Dynamical critical scaling of long-range interacting quantum magnets

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Slow variations (quenches) of the magnetic field across the paramagnetic-ferromagnetic phase transition of spin systems produce heat. In short-ranged systems the heat exhibits a universal power-law scaling as a function of the quench rate, known as Kibble-Zurek (KZ) scaling. Attempts to extend this hypothesis to long-range interacting systems have lead to seemingly contradicting results. In this work we analyse slow quenches of the magnetic field in the Lipkin-Meshkov-Glick model, which describes fully-connected quantum spins. We determine the quantum contribution to the residual heat as a function of the quench rate by means of a Bogoliubov expansion about the mean-field value and calculate the exact solution. For a quench which ends at the quantum critical point we identify two regimes: the adiabatic limit for finite-size chains, where the scaling is dominated by the Landau-Zener tunneling, and the KZ scaling. For a quench symmetric about the critical point, instead, there is no KZ scaling. Here we identify three regimes depending on the velocity of the ramp and on the size of the system: (i) the adiabatic limit for finite-size chains; (ii) the opposite regime, namely, the thermodynamic limit, where the residual heat is independent of the quench rate; and finally (iii) the intermediate regime, which is a crossover between the two solutions. We argue that this behaviour is a property of all-connected spin systems. Our findings agree with previous studies and identify the respective limits in which they hold.

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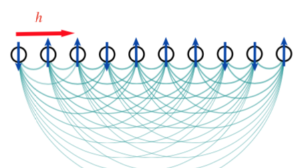
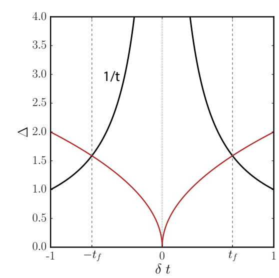
The development of a comprehensive statistical mechanics description of out-of-equilibrium systems is a quest of relevance across disciplines, including biology, physics, computer science and financial markets [1]. A specific, yet relevant question regards the connection between dynamical and equilibrium properties of quantum critical systems [2]. This would contribute to a systematic understanding of the subtle interplay between time evolution, interactions, quantum, and thermal fluctuations. Moreover, it is important for the development of quantum devices based on quantum annealing, where one aims at preparing many-body quantum states with adiabatic transformations [3]. Theoretical and experimental studies of many-body critical dynamics after sudden variations of control fields have identified features which are reminiscent of the behavior of thermodynamic functions at transition points [4, 5]. Yet, the relation between dynamical scaling and equilibrium critical phenomena is elusive and often only conjectured.

In this framework, it is believed that the thermodynamics of slow quenches across quantum critical points could be cast in terms of the so-called Kibble-Zurek (KZ) scaling [6–9]. The KZ scaling predicts that the heat produced by slow variations (quenches) of control fields across critical points scales with the quench rate through a power law determined by the equilibrium critical exponents [10–12]. This theory has a strong predictive power and has been experimentally verified for a large variety of physical systems [13–27]. Its validity, though, seems to be limited to systems where the coherence length diverges with a power law at the critical point but is well defined in the critical region. This hypothesis can be explained in a nutshell as follows. Assume a system of interacting spins in presence of a magnetic field h, as illustrated in Fig. 1(a). Let the magnitude of the magnetic field h be



(a)



(b)

FIG. 1. (color online) (a) The dynamics of a chain of spins 1/2 is analysed when the amplitude of the magnetic field h is slowly varied across the paramagnetic-ferromagnetic transition. The lines connecting the sites illustrate that each spin interacts with equal strength J with the rest of the chain. (b) The energy gap $\Delta(h)$ between the ground and the first excited state of the chain is displayed as a function of $h = 1+\delta t$ (solid line). The dashed line shows the rate $\gamma_h = |\dot{h}|/h$ with which the magnetic field is varied in time, such that at $t = \pm t_f$ $\gamma_h = \Delta(h)$. We determine the scaling of the heat generated by the quench on γ_h and compare our predictions with the KZ hypothesis, which relates this scaling to the universal critical exponents at equilibrium.

slowly varied from the paramagnetic to the ferromagnetic phase across the critical point h_c. The transformation is adiabatic when the rate of change $\gamma_h = |\dot{h}|/h$ is larger than the energy gap $\Delta(h)$, while in the other regime non-adiabatic effects are expected. Figure 1(b) displays the energy gap $\Delta(h)$ as a function of h: The gap vanishes as $\Delta(h) \sim |h - h_c|^{z\nu}$ at the critical point, with ν and z the equilibrium critical exponents. At the times $t = \pm t_f$ the equality $\Delta(h) = \gamma_h$ holds. The KZ theory assumes that in the time window $-t_f < t < t_f$ the dynamics are frozen and estimates the heat produced by the quench by the scaling $Q \sim 1/\xi_f^z$, where $\xi_f \sim 1/|h(t_f) - h_c|^\nu$ is the average size of the domains formed at the time $t = -t_f$ in the adiabatic approximation. This yields the scaling $Q \sim |h(t_f) - h_c|^{z\nu}$. For a quench where the magnetic field varies with time as $h = h_c + \delta t$ ($\delta > 0$), then [9–11, 28]

