Weak Gravity and Swampland Conjectures

Arthur Hebecker (Heidelberg)

emphasising some recent original work with

Y. Hamada / G. Shiu / P. Soler and S. Leonhardt and X. Gao / D. Junghans and T. Daus / J. March-Russell / S. Leonhardt

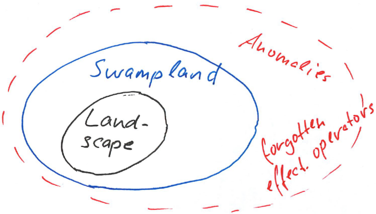
Outline

- General (Introductory) Comments on the Swampland
- From Weak Gravity to Inflation
- de Sitter Conjectures (and KKLT)
- From Weak Gravity to Global Symmetries and Wormholes

Introductory remarks on the Swampland

Vafa '05, Ooguri/Vafa '06

- The landscape of string compactifications is huge but discrete.
- The space of 4d EFTs is continuous → not all EFTs are in the landscape. But this is not yet very useful ....
- Key question: Are there general criteria for a given model not to be in the landscape?
- Does this logic work in 'quantum gravity' (rather than in string theory)? Can we learn about Quantum Gravity?



Some classical ‘Swampland Criteria’:

No exact global symmetries

see e.g. Banks/Dixon ’88, Kamionkowksi/March-Russell, Holman et al. ’92, Kallosh/Linde^2/Susskind ’95, Banks/Seiberg ’10, Harlow/Ooguri ’18

Infinite Moduli Space / Distance Conjecture *

Vafa ’05, Ooguri/Vafa ’06
More recent work by Palti, Grimm/Palti/Valenzuela, Lee/Lerche/Weigand, Xu ...

The weak gravity conjecture

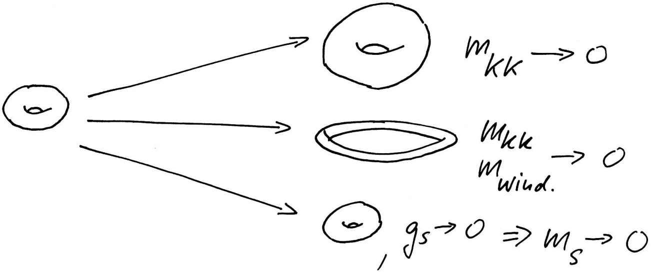
Arkani-Hamed/Motl/Nicolis/Vafa ’06

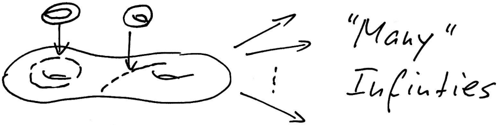
- Roughly speaking: ‘Gravity is always the weakest force.’
- More concretely:

  For any U(1) gauge theory there exists a charged particle with

$$\boxed{m < q\, M_P} \qquad \text{(with } q = gn\,\text{)}\,.$$

*) Infinities in moduli space and cutoff-suppression ...





Conjecture that either $m_{KK} \to 0$ or $m_s \to 0$ are always involved

Lee/Lerche/Weigand '19

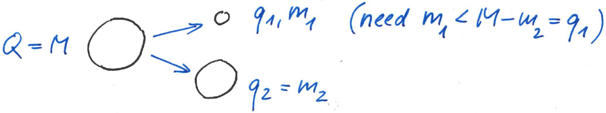
Weak gravity conjecture (continued)

- The historical supporting argument:

  In the absence of sufficiently light, charged particles, extremal BHs are stable. Such remnants are believed to cause inconsistencies.

  see e.g. Susskind '95

  Indeed, the boundary of stability of extremal black holes is precisely $q/m = 1$ for the decay products (here $M_P \equiv 1$).

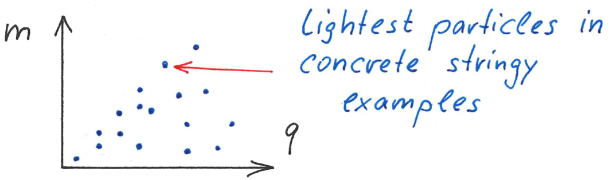


Weak gravity conjecture (continued)

- Another (possibly stronger?) supporting argument:

  Quantum gravity forbids global symmetries. We should not be able to take the limit of small gauge couplings.

  The WGC quantifies this on the basis of stringy examples.



