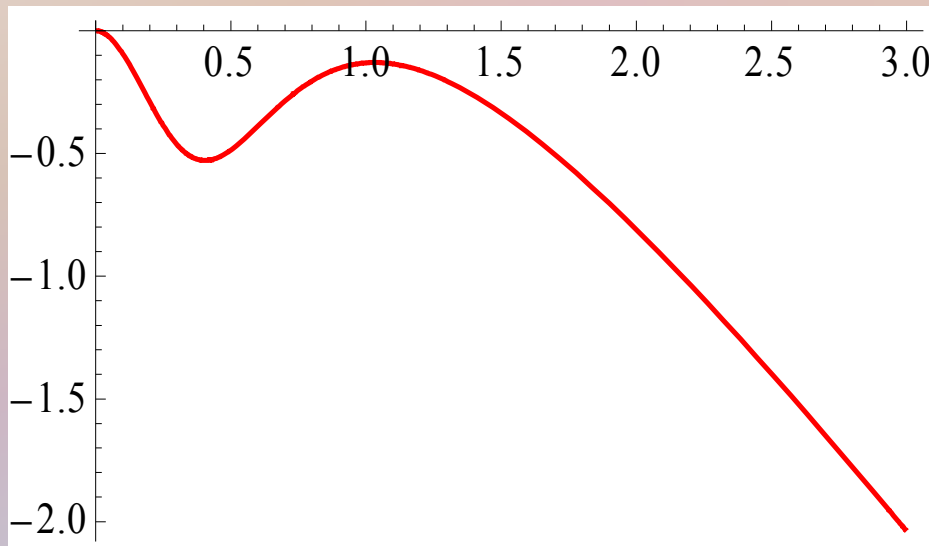
Walking potential (exact)