$$Q \sim \delta^{z\nu/(1+z\nu)} \,. \tag{1}$$

Although this separation between adiabatic and "impulse regime" may seem oversimplified, it describes the behaviour found in microcanonical systems, where the relaxation time is determined by the instantaneous gap between the ground and the first excited state [29]. The validity of the KZ scaling (1) has been extensively verified in integrable fermionic systems [28, 30–33]. Even at finite temperatures, where one has relevant corrections due to scattering of defects, the KZ scaling is a good working hypothesis [34, 35]. A conceptual problem arises, instead, when one applies the KZ scaling to critical systems where the coherence length is ill-defined [3, 36]. This is the case of systems with strong-long-range interactions, where the two-body interaction potential decays as a power law of the distance r between its microscopic components such as $V(r) \propto r^{-\alpha}$, with $0 \le \alpha < d$ and d the spatial dimension. In this case the energy is non-additive [36–38] and the scaling $Q \sim 1/\xi_f^z$ becomes meaningless.

In this work we consider a linear quench of the magnetic field across the paramagnetic–to–ferromagnetic transition in the Lipkin-Meshkov-Glick (LMG) model [39]. The LMG describes a one dimensional chain of N spins 1/2 and constant all-to-all ferromagnetic interactions in a transverse magnetic field h. The system is illustrated in Fig. 1(a) and it can be simulated by chains of trapped ions [40, 41]. Its Hamiltonian reads

$$H = -J\left(\frac{1}{N}\sum_{ij}\sigma_i^x\sigma_j^x + h(t)\sum_i \sigma_i^z\right) \,, \tag{2}$$

where σ_i^μ are the Pauli matrices of spin i and the prefactor $1/N$ in front of the interaction term warrants that the energy is extensive [36]. The parameter $J > 0$ scales the energy in units of the interaction strength. From now on energy and time are in units of J and J^{-1}, respectively.

When the magnetic field h is constant in time, in the thermodynamic limit the LMG model displays a quantum phase transition (QPT) between a symmetric state fully polarized along x and a symmetry–broken phase with two degenerate ground states of opposite macroscopic polarization along the z direction. The quantum critical point (QCP) is at $h_c = 1$, the universal behavior is the same as the Dicke model [42, 43] and is given by a mean-field theory with critical exponents $z = 1/3$ and $\nu z = 1/2$ [44–46].

The dynamical protocol we consider is a continuous ramp of the control field $h(t) = 1 + \delta t$, where $\delta > 0$ is the quench rate and $t \in [-t_0, t_0]$, such that $\delta t_0 = 1$, namely, the quench starts deep in the paramagnetic phase, crosses the QCP at $t = 0$ and finally ends far into the symmetry broken phase. Using the critical exponents ν and z in Eq. (1) one would expect the scaling $Q \sim \delta^{1/3}$ after the quench. This scaling was found using an heuristic application of adiabatic perturbation theory to the Rabi model for a quench to the critical point ($t \in [-t_0, 0]$), after showing that the Rabi model can be mapped to the Dicke model [47]. Nevertheless it does seem not agree with the numerical studies reported in Ref. [48, 49] for symmetric quenches ($t \in [-t_0, t_0]$). Moreover, it is inconsistent with the calculation performed in Ref. [50] for a system which is equivalent to the quantum dynamics of the LMG in the strict thermodynamic limit $N \to \infty$. The works we have just mentioned seem to find mutually contradicting results.

We now provide the solution of the Schrödinger equation governed by Hamiltonian (2) for $h(t) = 1 + \delta t$, which is valid for slow quenches and allows us to determine the spins' wave function during and after the quench. Our model allows us to show that the predictions of Refs. [47–50] are consistent with our solution in different regimes. These regimes are identified by analysing the scaling properties at the critical point and are due to the long-range nature of the interactions.

In order to solve the Schrödinger equation we first rewrite the Hamiltonian (2) introducing a single collective spin of length N, namely $S_\mu = \sum_i \sigma_i^\mu/2$ and $S_\pm = S_x \pm i\, S_y$ [51]:

$$H = -\frac{1}{2N}(\boldsymbol{S}^2 - S_z^2 - N/2) - 2h(t)S_z - \frac{1}{2N}(S_+^2 + S_-^2) \,. \tag{3}$$

We then perform a $1/N$ expansion around the ground-state of the mean-field model [52, 53], which is assumed to adiabatically follow the quench. The expansion is obtained by rotating the spin operators to align them with the semiclassical magnetization and then by applying an Holstein-Primakoff transformation up to order 1 [52]: $S_z = N/2 - a^\dagger a$, $S_+ = S_-^\dagger = \sqrt{N}a$, where the operators a and $a^\dagger$ satisfy the bosonic commutation relation $[a, a^\dagger] = 1$. The resulting Hamiltonian is quadratic and in diagonal form reads

$$H_0 = N\, e_0(h) + \delta e(h) + \Delta(h)\, a^\dagger a \,, \tag{4}$$

where e_0 is the thermodynamic mean field energy density, δe is a constant mean field shift, while the quantum fluctuations are described by the quadratic harmonic oscillator term whose frequency is the gap Δ [51]. We focus on the evolution of the quadratic term and observe that the quantum part of Hamiltonian (4) is obtained at leading order in $1/N$ expansion and is thus strictly valid in the thermodynamic limit. Nevertheless, by means of the continuous unitary transformation approach [54] the LMG Hamiltonian can be recast into the form (4) even for finite N [51, 55]. Then, the gap reads [51, 55]