Direct relation to compact geometries:

AH/Rompineve/Westphal/Witkowski ... Demirtas/Long/...
...McAllister/Stillman ... Cota/Klemma/Schimannek '20

- It is not obvious how the WGC could impact phenomenology.
- Interesting proposals have been made by

  Ooguri/Vafa, Palti, Ibanez/Valenzuela/Martin-Lozano/
  Montero, Reece, ... '16...'19.

- One of the widely accepted applications is to constraining large-field inflation by constrainig axions

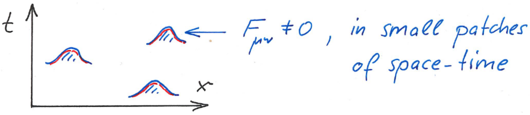
  Cheung/Remmen; de la Fuente/Saraswat/Sundrum . . . '14
  Rudelius; Ibanez/Montero/Uranga/Valenzuela; Brown/Cottrell/Shiu/Soler/..
  ..Staessens/Ye; Bachlechner/Long/McAllister; AH/Rompineve/Witkowski;
  Junghans; Heidenreich/Reece/Rudelius; Kooner/Parameswaran/Zavala;
  Harlow; AH/Rompineve/Westphal; . . . '15
  Ooguri/Vafa, Conlon/Krippendorf . . . '16
  Dolan/Draper/Kozaczuk/Patel; AH/Henkenjohann/Witkowski/Soler . . . '17

Axions

- Both in pheno-model-building and in string compactifications, axion-like fields are abundant:

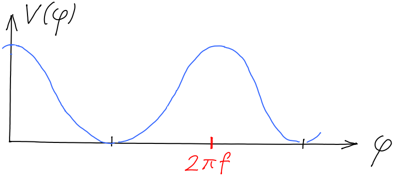
$$\mathcal{L} \supset -\frac{1}{2}(\partial\varphi)^2 - \frac{1}{32\pi^2}\left(\frac{\varphi}{f}\right)\mathrm{tr}(F\tilde{F})\,.$$

- Their shift symmetry is generically broken by instantons:



$$\Rightarrow \quad V_{\mathit{eff}} \sim \cos(\varphi/f)\ ,$$

$$\varphi \equiv \varphi + 2\pi f\,.$$



...to apply the WGC to axions, it has to be generalized:

Generalizations of the weak gravity conjecture

- The basic lagrangian underlying the above is

$$S \sim \int (F_2)^2 + m \int_{1-dim.} d\ell + g \int_{1-dim.} A_1 \,.$$

- This generalizes to charged strings, domain walls etc.

$$S \sim \int (F_{p+1})^2 + T \int_{p-dim.} dV + g \int_{p-dim.} A_p$$

with

$$F_{p+1} = dA_p \,.$$

Generalizations to instantons

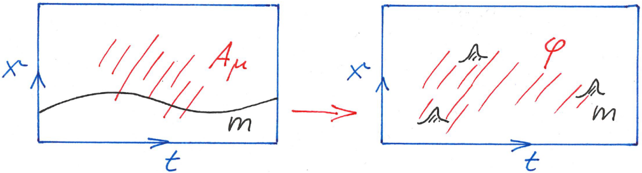
- One can also lower the dimension of the charged object, making it a point a in space-time:

$$S \;\sim\; \int (d\varphi)^2 \;+ m + g\,\varphi(x_{inst.})\,.$$

This should be compared with

$$S \sim \int (d\varphi)^2 + \int \mathrm{tr}(F^2) + \int \frac{1}{f}\,\varphi\,\mathrm{tr}(F\tilde{F})\,,$$

where $\int \mathrm{tr}(F^2) \sim S_{inst.} \sim m\,.$



WGC for instantons and axions

- The consequences for inflation are easy to derive.
- First, recall that the instantons induce a potential

$$V(\varphi) \;\sim\; e^{-S_{inst.}} \cos(\varphi/f)\,.$$

- Since, for instantons, $g \to 1/f$ and $m \to S_{inst.}$ we have

$$m < g\, M_P \qquad \Rightarrow \qquad \boxed{S_{inst.} < M_P/f}\,.$$

- Theoretical control (dilute instanton gas) requires $S_{inst.} > 1\,.$
- This implies $f < M_P$ and hence
  large-field 'natural' inflation is in trouble.
- Moreover, even for $f < M_P$ one gets a lower bound on the
  strength of instanton effects: $\exp(-S_{inst.}) > exp(-M_P/f)$.

