Holographic Flavor Transport

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Credits

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- 0812.3629 A. Karch, A. O'B., E. Thompson
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Outline:

- Motivation
- The Theory
- The Conductivity I
- The Gravity Dual
- The Conductivity II
- Summary and Outlook

Motivation

REAL Strongly-coupled Systems

Quantum Chromodynamics (QCD)

Relativistic Heavy-Ion Collider (RHIC)

QCD at $T \leq 2 \times T_c$

$T_c \approx 170$ MeV

Strongly-coupled, Nearly-ideal FLUID

Motivation

Well-described by hydrodynamics

Inputs for hydrodynamics:

I. Thermodynamics (equation of state)

II. Transport Coefficients

$\eta \quad \zeta \quad D \quad \sigma$

Question:

Can we compute

TRANSPORT COEFFICIENTS

for QCD

at RHIC temperatures?

Question:

Can we compute

TRANSPORT COEFFICIENTS

for QCD

at RHIC temperatures?

Answer:

NO.

Philosophy:
CHANGE the Question

Question:

Can we find ANY STRONGLY-COUPLED system for which we CAN compute TRANSPORT COEFFICIENTS?

Answer:

YES!

$\mathcal{N} = 4$ supersymmetric SU(N_c) Yang-Mills (SYM)

$$N_c \rightarrow \infty$$

$$\lambda \rightarrow \infty$$

$$\lambda = g_{YM}^2 N_c$$

Use Gauge-gravity Duality

Shear Viscosity

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

GOAL

Compute a
CONDUCTIVITY
associated with
“Quarks” or “Electrons”
using
Gauge-gravity Duality

RESULT

$$\sigma = \sqrt{\frac{N_f^2 N_c^2}{16\pi^2} T^2 \sqrt{e^2+1}\, f(m) + \frac{d^2}{e^2+1}}$$

Current $\langle J^x \rangle = \sigma E$ nonlinear in E

Pair Production

Drude Conductivity

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The Theory

$\mathcal{N} = 4$ supersymmetric SU(N_c) Yang-Mills (SYM)

$$N_c \to \infty$$

$$\lambda \to \infty$$

$$\beta = 0$$

The Theory

$\mathcal{N} = 4$ supersymmetric SU(N_c) Yang-Mills (SYM)

$N_c \rightarrow \infty$

$\lambda \rightarrow \infty$

No Quarks!

ADD

N_f $\mathcal{N} = 2$ hypermultiplets

$$\beta = +O\left(\frac{N_f}{N_c}\right)$$

"Probe Limit"