$$\Delta = \begin{cases} 2\sqrt{h(h-1)} + \mathcal{F}(N,h) & h > 1\,, \\ 2\sqrt{(1-h^2)} + \mathcal{F}(N,h) & h < 1\,, \end{cases} \quad (5)$$

where for large N the function $\mathcal{F}(N,h) \propto 1/N$ for $h \neq 1$ while at the critical point the gap scales as $\Delta \propto 1/N^{1/3}$ [51, 55]. The dynamics governed by Hamiltonian (4) corresponds now to the one of a single harmonic oscillator with the time-dependent frequency $\Omega(t) = \Delta(h(t))$. It is exactly solved in terms of the dynamical basis

$$\psi_n(x,t) = \left(\frac{e^{-i4\phi(t)}}{2\pi\xi^2(t)}\right)^{\frac{1}{4}} \frac{e^{-\tilde{\Omega}(t)\frac{x^2}{2}}}{\sqrt{2^n n!}} H_n\left(\frac{x}{\sqrt{2}\xi(t)}\right). \quad (6)$$

Here, H_n is the Hermite polynomial of degree n, $\phi(t)$ is a phase factor, $\tilde{\Omega}(t) = 1/2\xi^2 + i\dot{\xi}/\xi$ is the effective frequency and $\xi(t)$ is a time dependent scale factor which obeys the Ermakov-Milne equation [56–58]

$$\ddot{\xi}(t) + \Omega(t)^2\xi(t) = \frac{1}{4\xi(t)^3}\,. \quad (7)$$

By integrating Eq. (7) we exactly solve the quantum dynamics.

We consider the time interval $[-t_0, t_0]$ with $t_0 = 1/\delta$. The time evolution of the initial state $\psi_0(x, -t_0)$, which coincides with the ground state of the instantaneous Hamiltonian $H(-t_0)$ at $h = 0$, is to good approximation the wave function $\psi_0(x,t)$, Eq. (6). The evolution is adiabatic when $\psi_0(x,t)$ coincides with the instantaneous ground state of Hamiltonian $H(t)$. This corresponds to the fidelity $f(t) = 1$, where $f(t) = |c_0(t)|^2$ and $c_n(t) = \int \psi_n^{\mathrm{ad}*}(x,t)\psi_0(x,t)dx$ is the overlap integral between $\psi_0(x,t)$ and the eigenfunctions $\psi_n^{\mathrm{ad}}(x,t)$ of the adiabatic basis of the oscillator with frequency $\Omega(t)$. Specifically, $\psi_n^{\mathrm{ad}}(x,t)$ are the solutions of Eq. (6) after setting $\dot{\xi} = \ddot{\xi} = 0$ in Eq. (7) and thus $\xi(t)^2 = 1/(\sqrt{2}\Omega(t))$ in Eq. (6). The explicit expression for the overlap integral is derived in the Supplementary Material (S.M.) and Ref. [59] [1]. The residual energy (heat) at time $t > -t_0$ is proportional to the number of the oscillator's excitations $n_{\mathrm{exc}}(t)$, namely, $Q(t) = \hbar\Omega(t)n_{\mathrm{exc}}(t)$, with

$$n_{\mathrm{exc}}(t) = \sum_{n=1}^{\infty} n|c_n(t)|^2\,. \quad (8)$$

The results we present in Fig. 2 are obtained by numerically solving Eq. (7).

In order to verify that our model delivers a reliable description of the LMG model, we compare the predictions of Eqs. (6)-(7) with the results obtained in Ref. [49] by numerically integrating the dynamics of $2^9 - 2^{11}$ spins with Hamiltonian (2). Figure 2(a) displays the time evolution of the fidelity $f(t)$ and of the heat $Q(t) = \hbar\Omega(t)n_{\mathrm{exc}}(t)$ obtained from Eqs. (6)-(7) and for the parameters of Ref. [49]. The curves in Fig. 2(a) reproduce the ones numerically found in Ref. [49] confirming that our approach, based on integrating the Schrödinger equation of a single time-dependent harmonic oscillator, reliably describes the behaviour of slow quenches in the LMG model. As in that work, we observe the collapse of the fidelity at and after the critical point. For the choice of the parameters the dynamics are close to adiabatic with fidelity $f > 80\%$. In Ref. [49], though, no numerical evidence of KZ scaling was found. It was there conjectured that this may be due to the finite-size universal functions at the critical point. In particular, they introduced the parameter