- There are (at least) three ideas about how to enlarge the axionic field range without losing calculational control:

  (a) KNP Kim/Nilles/Peloso '04

  (b) N-flation Dimopoulos/Kachru/McGreevy/Wacker '05

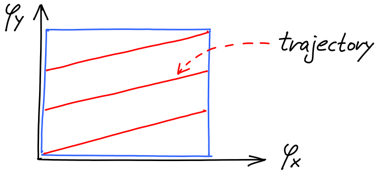
  (c) Axion-Monodromy McAllister/Silverstein/Westphal '08

- I will focus on **(a)** and a proposal derived from it, which we called Winding Inflation.

(a) KNP / Winding inflation

Kim/Nilles/Peloso '04; Berg/Pajer/Sjors '09; Ben-Dayan/Pedro/Westphal '14

- Consider a 'winding' trajectory on a 2d periodic field space:



- Clearly, such a trajectory can be much longer than the (naive) field range
- But: Realising the required potential through the interplay of multiple instanton effects is non-trivial!
- Thus, even getting only an effective trans-planckian axion appears to be difficult.

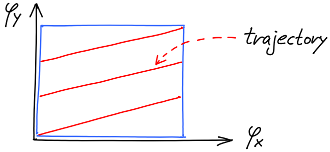
Winding Inflation

AH/Mangat/Rompineve/Witkowski '15

- φ_x , φ_y are two 'stringy axions', both with $f < 1$ ($M_P \equiv 1$).
- They are also moduli. Hence, fluxes (e.g. $\langle F_3 \rangle \neq 0$ on the compact space) can be used to stabilize them.
- A judicious flux choice allows for stabilizing just one linear combination, forcing the remaining light field on the winding trajectory:

$$V \supset (\varphi_x - N\varphi_y)^2 + e^{-M} \cos(\varphi_x / f) + e^{-m} \cos(\varphi_y / F)$$

with $N \gg 1$.



Concrete realization at (partially) large complex stucture

- Let $z_1, \cdots, z_n, u, v$ be complex structure moduli of a type-IIB orientifold, let $\mathrm{Im}(u) \gg \mathrm{Im}(v) \gg 1$.

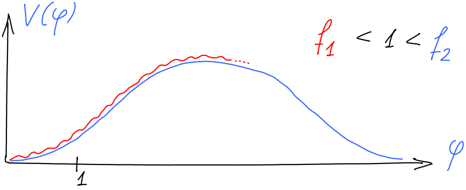
$$K = -\log\left(\mathcal{A}(z, \bar{z}, u - \bar{u}, v - \bar{v}) + \mathcal{B}(z, \bar{z}, v - \bar{v})e^{2\pi i v} + \mathrm{c.c.}\right)$$

$$W = w(z) + f(z)(u - Nv) + g(z)e^{2\pi i v}$$

- Without exponential terms, it is clear that W leaves one of the originally shift-symmetric directions $\mathrm{Re}(u)$ and $\mathrm{Re}(v)$ flat
- If $N \gg 1$, this direction is closely aligned with $\mathrm{Re}(u)$
- The exponential terms induce a long-range cosine potential for this light field φ:

$$e^{2\pi i v} \quad \rightarrow \quad \cos(2\pi\varphi/N)$$

- For appropriate choice of m, M (i.e. of $\mathrm{Im}(u)$, $\mathrm{Im}(v)$), the potential takes the form



- Intriguingly, this realises a loophole in the WGC-argument against axionic inflation: Rudelius, Brown/Cottrell/Shiu/Soler, '15

- A heavy, 'sub-planckian' instanton maintains the WGC, while a lighter 'super-planckian' instanton realises inflation.

  [One says that only the mild form of the WGC holds.]

  For other arguments and loopholes see e.g.
  de la Fuente/Saraswat/Sundrum '14 Bachlechner/Long/McAllister '15.