N_f fixed << $N_c \to \infty$

Scales

Temperature T

Mass m

$U(1)_B$ symmetry current J^{μ}

Baryon Number Density $\langle J^t \rangle$

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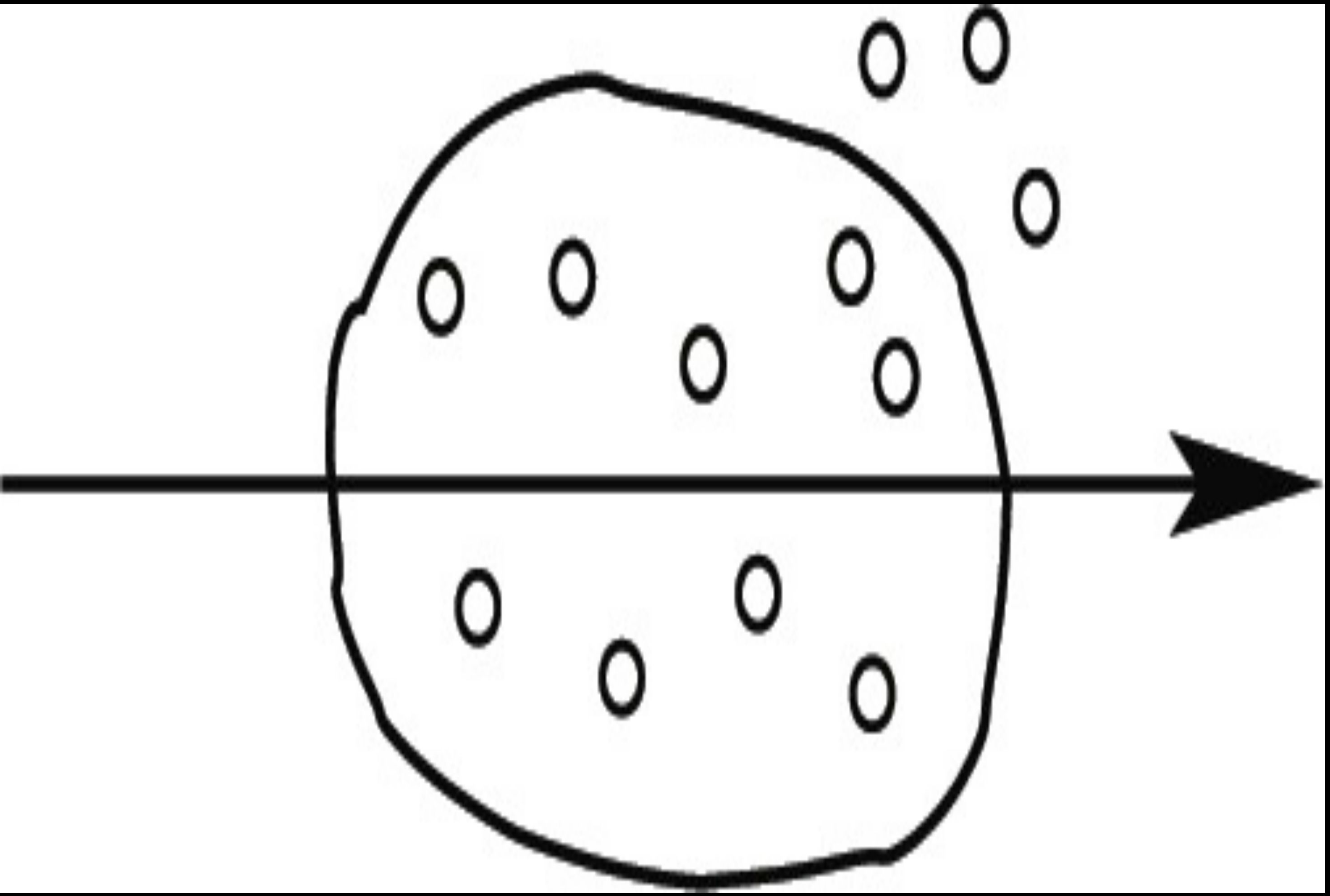
Electric Field E

"Two-fluid" picture

Lorentz force = Drag Force



$$\left\langle J^x \right\rangle = \sigma E$$

Steady-state???

Translation Invariance $\Rightarrow$ Momentum Conservation

Net Charge + Constant Electric Field $\Rightarrow$ Net Work

ENTIRE SYSTEM ACCELERATES FOREVER

NO

DISSIPATION!

Probe Limit MIMICS Dissipation

$$\langle T_{\mu\nu} \rangle = \mathrm{O}(N_c^2)_{\mu\nu} + \mathrm{O}(N_f N_c)_{\mu\nu}$$

$$\partial_\mu \langle T^{\mu\nu} \rangle = F^{\nu\sigma} \langle J_\sigma \rangle$$

$$\partial_t \langle T^{tt} \rangle = E \langle J_x \rangle \qquad \partial_t \langle T^{tx} \rangle = -E \langle J_t \rangle$$

$$\langle J_\mu \rangle = \mathrm{O}(N_f N_c) \qquad E = \mathrm{O}(1)$$

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The Gravity Dual

$\mathcal{N}$ = 4 SYM $N_c, \lambda \to \infty$	=	Supergravity $AdS_5 \times S^5$
Finite temperature	=	AdS-Schwarzschild
N_f $\mathcal{N}$ = 2 hypers.	=	N_f probe D7-branes $AdS_5 \times S^3$
m	=	Embedding
J^μ	=	A_μ