$$\Lambda = N\delta\,, \quad (9)$$

which was identified on the basis of scaling properties of transition amplitudes in adiabatic perturbation theory. The parameter Λ plays indeed an important role in the dynamics of our model and has a specific physical meaning, as we argue in the following using scaling arguments. For this purpose, we approximate the oscillator frequency with $\Omega(t)^2 = -4\delta t + 1/N^{2z_{\mathrm{eff}}}$ for $t < 0$ and $\Omega(t)^2 = 8\delta t + 1/N^{2z_{\mathrm{eff}}}$ for $t \geq 0$, where the expression is reported apart from scalars which are intensive and not universal. We numerically verified that the remaining terms are irrelevant, since they become sub-leading in the critical $t \simeq 0$ stage of the dynamics. The overall effect of finite-size fluctuations is now summarized in an effective finite-size scaling exponent z_{eff}, such that $1/3 < z_{\mathrm{eff}} < 1$. The full numerical solution of Eq. (7) indicates that finite-size corrections only become important at $t \simeq 0$, therefore in this discussion we assume $z_{\mathrm{eff}} = 1/3$. We identify the scaling relations by performing the transformation $\xi = \delta^{-1/6}\tilde{\xi}$ and $t = \delta^{-1/3}s$ (note that s is the same rescaled time variable as in Fig. 2(c) of Ref. [49] apart from the factor $\Lambda^{2/3}$). This transformation leads to the Schrödinger equation governed by the Hamiltonian of a quantum harmonic oscillator with effective frequency $\Omega(s)$, such that

$$\Omega(s)^2 = \begin{cases} -4s + \Lambda^{-2/3} & s < 0\,, \\ 8s + \Lambda^{-2/3} & s > 0\,. \end{cases} \quad (10)$$

Remarkably, Λ is now the sole physical parameter that depends on the quench rate δ and is the sole scale which determines the dynamical behaviour at the critical point. We can now identify three regimes: (i) the limit $\Lambda \ll 1$, where the quench rate is much smaller than the gap and thus the dynamics are expected to be adiabatic except for small corrections. This regime is expected to provide the Landau-Zener scaling, where $n_{\mathrm{exc}} \sim \delta^2$ and the corrections to adiabaticity scale with Λ^2 [10, 28, 31, 33]. (ii) In the limit $\Lambda \gg 1$, on the other hand, the system approaches the thermodynamic limit where the dynamics are independent of Λ to leading order in an expansion

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[1] See the supplementary materials

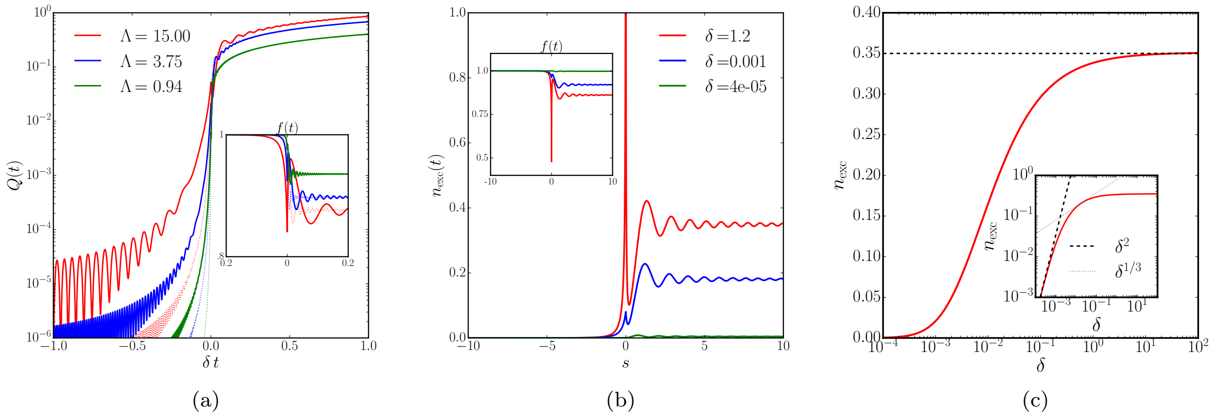

FIG. 2. (color online) (a) Heat generated by the quench, $Q(t) = \hbar\Omega(t)n_{\rm exc}(t)$, in units of J, as a function of t, in units of $1/(J\delta)$. The heat is determined from Eq. (8) using Eq. (6)-(7) and for different values of Λ (see legend), which are the same as in Ref. [49]. The solid (dashed) lines correspond to $N = 2^9$ ($N = 2^{12}$). The inset reports the corresponding fidelity $f(t)$. Larger values of Λ are reported in panel (b), where the average number of excitation $n_{\rm exc}(t)$ and the fidelity $f(t)$ are reported as a function of the rescaled time $s = \delta^{1/3}t$ for $N = 2^{12}$ and quench rate $\delta = 4 \times 10^{-5}, 10^{-3}, 1.2$, corresponding to $\Lambda = 2\times 10^{-1}, 6, 6\times 10^3$, respectively. (c) The number of excitations at the end of the quench, $n_{\rm exc}(t_0)$, is reported as a function of δ for $N = 500$; the horizontal dashed line indicates the constant value $n_{\rm exc}(t_0) = 0.35$ of the thermodynamic limit. The inset displays $n_{\rm exc}(t_0)$ in logarithmic scale: the dashed line indicates the region of the LZ scaling δ^2, the dotted line the region of the KZ scaling $\delta^{1/3}$. The behaviour for a different system size N' is obtained by rescaling the δ-axis by the factor N'/N.

in $1/\Lambda$. In this limit, thus, the dynamics, and in particular the excitations and the fidelity, are expected to be independent of δ. This result is consistent with the prediction of Ref. [50], which considered a slow quench of the frequency of a single harmonic oscillator, albeit with a different power law in time. The non-analytic regime is expected for intermediate values of Λ.