$$V_0(\lambda)$$

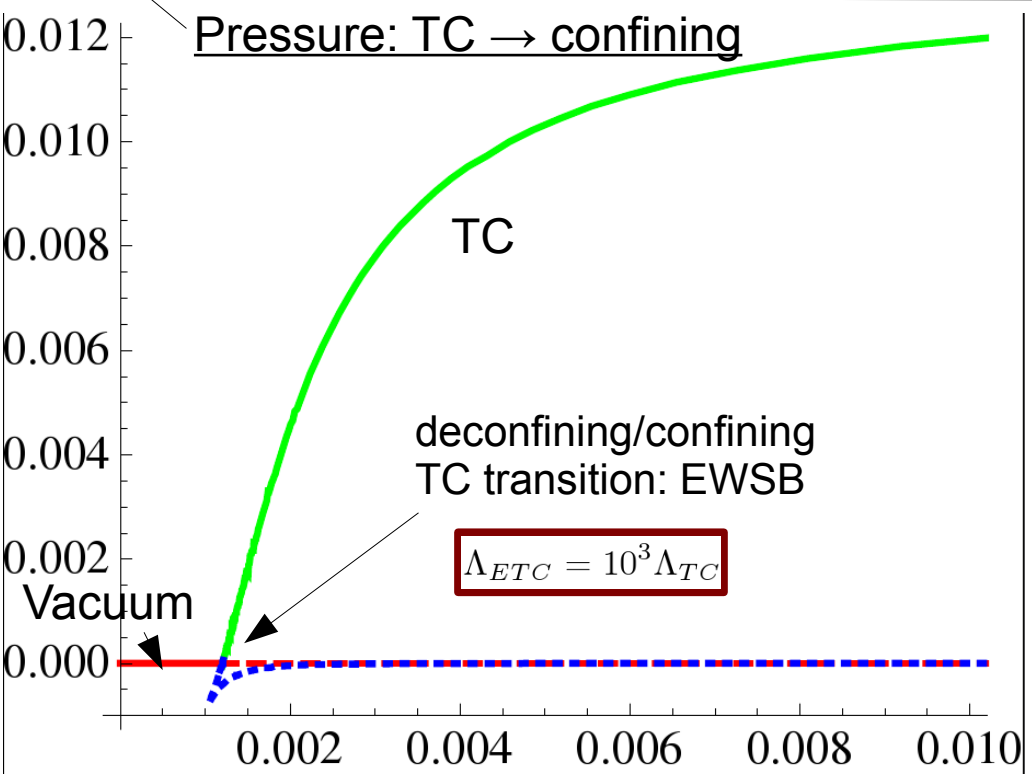
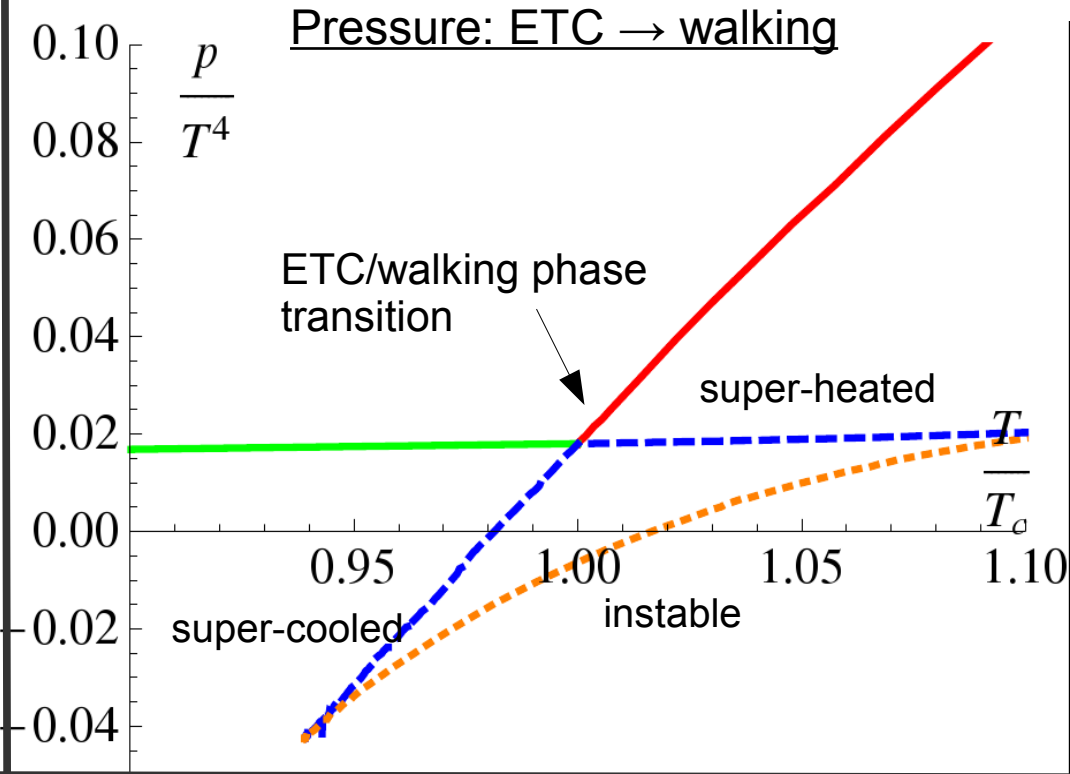