Anti-de Sitter Space

$$ds^2_{AdS_5} = g_{rr}(r)dr^2 + g_{tt}(r)dt^2 + g_{xx}(r)d\vec{x}^2$$

Boundary $r \longrightarrow \infty$

$$g_{rr} \sim 1/r^2 \qquad |g_{tt}| \sim g_{xx} \sim r^2$$

Horizon $g_{tt}(r_H) = 0 \quad r_H = \pi T R^2$

$$ds^2_{S^5} = d\theta^2 + \sin^2\theta ds^2_{S^1} + \cos^2\theta ds^2_{S^3}$$

The Gravity Dual

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D7-Brane Action

$$S_{D7} = -N_f T_{D7} \int d^8x \sqrt{-\det\left(g_{ab}^{D7} + (2\pi\alpha')\, F_{ab}\right)}$$

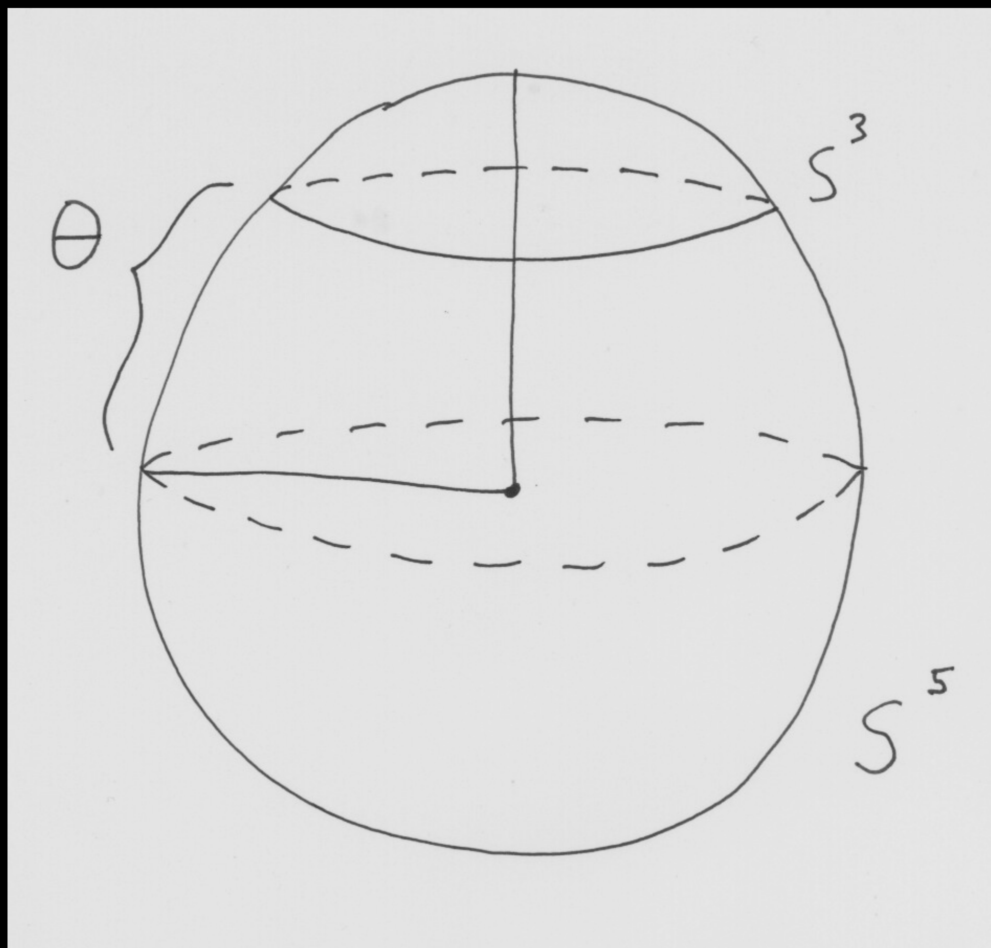
Extended along $AdS_5 \times S^3$

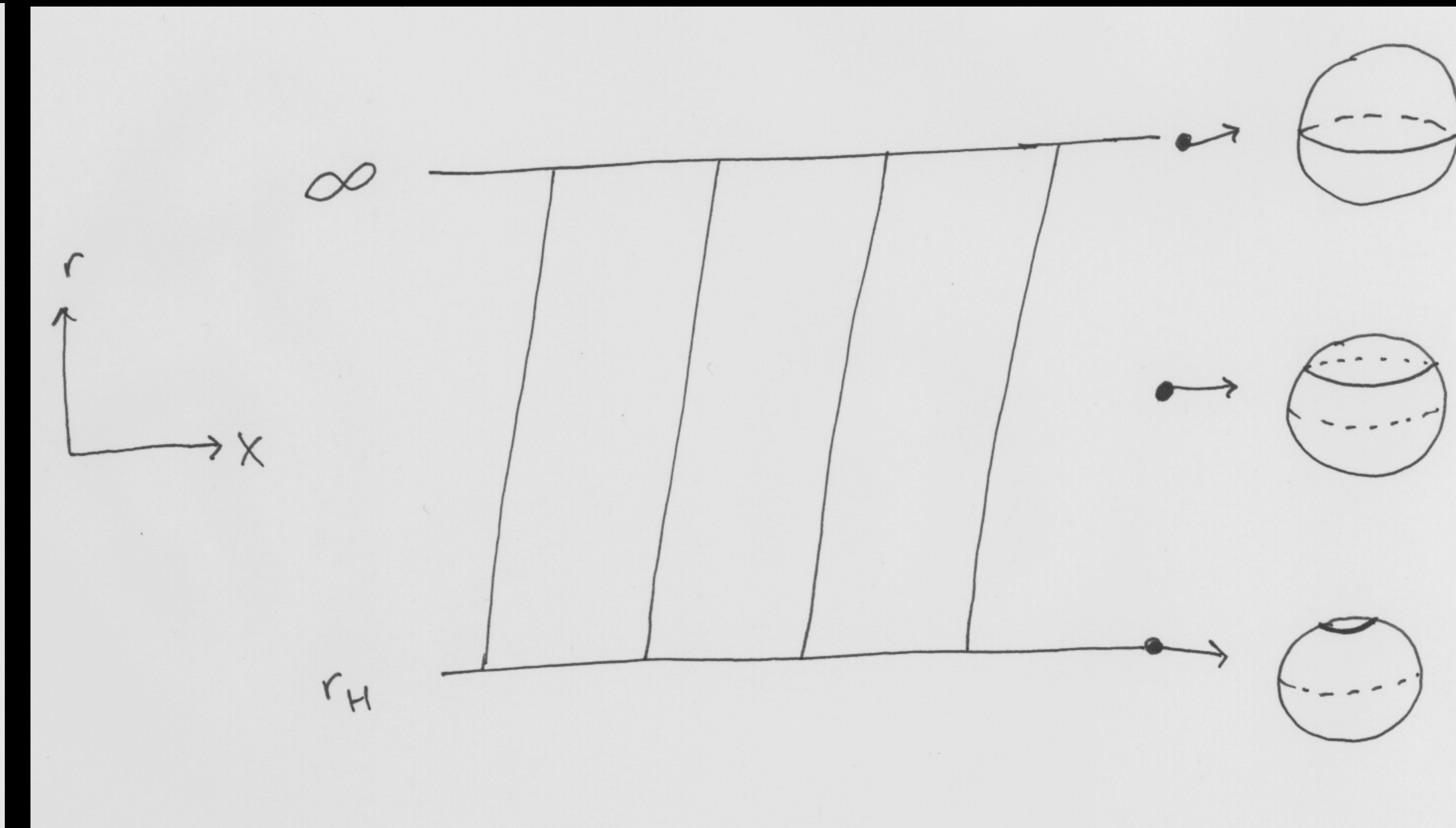
Can move along θ and S^1 directions

Ansatz: $\theta(r)$

$$ds_{D7}^2 = \left(g_{rr} + \theta'(r)^2\right) dr^2 + g_{tt} dt^2 + g_{xx} d\vec{x}^2 + \cos^2\theta(r) ds_{S^3}^2$$