Figure 2(b) displays the time evolution of $f(t)$ and $n_{\rm exc}(t)$ by integrating Eqs. (6)-(7) for values of Λ in the three different regimes. The value of $n_{\rm exc}(t_0)$ that we extract from these calculations is reported in Fig. 2(c) as a function of δ for N constant. Here we observe the Landau-Zener scaling $n_{\rm exc} \sim \delta^2$ for $\delta \ll 1/N$, in agreement with our conjecture based on scaling arguments. For $\delta \gg 1/N$ the excitation number tends to the constant value predicted by the thermodynamic limit, $n_{\rm ex,\infty} \approx 0.35$ for the linear quench in the LMG model. Even though in the thermodynamic limit there is no power law scaling, the final number of defects $n_{\rm exc}(t_0)$ still depends on the scaling of the gap at $s \to 0$. It is therefore a universal value, and hints towards a close relation between the out-of-equilibrium dynamics and the equilibrium universal properties. Since the slope of the curve $n_{\rm exc}$ as a function of δ varies continuously, it contains also an interval of values δ with scaling $\delta^{1/3}$. This scaling, which would agree with the KZ prediction, is clearly in a crossover regime and is not found in the thermodynamic limit. We find it instead for a semi-ramp which ends (starts) at the QCP. As we show in the S.M. [2], away from the adiabatic regime our model delivers the KZ scaling $\delta^{1/3}$ in the thermodynamic limit. This is in agreement with the predictions of Refs. [28, 47].

In conclusion, we have shown that the slow quench dynamics presents different qualitative behaviours depending on whether the protocol ends at the critical point (half ramp) or deep in the other phase (full ramp). While the half ramp exhibits KZ scaling in the thermodynamic limit, for the full ramp the residual heat is constant and independent of δ. These predictions apply also to the Dicke model, which belongs to the same universality class as the LMG model and whose finite-size corrections to the gap have the same scaling properties with N [60]. We remark that our predictions concern the quantum contribution to the heat when the mean-field spin follows adiabatically the magnetic field and thus hold when the non-adiabatic corrections of the mean-field energy are smaller than the quantum heat. Assuming that the semiclassical evolution is analytical in δ and no work is done on the system in a cyclic process in the adiabatic limit $\delta \to 0$, the semiclassical contribution to the specific heat would scale as $N\delta^2$ [61]. The KZ scaling could then already be observed for $\delta \lesssim N^{-2/5}$, while the Landau-Zener scaling just depends on model-specific, non-universal parameters. These observations also suggest that the scaling $N\delta^2$ found in Ref. [48] for the slower quenches is dominated by the mean-field dynamics, where the quantum contribution to the heat is not yet visible.

[2] See the supplementary materials

Our predictions could be experimentally verified in chains of hundred ions with tailored all-to-all interactions [5]. The regime corresponding to $\Lambda \gg 1$, where the quantum residual energy tends to a constant, is going to be a very small correction to the mean-field scaling, yet it could be observed in simple systems such as the Rabi model [47]. Our study, moreover, provides insight into the behavior observed for quenches in systems of ultracold atoms in cavity quantum electrodynamics [62, 63], where similar results as the ones shown in Fig. 2(a) were reported for the order parameter of the Dicke phase transition. A systematic comparison with these works requires the development of a model including noise and dissipation [64, 65]. These are essential features of cavity quantum electrodynamics setups [66] and will be the subject of future work.

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Appendix A: Exact solution and Defect density

Before outlining the solution strategy for our problem a comment is in order here. The dynamics of each oscillator can be solved exactly [67–69] and any dynamical state $\psi(x,t)$ in the representation of the coordinate x can be expressed as

$$\psi(x,t) = \sum \alpha_n \psi_n(x,t)\,, \tag{A1}$$

where α_n are time independent constants and the dynamical eigenstates are given by

$$\begin{aligned}\psi_n(x,t) =& \frac{1}{\sqrt{2^n n!}}\left(\frac{1}{2\pi\xi^2(t)}\right)^{\frac{1}{4}} e^{-\tilde{\Omega}(t)\frac{x^2}{2}} \\ & H_n\left(\frac{x}{\sqrt{2}\xi(t)}\right) e^{-i\left(n+\frac{1}{2}\right)\lambda(t)}\,.\end{aligned} \tag{A2}$$