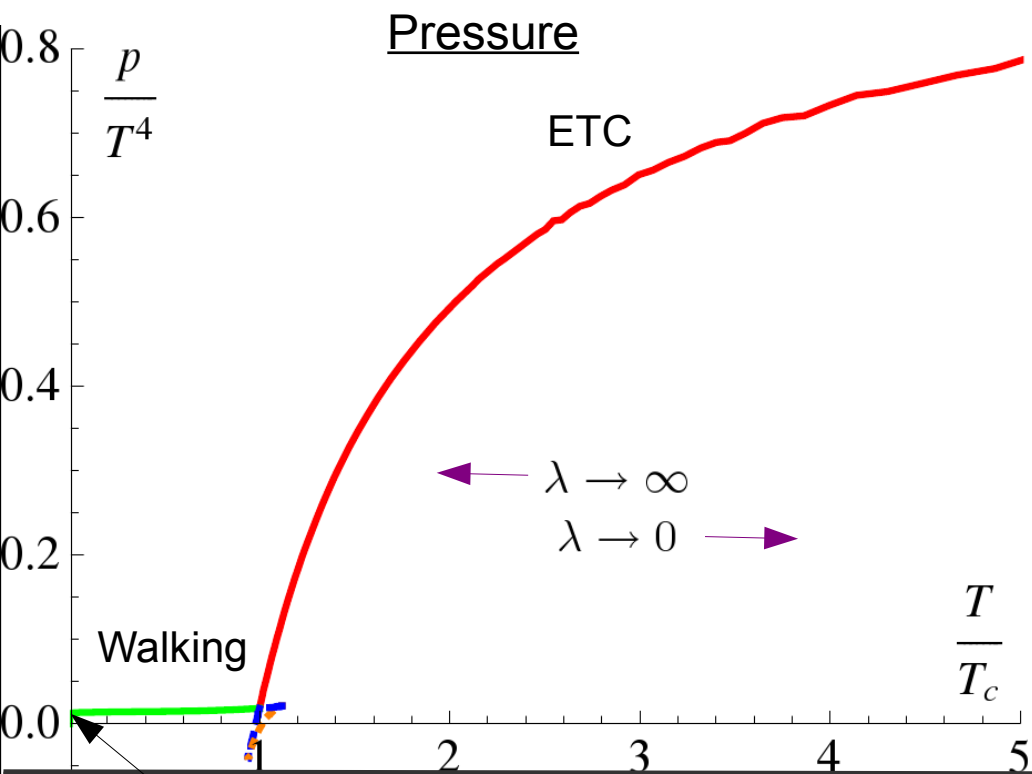
Walking + confining potential