Boundary Conditions





$$\theta(r_H) \in \left[0, \frac{\pi}{2}\right] \qquad \theta'(r_H) = 0$$

The Gravity Dual

$\mathcal{N} = 4$ SYM $N_c, \lambda \to \infty$	=	Supergravity $AdS_5 \times S^5$
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Dual Operators

$$\mathcal{O}_m = \frac{\partial}{\partial m}\mathcal{L} = \bar{\psi}\psi + (\text{scalars})$$

$$\theta(r) = \frac{m}{r} + \frac{\langle \mathcal{O}_m \rangle}{r^3} + \ldots$$

$$\theta(r_H) = 0 \quad \Rightarrow \quad m = 0$$

$$\theta(r_H) \to \pi/2 \quad \Rightarrow \quad m \to \infty$$

The Gravity Dual

$\mathcal{N} = 4$ SYM $N_c, \lambda \to \infty$	=	Supergravity $AdS_5 \times S^5$
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Dual Operators

We want μ_B $\langle J^t \rangle$

$$A_t(r) = \mu_B + \frac{\langle J^t \rangle}{r^2} + \ldots$$

We also want E $\langle J^x \rangle$

$$A_x(t,r) = -Et + \frac{\langle J^x \rangle}{r^2} + \ldots$$

Calculating the Conductivity

$$S_{D7} \sim -N_f T_{D7} \int_{r_H}^{\infty} dr\, g_{xx} \cos^3\theta(r)$$

$$\sqrt{|g_{tt}| g_{xx} \left(g_{rr} + \theta'(r)^2\right) - (2\pi\alpha')^2 \left(g_{xx} A_t'^2 + g_{tt} A_x'^2 + \left(g_{rr} + \theta'(r)^2\right) \dot{A}_x^2\right)}$$

$$\partial_r \frac{\delta S_{D7}}{\delta A'_\mu} = 0 \qquad \frac{\delta S_{D7}}{\delta A'_\mu} = \langle J^\mu \rangle$$

Solve for $A'_t \quad A'_x$ in terms of $\langle J^t \rangle \quad \langle J^x \rangle$

$E \quad \theta(r)$

Calculating the Conductivity

Insert the solutions back into the action:

$$S_{D7} \sim - \int_{r_H}^{\infty} dr\, g_{xx}^{5/2} |g_{tt}|^{1/2} \left(g_{rr} + \theta'(r)^2 \right)^{1/2} \cos^6 \theta(r)$$

$$\sqrt{\frac{|g_{tt}| g_{xx} - (2\pi\alpha')^2 E^2}{c^2 |g_{tt}| g_{xx}^3 \cos^6 \theta(r) + |g_{tt}| \langle J^t \rangle^2 - g_{xx} \langle J^x \rangle^2}}$$

$$c^2 = \frac{\lambda N_f^2 N_c^2}{(2\pi)^6}$$

Calculating the Conductivity

Insert the solutions back into the action:

$$S_{D7} \sim -\int_{r_H}^{\infty} dr\, g_{xx}^{5/2} |g_{tt}|^{1/2} \left(g_{rr} + \theta'(r)^2\right)^{1/2} \cos^6\theta(r)$$

$$\sqrt{\frac{|g_{tt}| g_{xx} - (2\pi\alpha')^2 E^2}{c^2 |g_{tt}| g_{xx}^3 \cos^6\theta(r) + |g_{tt}| \langle J^t \rangle^2 - g_{xx} \langle J^x \rangle^2}}$$

Numerator changes sign!

Calculating the Conductivity

Imaginary Action $\Rightarrow$ INSTABILITY

Minkowski Space

$$\sqrt{1-(2\pi\alpha')^2 E^2}$$

$$E > \frac{1}{2\pi\alpha'}$$

Strings ripped apart!