Let us take a

Structural view on the above 'Winding Inflation' model

Recall how gauging/Higgsing works in general:

(p) $\mathcal{L}_p \;=\; \int_d |F_{p+1}|^2$ with $F_{p+1} = dA_p$

($p-1$) $\mathcal{L}_{p-1} \;=\; \int_d |F_p|^2$ with $F_p = dA_{p-1}$

(Higgsed) $\mathcal{L}_{p\,/\,p-1} \;=\; \int_d \; |F_{p+1}|^2 + |F_p + A_p|^2\,.$

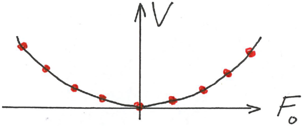
The most familiar example is, of course, $p = 1$ and $p - 1 = 0$:

(Higgsed) $\mathcal{L}_{1\,/\,0} \;=\; \int_d \; |F_2|^2 + |d\varphi + A_1|^2\,.$

- The above includes the slightly special case of (-1)-forms:

$(p=-1) \quad \mathcal{L}_{-1} = \int_d |F_0|^2$ where, by flux quantization,

$F_0 \in \alpha \times \mathbb{Z}$



- All the dynamics is a discrete set of vacua with domain walls coupled to A_3 of $*F_0 = F_4 = dA_3$.

- Crucially, one can use this theory to Higgs a 0-form, i.e. an axion

Dvali '05; Kaloper/Sorbo '08
(also: Quevedo/Trugenberger '97)
AH/Henkenjohann/Witkowski '17
Heidenreich/McNamara/Montero/Reece/Rudelius/Valenzuela '20

$$\mathcal{L}_{0\,/\,-1} = \int_d |F_1|^2 + |F_0 + A_0|^2 = \int_d (\partial\varphi)^2 + |F_0 + \varphi|^2$$

$$\mathcal{L}_{0\,/\,-1} \;=\; \int_d (\partial\varphi)^2 + |F_0 + \varphi|^2$$

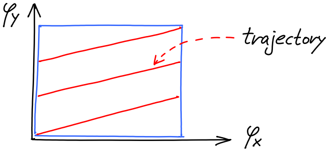
- This is of course just ‘the gauge-theory perspective’ on axion monodromy.
- Since ‘turning on fluxes corresponds to gauging’, the flux landscape gives mass to the (axionic components of) moduli in precisely this way.
- In ‘Winding Inflation’ we used $W \sim u - Nv$, with complex-structure moduli u and v.
- This corresponds to Higgsing a specific linear combination of $\varphi_x = \text{Re}\, u$ and $\varphi_y = \text{Re}\, v$:

$$\mathcal{L}_{0\,/\,-1} \;=\; \int_d (\partial\varphi_x)^2 + (\partial\varphi_y)^2 + |F_0 + \varphi_x + N\varphi_y|^2$$

Conceptual summary of Winding Inflation

$$\mathcal{L}_{0\,/\,-1} \;=\; \int_d (\partial\varphi_x)^2 + (\partial\varphi_y)^2 + |F_0 + \varphi_x + N\varphi_y|^2$$

- The underlying idea is to generate a transplanckian axion by Higgsing a linear combination of two subplanckian axions.



- Whether this will actually work for inflation is still under disussion

see e.g. Palti '15 and Blumenhagen/Herschmann/Wolf '16
Shiu/Staessens/Ye '15, Shiu/Staessens '18
AH/Junghans/Schachner '19
Carta/Mininno/Righi/Westphal '21

An Aside:

- Return, for a moment, to the more conventional WGC with 1-forms rather than 0-forms (axions).

- Here, the same gauging idea can apparently be used.

Saraswat '16

$$\mathcal{L}_{1\,/\,0} \;=\; \int_d (F_x)^2 + (F_y)^2 + |d\varphi + A_x + NA_y|^2$$

with $F_x = dA_x$, $F_y = dA_y$,

and with the surviving light gauge field $A_{\rm light} \sim NA_x - A_y$ having gauge coupling

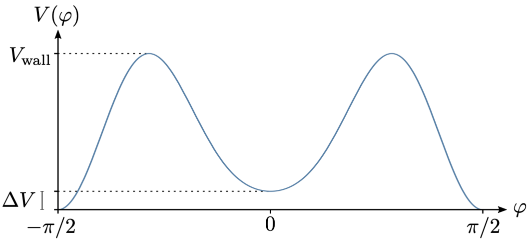
$$g_{\rm light} \sim \frac{1}{\sqrt{N^2+1}} \; .$$

- So far, not even a ‘believable’ stringy implementation (like the Winding proposal) has been suggested.
- Technically, this appears to be purely accidental (related to the available cycles in the 0-form and 1-form case).
- If a model can be found, one might also expect that an iteration of appropriate gaugings can be realized
- Thus, if Higgsing can indeed avoid the WGC, it might even do so exponentially due to the clockwork mechanism.