$$V(\lambda) = V_0(\lambda) \left[ 1 + \frac{e}{10} \sqrt{\log(1 + \lambda^4)} \right]$$

→  $\beta(\lambda)$  Not known exactly



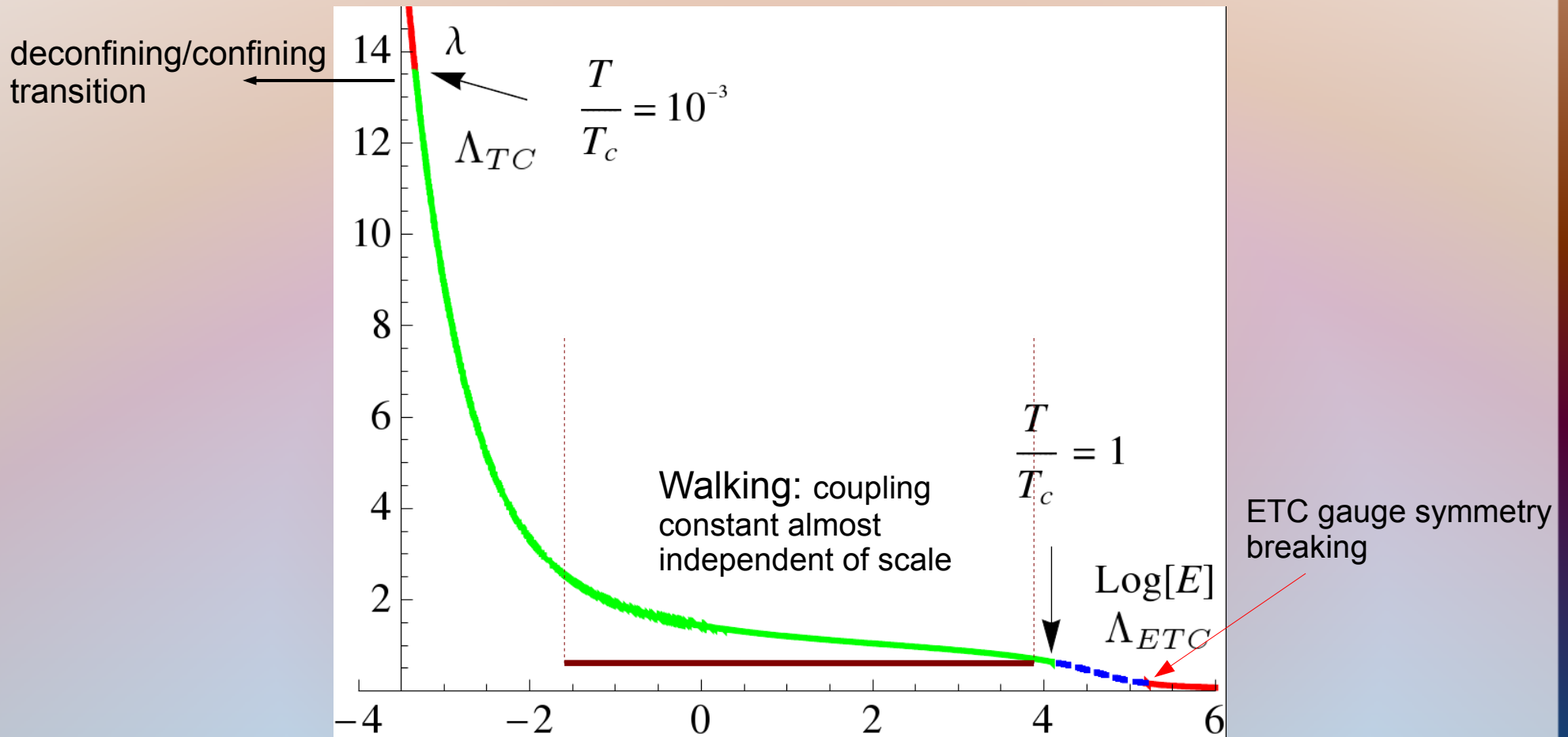
• Search black hole solutions with  $f(r) \neq 1$  → **Thermodynamics**



- **Two first order transitions**
- ETC at UV: technigluons + technifermions.
- Transition from ETC  $\rightarrow$  walking region: Massive technigluons.
- Transition from walking  $\rightarrow$  confining TC: Technifermions form bound states. EWSB.