AdS-Schwarzschild

$$\sqrt{|g_{tt}|g_{xx}-(2\pi\alpha')^2 E^2}$$

Calculating the Conductivity

$$\sqrt{\frac{|g_{tt}|g_{xx} - (2\pi\alpha')^2 E^2}{c^2|g_{tt}|g_{xx}^3 \cos^6\theta(r) + |g_{tt}|\langle J^t\rangle^2 - g_{xx}\langle J^x\rangle^2}}$$

$$\left[|g_{tt}|g_{xx} - (2\pi\alpha') E^2\right]_{r_*} = 0$$

To AVOID the instability: DEMAND

$$\left[c^2|g_{tt}|g_{xx}^3 \cos^6\theta(r) + |g_{tt}|\langle J^t\rangle^2 - g_{xx}\langle J^x\rangle^2\right]_{r_*} = 0$$

Solve for $\langle J^x\rangle = \sigma E$

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The Conductivity

$$\sigma = \sqrt{\frac{N_f^2 N_c^2}{16\pi^2} T^2 \sqrt{e^2+1}\, f(m) + \frac{d^2}{e^2+1}}$$

$$e = \frac{E}{\frac{\pi}{2}\sqrt{\lambda}T^2}$$

$$d = \frac{\langle J^t \rangle}{\frac{\pi}{2}\sqrt{\lambda}T^2}$$

$$f(m) = \cos^6\theta(r_*)$$

The Conductivity

$$\sigma = \sqrt{\frac{N_f^2 N_c^2}{16\pi^2} T^2 \sqrt{e^2+1} f(m) + \frac{d^2}{e^2+1}}$$

Depends on E!

$$\langle J^x \rangle = \sigma(E) E$$

Linearize in E

$$\langle J^x \rangle = \sigma(0) E$$

The Conductivity

$$\sigma = \sqrt{\frac{N_f^2 N_c^2}{16\pi^2} T^2 \sqrt{e^2+1} f(m) + \frac{d^2}{e^2+1}}$$

$$\langle J^t \rangle = 0 \quad \text{BUT} \quad \sigma \neq 0$$

Pair Production

The Conductivity

$$\sigma = \sqrt{\frac{N_f^2 N_c^2}{16\pi^2} T^2 \sqrt{e^2+1}\, f(m) + \frac{d^2}{e^2+1}}$$

$$m \to \infty \quad \Rightarrow \quad f(m) \to 0$$

$$m \to 0 \quad \Rightarrow \quad f(m) \to 1$$

The Conductivity

$$\sigma = \sqrt{\frac{N_f^2 N_c^2}{16\pi^2} T^2 \sqrt{e^2+1}\, f(m) + \frac{d^2}{e^2+1}}$$

$$\langle T^{tx} \rangle \propto \langle J^t \rangle$$

NO momentum flow at zero density

The Conductivity

$$\sigma = \sqrt{\frac{N_f^2 N_c^2}{16\pi^2} T^2 \sqrt{e^2+1} f(m) + \frac{d^2}{e^2+1}}$$

Drude Conductivity

Linearize in E $\sigma(0)$ Take $m \rightarrow \infty$

$$\sigma \rightarrow d = \frac{\langle J^t \rangle}{\frac{\pi}{2}\sqrt{\lambda} T^2}$$

Why $m \to \infty$?

Charges behave as semi-classical quasi-particles:

$$\frac{dp}{dt} = -\mu p + E$$

Separate calculation

$$\boxed{\mu m = \frac{\pi}{2}\sqrt{\lambda}T^2}$$

$$\mu m = \frac{\pi}{2}\sqrt{\lambda}T^2$$

$$\sigma \to d = \frac{\langle J^t \rangle}{\frac{\pi}{2}\sqrt{\lambda}T^2} = \frac{\langle J^t \rangle}{\mu m}$$

Drude Conductivity

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Summary + Outlook

Probe Branes are Great!