Choi/Kim/Yun/Im, Kaplan/Rattazzi, ...
(see, however, Ibanez/Montero '17)

- Winding uplifts: One can use the 'Winding Inflation' method to produce a parametrically small uplift (by tuning the complex-structure dependent prefactors).

AH/Leonhardt '20



- Gopakumar-Vafa-hierarchies: The cosine-prefactors are governed by GV-invariants. Thus, much of the tuning involved can be avoided in models with large GV-hierarchies.

Carta/Mininno/Righi/Westphal '21

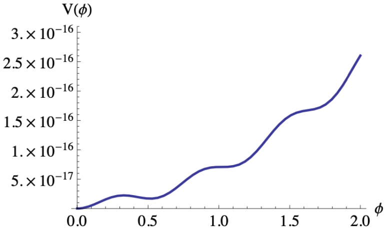
Mini-Summary:

I think some of the simplest stringy inflation models are of the type 'interplay of several cosine terms'.

(This could be KNP, Winding or a (modest-range) monodromy model – or some mix thereof.)

see e.g. Kappl/Nilles/Winkler '15

Figure from Kadota/Kobayashi/Oikawa/Omoto/Hajime/Otsuka/Tatsuishi '16:



$f = 0.1$

De Sitter Swampland Conjectures

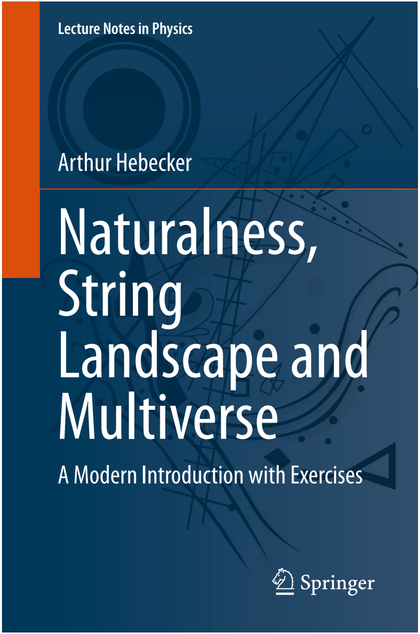
- One possible Swampland constraint (with very far-reaching implications!) is $\Lambda_{cosm.} \leq 0$.
- Indeed, a longstanding unease about the status of de Sitter space in quantum gravity exists.

Woodard, Danielsson, Van Riet, Bena, Grana, Sethi, Dvali, ...

- More recently, concrete formulations of varying strength have been considered within the Swampland program
  (e.g. $V'/V > \mathcal{O}(1)$ or $V''/V < -\mathcal{O}(1)$)

Danielsson/Van Riet
Obied/Ooguri/Spodyneiko/Vafa
Garg/Krishnan, Andriot
Ooguri/Palti/Shiu/Vafa '18

(see also the critical or at least cautionary remarks in AH/Denef/Wrase and AH/Wrase '18)



Lecture Notes in Physics

Arthur Hebecker

Naturalness, String Landscape and Multiverse

A Modern Introduction with Exercises

Springer

Is the whole paradigm in danger?

Do we have a good alternative ?

Problems with de Sitter in string compactifications

- Let us briefly pause to explain one of the reasons why realizing de Sitter is difficult.

- The generic result of a compactification with volume $\mathcal{V}$ (and some positive-energy source in the compact space) is

$$\mathcal{L} \ \sim \ \mathcal{V} \left[\mathcal{R}_4 - \frac{(\partial \mathcal{V})^2}{\mathcal{V}^2} - E \right] \ .$$

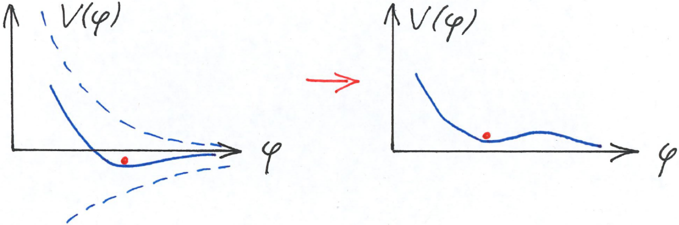
- After Weyl-rescaling to the Einstein frame and introducing the canonical field $\varphi = \ln(\mathcal{V})$, one finds

$$\mathcal{L} \ \sim \ \left[\mathcal{R}_4 - (\partial \varphi)^2 - E \, e^{-\varphi} \right] \ .$$

- The exponent is usually $\mathcal{O}(1)$, so the simplest compactifications lead to steep potentials: $|V'|/V \sim \mathcal{O}(1)$.