The effective frequency $\tilde{\Omega}(t)$ can be expressed in terms of the effective width $\xi(t)$ as

$$\tilde{\Omega}(t) = -i\frac{\dot{\xi}(t)}{\xi(t)} + \frac{1}{2\xi^2(t)}\,, \tag{A3}$$

while $\lambda(t)$ is the total phase accumulated at time t and reads

$$\lambda(t) = \int^t \frac{dt'}{2\xi^2(t')}. \tag{A4}$$

The exact time evolution of each harmonic oscillator can be then fully described by a single complex parameter, i.e. the effective width $\xi(t)$, which satisfies the Ermakov–Milne equation:

$$\ddot{\xi}(t) + \Omega(t)^2\xi(t) = \frac{1}{4\xi^3(t)}\,. \tag{A5}$$

If the initial state is a pure state of the basis (A2), say, the ground state, then all the coefficients α_n of Eq. (A1) vanish except for the coefficient α_0. This holds also at all later times, and thus in the exact dynamical basis (A2) no excited states will be generated. However at each time $t > t_0$ the dynamical pure state $\psi_0(x,t)$ will, in general, be different from the instantaneous equilibrium ground state since the effective width will not coincide with the equilibrium basis $\psi_n^{\rm ad}(x,t)$, whose wave functions read

$$\psi_n {\rm ad}(x,t) = \frac{1}{\sqrt{2^n n!}}\left(\frac{\Omega(t)}{\pi}\right)^{\frac{1}{4}} e^{-\Omega(t)\frac{x^2}{2}} H_n\left(x\sqrt{\Omega(t)}\right)\,. \tag{A6}$$

Therefore, if we decompose any pure state $\psi_n(x,t)$ of the dynamical basis using the instantaneous equilibrium basis, it will generally contain a number of equilibrium excitations. Then, assuming to start the evolution in the equilibrium ground state at $t=-t_0$, the number of excitations in the instantaneous equilibrium basis at time t is given by [59]

$$n_{\rm exc}(t) = \sum_{n\in 2\mathbb{N}} n|c_n(t)|^2 \tag{A7}$$

where the coefficients $c_n(t)$ are the transition amplitudes between the dynamical state and the instantaneous equilibrium basis

$$c_n(t) = \int_{-\infty}^{+\infty} dx \psi_n^*(x,t)\psi_0(x,t)\,. \tag{A8}$$

The definition (A7) can be calculated by choosing different basis sets for the evaluation of transition amplitudes rather than the eigenstates given in (A6) [59]. However the basis of the eigenstates in (A6) is the most natural choice in the context of the Kibble-Zurek mechanism.

Using definition (A7) together with Eq. (A8) one can derive an explicit expression for the excitation number $n_{\rm exc}(t)$. For this purpose we evaluate the transition amplitudes

$$c_n(t) = \int_{-\infty}^{+\infty} dx \psi_n^{\rm ad*}(x,t)\psi_0(x,t) = \frac{1}{\sqrt{2^n n!\pi}}\left(\frac{\Omega(t)}{2\xi^2(t)}\right)^{\frac{1}{4}} \int_{-\infty}^{+\infty} dx e^{-(\Omega(t)+\tilde{\Omega}(t))\frac{x^2}{2}} H_n\left(\sqrt{\Omega(t)}x\right)\,. \tag{A9}$$

We perform a change of variable and cast the integral as follows:

$$\int_{-\infty}^{+\infty} dx e^{-(\Omega(t)+\tilde{\Omega}(t))x^2} H_n\left(\sqrt{\omega\tilde{(t)}}x\right) = (\Omega(t))^{-\frac{1}{2}} \int_{-\infty}^{+\infty} e^{-\left(\frac{\tilde{\Omega}(t)}{\Omega(t)}+1\right)\frac{s^2}{2}} H_n\left(s\right) ds\,.$$

We then employ the generating function for Hermite polynomials in the integral:

$$\begin{aligned}\int_{-\infty}^{+\infty} e^{-\left(\frac{\tilde{\Omega}(t)}{\Omega(t)}+1\right)\frac{s^2}{2}} H_n\left(s\right) ds &= \lim_{t\to 0}\frac{d^n}{dt^n}\int_{-\infty}^{+\infty} e^{-\left(\frac{\tilde{\Omega}(t)}{\Omega(t)}+1\right)\frac{s^2}{2}} e^{2st-t^2} ds = \sqrt{\frac{2\pi}{\left(\frac{\tilde{\Omega}(t)}{\Omega(t)}+1\right)}}\lim_{t\to 0}\frac{d^n}{dt^n} e^{-t^2\frac{(\tilde{\Omega}(t)-\Omega(t))}{(\Omega(t)+\tilde{\Omega}(t))}} \\ &= \begin{cases} \sqrt{\frac{2\pi}{\left(\frac{\tilde{\Omega}(t)}{\Omega(t)}+1\right)}}\frac{n!}{\frac{n}{2}!}\left(\frac{\tilde{\Omega}(t)-\Omega(t)}{\tilde{\Omega}(t)+\Omega(t)}\right)^{n/2} & n\in 2\mathbb{Z} \\ 0 \quad n\in 2\mathbb{Z}+1 \end{cases}\end{aligned} \tag{A10}$$

Thus the probability of having n excitations in the evolved state at the time t is given by

$$|c_{n0}(t)|^2 = \frac{(n-1)!!}{n!!}\frac{\sqrt{2\Omega(t)}}{\xi(t)\left|\tilde{\Omega}(t)+\Omega(t)\right|}\left|\frac{\tilde{\Omega}(t)-\Omega(t)}{\tilde{\Omega}(t)+\Omega(t)}\right|^n\,. \tag{A11}$$