Computed CONDUCTIVITY

for a "DISSIPATIVE"

STRONGLY-COUPLED

Non-Abelian Gauge Theory

FUTURE DIRECTIONS

MORE TRANSPORT COEFFICIENTS:

Magnetic Fields

Anomalous currents

Condensed Matter Applications:

Thermo-electric Transport

Quantum Hall Effect

Non-relativistic Theories

Superfluidity and Fermi Surfaces

$$N_f = 2 \quad \Rightarrow \quad U(2) = U(1) \times SU(2)$$

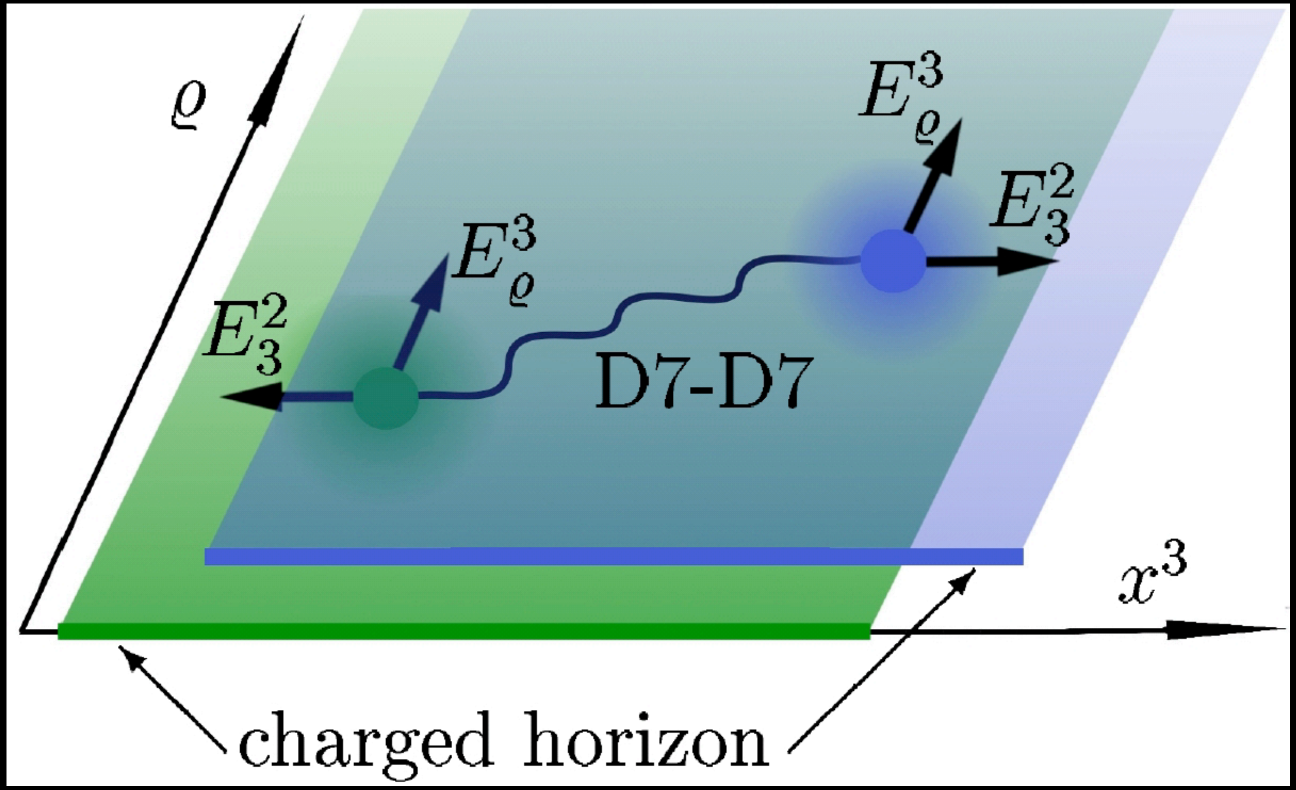
$$J_a^\mu \qquad a = 1, 2, 3$$

Chemical potential for $U(1) \subset SU(2)$ $\langle J_3^t \rangle$

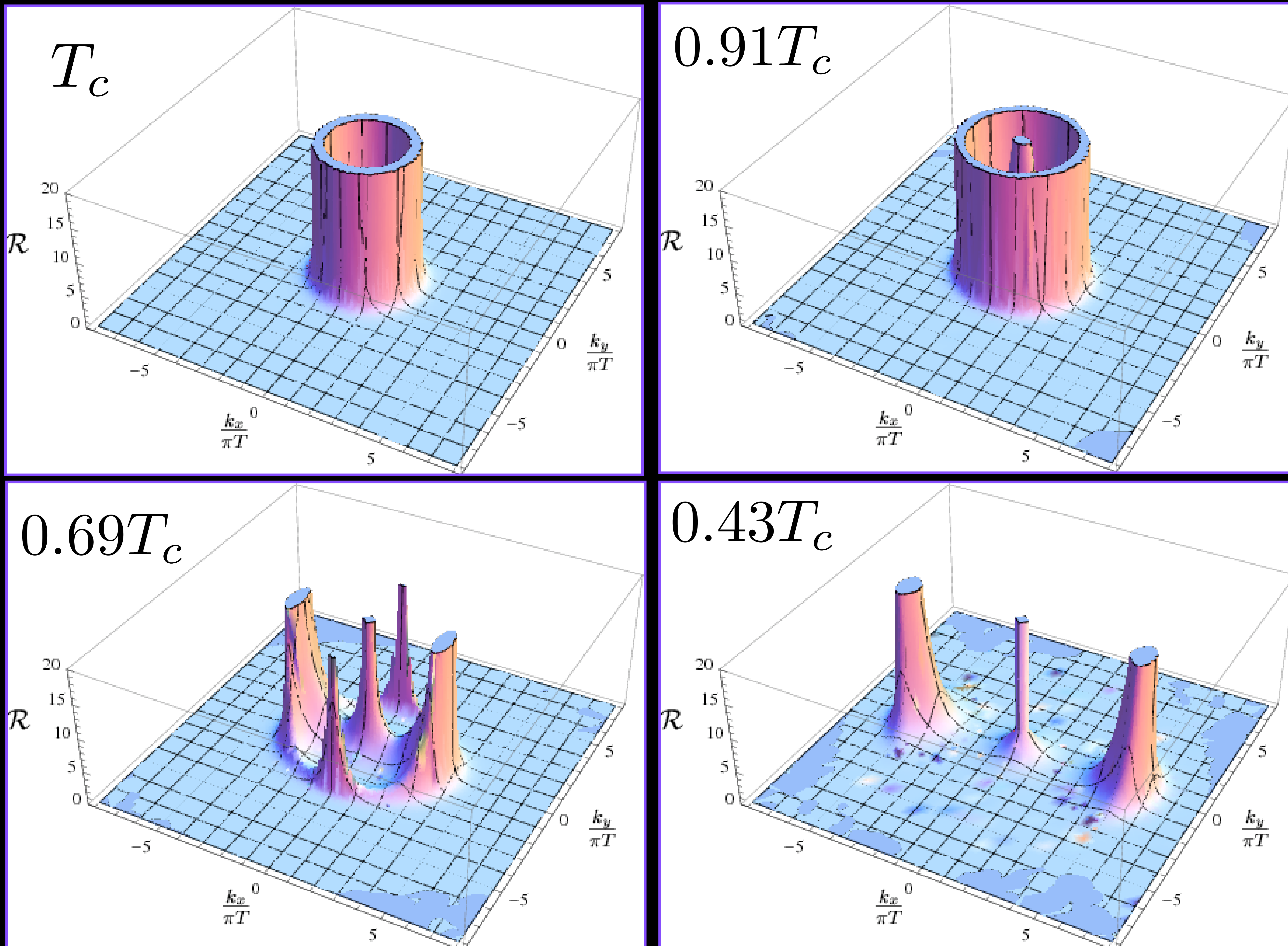
Ground state: nonzero

$$\langle J_1^x \rangle$$

p-wave superfluid!



What does the Fermi Surface look like?



T_c
$0.91T_c$
$0.69T_c$
$0.43T_c$
$\mathcal{R}$
0
5
10
15
20
$\frac{k_x}{\pi T}$
$\frac{k_y}{\pi T}$
−5

Thank You.