String compactifications: flux landscape

- Combining two such runaway potentials with different sign allows in principle for AdS solutions.
- At least 3 potential terms with different falloff and appropriate coefficients are needed to get dS.



If all parameters involved are $\mathcal{O}(1)$, this can never happen in parametric control.

Dine/Seiberg '85
Ooguri/Palti/Shiu/Vafa '18

However, with some tuning of fluxes ‘accidentally’ small and large parameters can be realized.

The earliest such scenario for realizing dS was

KKLT

Kachru/Kallosh/Linde/Trivedi ’03

An alternative is the ‘large volume scenario’ or LVS

Balasubramanian/Berglund/Conlon/Quevedo ’05

We will first recall how KKLT works and discuss recent criticism by

Moritz/Retolaza/Westphal ’17

which was historically important in the above debate.

But then we will come to a rather different concern, which at the moment appears to threaten KKLT more seriously ....

(2-slide reminder of) KKLT

- CY with all complex-structure (shape) moduli fixed by fluxes; The only field left: Kahler modulus $T = \tau + ic$ with $\tau \sim \mathcal{V}^{2/3}$.

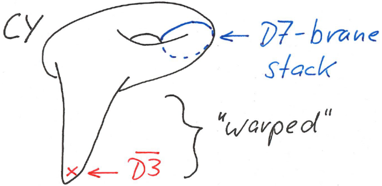
- $K = -3\ln(T + \overline{T})$; fluxes give $W = W_0 = \text{const.}$,

  $\Rightarrow \quad V \equiv 0$ ('no scale') .

- Gaugino condensation on D7 brane stack: $W = W_0 + e^{-T}$.

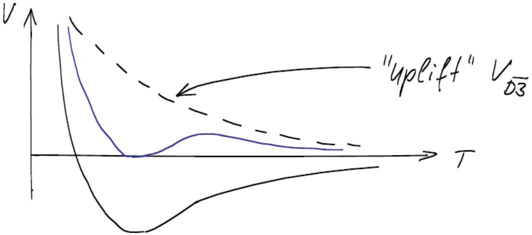
  Derendinger/Ibanez/Nilles, Dine/Rohm/Seiberg/Witten '85

- Small uplift by $\overline{\text{D3}}$-brane in a warped throat: $V \to V + c/\tau^2$.



KKLT

- The scalar potential is changed first to SUSY-AdS, then to an 'uplifted' meta-stable de Sitter potential:



- A longstanding critical debate has targeted the metastability of the $\overline{D3}$ in view of flux-backreaction.

  (My take on this is that metastability remains plausible.)

Bena, Grana, Danielsson, Van Riet, ...

KKLT under attack

Moritz/Retolaza/Westphal '17
Gautason/Van Hemelryck/Van Riet '18

- Recent criticism was rooted in a possibly too simplistic treatment of D7-gaugino–bulk-coupling:

$$\mathcal{L}_{10} \;\supset\; |G_3|^2 \,+\, G_3 \cdot \Omega_3 \,\langle \lambda\lambda \rangle \,\delta_{D7} \;.$$

Camara/Ibanez/Uranga '04, Koerber/Martucci '07
Baumann/Dymarsky/Klebanov/Maldacena/McAllister '06
Heidenreich/McAllister/Torroba '10

- It is clear what to expect:
  G_3 backreacts, becoming itself singular at the brane.

- Plugging this back into the action,
  one gets a divergent effect of type $(\delta_{D7})^2$.

- Now anything can happen....

KKLT rescued

Hamada/AH/Shiu/Soler '18,'19; Kallosh '19; Carta/Moritz/Westphal '19

- Singular gaugino effects have been observed before, in other string models.

  Horava/Witten '96

- It has been shown that a highly singular $\langle\lambda\lambda\rangle^2$-term saves the day by 'completing the square'. Applied to our case:

$$\mathcal{L}_{10} \;\supset\; \Big| G_3 \,+\, \Omega_3 \,\langle\lambda\lambda\rangle\, \delta_{D7} \Big|^2 \,.$$

- Very roughly speaking, one now writes $G_3 = G_3^{flux} + \delta G_3$ and lets the second term cancel (most of) the δ-function.