We insert this expression in Eq. (A7) and obtain the number of excitations at time t:

$$n_{exc}(t) = \frac{\xi^2}{2\Omega(t)}\left[\left(\frac{1}{2\xi^2}-\Omega(t)\right)^2 + \left(\frac{\dot{\xi}}{\xi}\right)^2\right]\,. \tag{A12}$$

From this expression we can also determine the ground state fidelity $f(t) = |c_0(t)|^2$, which reads:

$$f(t) = |c_{00}(t)|^2 = \frac{2\Omega(t)}{\xi(t)}\left[\left(\frac{1}{2\xi^2}-\Omega(t)\right)^2 + \left(\frac{\dot{\xi}}{\xi}\right)^2\right]^{-1/2}\,. \tag{A13}$$

Appendix B: Slow quench dynamics starting at the critical point

We here consider the semi ramp case, when the magnetic field is continuously quenched from the critical point far into the symmetry broken phase. The dynamical protocol is still a linear ramp of the magnetic field $h = 1+\delta t$ and the evolution begins at criticality ($t = 0$) with the system lying in its instantaneous ground state. For $t > 0$ the harmonic oscillator frequency varies as

$$\Omega(t)^2 = \delta t + 1/N^{2/3} \tag{B1}$$

where, once again, we discarded time dependent finite size corrections and sub-leading terms which do not modify the universal behavior as well as unimportant numerical factors.

It is convenient to employ the rescaling

$$\xi = \delta^{-1/6}\tilde{\xi} \qquad t = \delta^{-1/3}s \tag{B2}$$

already introduced in the main text. The Ermakov-Milne equation now reads

$$\ddot{\xi}(s) + \Omega(s)^2\xi(t) = \frac{1}{4\xi^3(s)}. \tag{B3}$$

with the rescaled frequency given by

$$\tilde{\Omega}(s)^2 = s + \Lambda^{-2/3} \tag{B4}$$

where $\Lambda = (\delta\, N)$. From now one we will discard the $\tilde{}$superscript over rescaled quantities, since they are the only ones appearing in the following calculations. The solution of latter equation can be constructed from the motion of the associated classical Harmonic oscillator

$$\ddot{x}(s) + \Omega(s)^2 x(s) = 0. \tag{B5}$$

This equation admits the two independent solutions

$$x_1(s) = \mathrm{Ai}\left(-\Omega^2\right) \tag{B6}$$
$$x_2(s) = \mathrm{Bi}\left(-\Omega^2\right) \tag{B7}$$

where we omitted the s dependence on Ω. The functions $\mathrm{Ai}(x)$ and $\mathrm{Bi}(x)$ are Airy functions. The two functions $x_1(s)$ and $x_2(s)$ have the constant and finite Wronskian

$$\mathrm{Wr}(x_1, x_2) = \frac{1}{\pi}. \tag{B8}$$

It is convenient to rewrite the solutions of equation (B3) as a pair of complex conjugate solutions w and w^* with

$$w = a x_1(s) + b x_2(s) \tag{B9}$$

where $a \in \mathbb{C}$ and $b \in \mathbb{R}$ are constants. Since Eq. (B5) is homogeneous one can rescale the two solution by a constant factor, then, without loss of generality, one can impose $b = 1$. The function

$$\xi(s) = \sqrt{w w^*} \tag{B10}$$

is solution of the Ermakov-Milne equation (A5) if

$$\mathrm{Wr}(w, w^*) = 2i\mathrm{Im}(a)\mathrm{Wr}(x_1, x_2) = i, \tag{B11}$$

which univocally fixes the imaginary part of a. In order to completely define the solution solution one should find the appropriate value of $\mathrm{Re}(b)$ which satisfy the boundary condition

$$\frac{1}{2\xi(0)^2} = \Lambda^{-1/3} \tag{B12}$$

By inverting this expression one readily obtains

$$2\xi(0)^2 = \Lambda^{1/3}\,. \tag{B13}$$

When the thermodynamic limit is taken first then the right-hand side (r.h.s.) of this expression diverges. We then consider the limit when $\Lambda^{-2/3}$ can be neglected in the argument of the Airy functions but the r.h.s. of Eq. (B13) remains finite. We obtain the expression

$$\mathrm{Re}(a) = \frac{x_2(0)}{x_1(0)} \pm \sqrt{\frac{\Lambda^{1/3}}{2x_1(0)} + \mathrm{Im}(a)}\,. \tag{B14}$$

Therefore the quantity $\mathrm{Re}(a)$ diverges in the thermodynamic limit for finite values of the ramp velocity δ. Employing the asymptotic expression for the Airy functions and neglecting oscillatory terms as it was done in [70], the asymptotic time limit of the scale parameter ξ is

$$\lim_{s\to\infty}\xi(s)^2 = \frac{1+|a|^2+2\mathrm{Re}(a)}{\pi\Omega(s)} \tag{B15}$$

which, once inserted into Eq. (A12), leads to the relation:

$$n_{\mathrm{exc}}(t) \propto \mathrm{Re}(a)^2\,, \tag{B16}$$

which scales as $\Lambda^{1/3}$. This expression is equivalent to the one obtained in [70] for a gapless system and suggests a KZ scaling for large sizes.