# Coupling/Energy

This dilaton potential really produces walking behavior for the coupling



# Beta function

Walking potential

$$\beta_0(\lambda) = -c\lambda^2 \frac{[(1-\lambda)^2 + e]}{1 + a\lambda^3}$$



$$V_0(\lambda)$$

Walking + confining potential

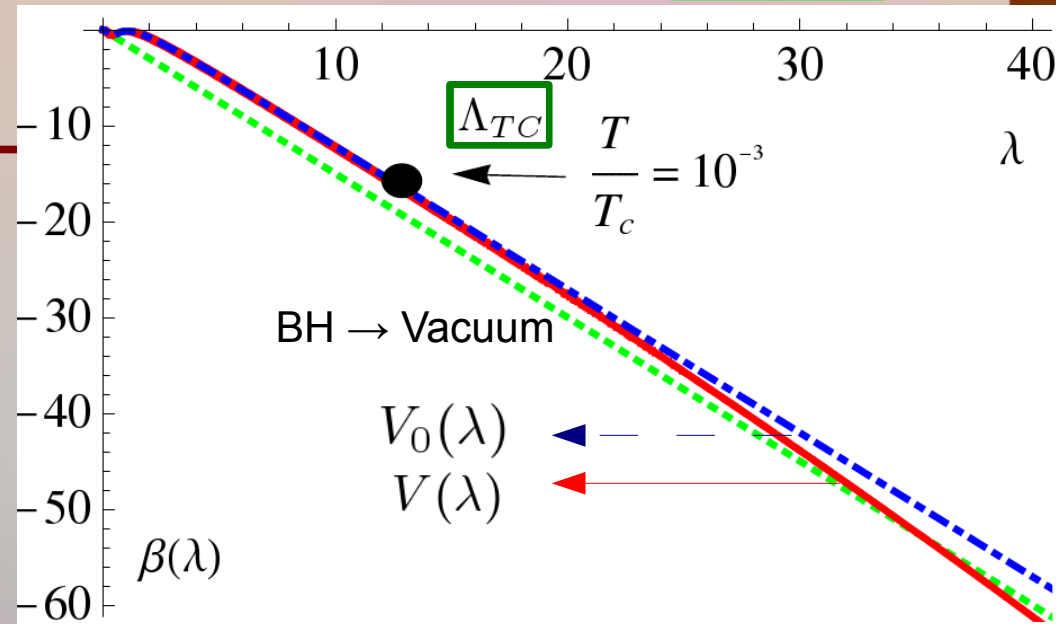
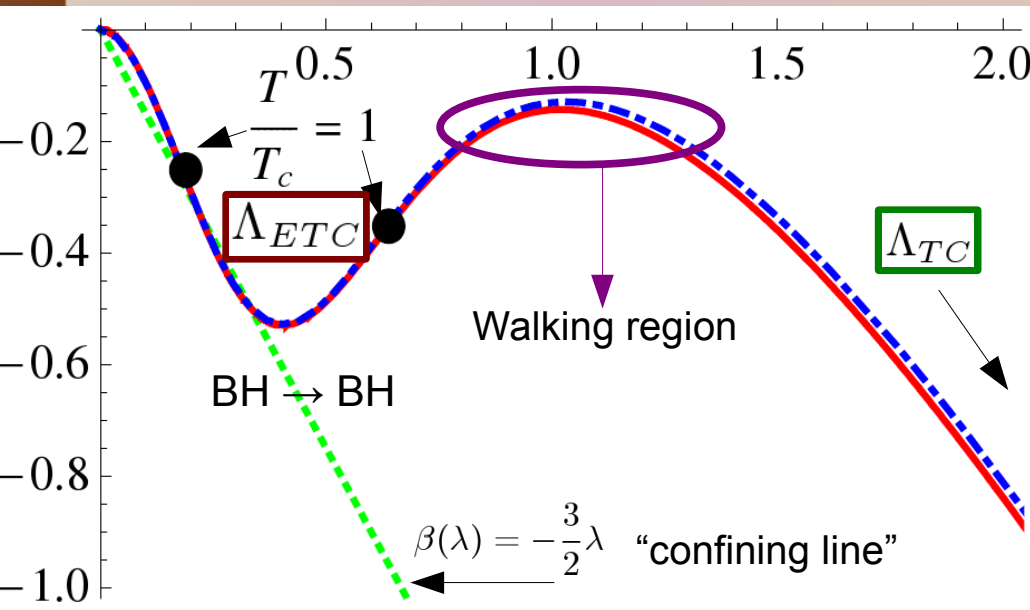
$$V(\lambda) = V_0(\lambda) \left[ 1 + \frac{e}{10} \sqrt{\log(1 + \lambda^4)} \right]$$



$$\beta(\lambda)$$

Log term is introduced to get beta function to cut “confining line” at IR

$$\beta(\lambda) = -\frac{3}{2}\lambda$$



# 5. Conclusions

- We have presented a scheme for converting a beta function of a gauge theory to its thermodynamics via gauge/gravity duality
- The scheme is a phenomenological bottom-up one; real effect of the fermions?
- Many further topics for study:
  - Scanning over parameters, different betas/potentials, case of several couplings, transport coefficients, mass spectra...