  The result is (**very** roughly):

$$\mathcal{L}_{10} \;\supset\; \Big| G_3^{flux} \,+\, \langle\lambda\lambda\rangle \Big|^2 \quad \rightarrow \quad \Big| D_T W_0 \,+\, \partial_T e^{-T} \Big|^2 .$$

The perfect square structure in M-theory

- The established part of the story is in M-theory (with x^{11} compactified on $S^1/\mathbb{Z}_2$). There, one has

$$S \sim -\int_{11} \left(G_4^2 \; - \; \delta(x^{11})(G_4)_{ABC\,11}\, j^{ABC} \right),$$

where $j^{ABC} \sim \overline{\lambda}\,\Gamma^{ABC}\lambda$.

- It is well-known that the divergence problem is resolved by the proposal (enforced by SUSY)

Horava/Witten

$$S \sim -\int_{11} \left(G_4 - \frac{1}{2}\delta(x^{11})\, j \right)^2 .$$

- Our proposal basically describes how an analogous quartic gaugino term on the brane must be added in type IIB.

(cf. Hamada/AH/Shiu/Soler '18/'19 for details)

In summary:

10d perfect square structure leads to
4d SUGRA perfect square structure
and to KKLT, including possible uplift.

$$e^K K^{T\overline{T}} \Big| D_T(W_0 + e^{-T}) \Big|^2$$

Recent related work by other groups

agreement with Carta/Moritz/Westphal,
still (partial) disagreement with Gautason/Van Hemelryck/Van Riet/Venken

Using Generalized Complex Geometry, the AdS parameter can be related to a parameter in 10d SUSY conditions.
⇒ fully 10d-local check of pre-uplift KKLT

Bena/Grana/Kovensky/Retolaza

Related attempt of component-level check w/o SUSY:

Kachru/Kim/McAllister/Zimet

However, non-local D7 action introduced ad hoc;
divergence cancellation in G_3 kinetic term remains unclear.

The advertised new concern starts with the

The Throat Glueing Problem

Carta/Moritz/Westphal '19

- Recall basic parametrics of KKLT:

$$V_{AdS} \sim -e^{-\mathrm{Re}(T)} \qquad \text{vs.} \qquad V_{Uplift} \sim e^{-K/g_s M}\,.$$

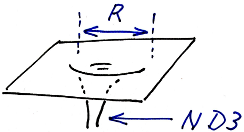
(Here K and M are the flux numbers of the two 3-cycles of the KS throat.)

- For a metastable uplift to dS, the two potentials must match:

$$\Rightarrow \qquad \mathrm{Re}(T) \simeq K\,/\,g_s M\,.$$

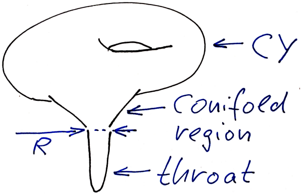
- At the same time, the throat carries $N = KM$ units of D3 charge, giving it a radius

$$R^4_{throat} \simeq g_s N\,.$$



Throat Glueing Problem (continued)

- However, at least most naively, $g_s \, \mathrm{Re}(T) \sim R^4_{CY}$ and the standard picture

  implies $R^4_{throat} < R^4_{CY}$.

- With the previous estimates, this leads to the problematic inequality

  $$g_s N \lesssim K/M$$

  or (using $K = N/M$)

  $$\mathcal{O}(1) \lesssim 1/g_s M^2 \,.$$

Throat Glueing Problem (continued)

- The problem is that $g_s M \simeq R^2_{S^3} \gtrsim 1$

  KS, KPV, Klebanov/Herzog/Ouyang '01

  for supergravity control and $M \gtrsim 12$

  KPV (see also Bena/Dudas/Grana/Lüst, Blumenhagen/Kläwer/Schlechter)

  for metastability of the anti-D3-brane.

- Thus, the standard picture of a small throat glued into the large bulk of a CY can not be maintained.

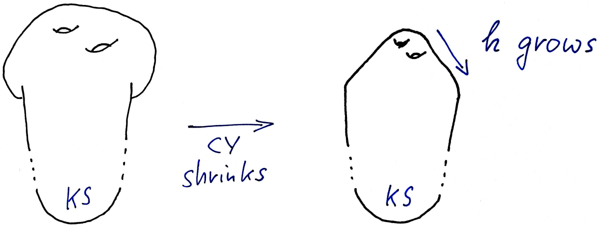
Is the Throat Glueing Problem deadly ?