Appendix C: Slow quench dynamics ending at the critical point

Let us now consider the case of a slow quench starting in the paramagnetic phase and ending at the critical point. The magnetic field is given by the expression $h = h_c + \delta\, t$, the dynamics starts at $t = -t_0$ in the paramagnetic phase. The magnetic field slowly approaches the critical point at $t = 0$. The solution to Eq. (B3) is still given by Eqs. (B9) and (B10) but with the boundary conditions

$$\lim_{s\to-\infty}\frac{1}{2\xi(s)^2} = \Omega(s), \tag{C1}$$

$$\lim_{s\to-\infty}\dot{\xi}(s) = 0. \tag{C2}$$

These conditions are consistent with the system being in the adiabatic ground state at large $|t|$. In the $s\to\infty$ limit Ω^2 diverges and one must use the asymptotic expansion for the Airy functions

$$\lim_{s\to-\infty} x_1(s) \approx \frac{\cos\left(\frac{2}{3}\Omega^3 - \frac{\pi}{4}\right)}{\sqrt{\pi}\Omega^{1/4}}, \tag{C3}$$

$$\lim_{s\to-\infty} x_2(s) \approx \frac{\sin\left(\frac{2}{3}\Omega^3 - \frac{\pi}{4}\right)}{\sqrt{\pi}\Omega^{1/4}}. \tag{C4}$$

In order to fulfill conditions (C1), the oscillatory terms in the expression for ξ must cancel for large s, leading to

$$\mathrm{Re}(a) = 0, \tag{C5}$$

$$\mathrm{Im}(a) = b. \tag{C6}$$

Moreover one has to impose the condition

$$\mathrm{Wr}(w, w^*) = 2i\mathrm{Im}(a)b\mathrm{Wr}(x_1, x_2) = i, \tag{C7}$$

which fully determines the coefficients in Eq. (B9)

$$\mathrm{Im}(a) = b = \sqrt{\frac{\pi}{2}}. \tag{C8}$$

The resulting expression for the scale factor is

$$\xi(s)^2 = \frac{\pi}{2}\mathrm{Ai}\left(-\Omega(s)^2\right)^2 + \frac{\pi}{2}\mathrm{Bi}\left(-\Omega(s)^2\right)^2 \tag{C9}$$

the number of defects is given by the formula

$$n_{exc}(s) = \frac{\xi(s)^2}{2\Omega(s)}\left[\left(\frac{1}{2\xi(s)^2} - \Omega(s)\right)^2 + \left(\frac{\dot{\xi(s)}}{\xi(s)}\right)^2\right] \tag{C10}$$

which is identical to Eq. (A12), since this quantity is invariant under the rescaling in Eq. (B2). The number of defects at the final point of the ramp (which is the critical point) is obtained by evaluating Eq. (C10) at $s = 0$. At this instant the rescaled frequency is given by its finite size correction $\Omega(0) = \Lambda^{-1/3}$, while the scale factor ξ reads,

$$\xi(0)^2 = \frac{\pi}{2}\mathrm{Ai}\left(-\Lambda^{-2/3}\right)^2 + \frac{\pi}{2}\mathrm{Bi}\left(-\Lambda^{-2/3}\right)^2 . \tag{C11}$$

Let us consider the thermodynamic limit first $\Lambda \to \infty$. In this case the argument of the Airy functions goes to zero and the terms in the square brackets of Eq. (C10) read

$$\frac{1}{4\xi(0)^4} = \frac{3^{8/3}\Gamma(2/3)^4}{16\pi^2} \tag{C12}$$

$$\left(\frac{\dot{\xi}(0)}{\xi(0)}\right) = \frac{3^{2/3}\Gamma(2/3)^2}{\Gamma(1/3)^2} \tag{C13}$$

which inserted into the defect density in Eq. (C10) lead to the result

$$n_{\mathrm{exc}}(0) = \frac{\pi\,\Lambda^{1/3}}{3^{2/3}\Gamma(1/3)^2} \tag{C14}$$

where we restricted to the leading term in the $\Lambda \to \infty$ limit. Therefore the result for the number of excitations diverges in the thermodynamic limit with a power $N^{1/3}$. However the residual heat is finite since it is obtained by multiplying the divergent defect density by the vanishing oscillator frequency $Q(0) = \Delta(0) n_{\mathrm{exc}}(0)$ leading to

$$Q = \frac{\pi\,\delta^{1/3}}{3^{2/3}\Gamma(1/3)^2} \tag{C15}$$

which agrees with the KZ scaling of Ref. [47]. For a finite size system $N < \infty$ the slow ramp limit $\delta \to 0$ coincides with the $\Lambda \to 0$ limit of Eq. (C10) evaluated at $s = 0$. The leading term in this case is generated by the velocity correction to the effective frequency

$$\lim_{\Lambda\to 0}\frac{\dot{\xi}(0)}{\xi(0)} = -\frac{5}{24}\Lambda^{2/3}, \tag{C16}$$

which substituted into Eq. (C10) evaluated at $s = 0$ gives

$$n_{\mathrm{exc}}(0) = \frac{25}{2304}\Lambda^2 \propto \delta^2 \tag{C17}$$

which leads to the expected adiabatic correction for the residual heat $Q \propto \delta^2$ in a finite size system [61, 70].

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