- Not obviously, since a priori the warp factor $h(y)$ of

  $$ds_{10}^2 = h(y)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + h(y)^{1/2}\tilde{g}_{mn}dy^m dy^n$$

  is just some function on the CY.

- The Kahler modulus corresponds to $h(y) \rightarrow h(y) + \text{const.}$ It is a flat direction 'at the level of GKP'. So we may simply make the bulk smaller than the throat!



The singular-bulk problem

- An actual problem is not that the geometry defies our standard intuition, it is that the CY may be forced into a singular regime, since $h < 0$.

- The danger of growing singularities as $h \to h - \text{const.}$ has already been discussed in the Appendix of Carta et a;., but without turning this into a quantitative problem for KKLT.

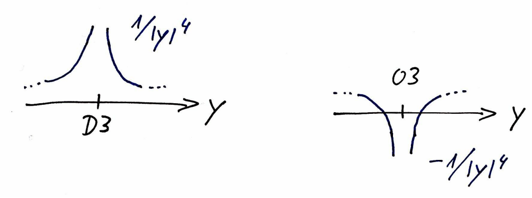
- The goal of next few slides is exactly this:

  Demonstrate that, generically, the regime of KKLT is enforcing $h < 0$ in a large portion of the CY geometry.

The singular-bulk problem (continued)

- Before starting, let us recall the standard behavior of h near D3-branes/O3-planes:

$h(y)$:



- The string-sized negative regions near O3s are not a problem
- Also having many O3s is a priori not a problem as long they are scattered, each with it's small negative region.
- The bulk singularity problem arises from the 'macroscopic' behaviour of $h(y)$.

The singular-bulk problem (continued)

- For quantifying the problem, a key insight is that the warped D7 size $\mathcal{V}_\Sigma$ determines the exponential effect:

$$\mathrm{Re}(T) \sim N/g_s M^2 \qquad \Rightarrow \qquad \mathcal{V}_\Sigma \sim N/M^2$$

with

$$\mathcal{V}_\Sigma = \int_\Sigma \sqrt{\tilde{g}}\, h(y) = \tilde{\mathcal{V}}_\Sigma \, \langle\, h \,\rangle_\Sigma \, .$$

- W.l.o.g., we use a CY such that $\tilde{\mathcal{V}} = \int_{CY} \sqrt{\tilde{g}} = 1$. Hence $\tilde{\mathcal{V}}_\Sigma$ is an $\mathcal{O}(1)$ number.

  $\Rightarrow$ We are in effect constraining the warp factor on Σ:

$$\langle\, h \,\rangle_\Sigma \sim N/M^2 \tilde{\mathcal{V}}_\Sigma \sim N/M^2 \, .$$

The singular-bulk problem (continued)

- In summary, for a large part of the D7 locus Σ we have

$$h \lesssim N/M^2 \, .$$

- We also know from GKP that h represent a form of 'electrostatic potential' for the D3 charge density on the CY:

$$-\tilde{\nabla}^2 h \, = \, g_s \, \tilde{\rho}_{D3} \, .$$

  Our normalization is such that $\tilde{\rho}_{D3}$ is a CY-metric δ-function for a single D3 brane.

- We see that h is a compact-space Green's function for a charge distribution of

  $g_s N$ units of positive charge, localized at conifold

  $-g_s N$ units of negative charge, scattered in the CY.

The singular-bulk problem (continued)

- If the parameter $g_s N$ were $\mathcal{O}(1)$, we would have $|\tilde{\partial} h| \sim 1$. (The details of the function are fixed by geometry and charge distribution. An additive constant is undetermined.)

- But in our case the variation is scaled up by $g_s N \gg 1$. At the same time h is bounded on the D7: $h \lesssim N/M^2$.

$$\Rightarrow \qquad \boxed{\frac{|\tilde{\partial} h|}{h} \gtrsim g_s M^2 \gtrsim M \gg 1}$$

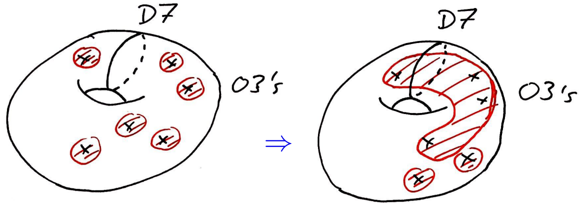
Now, by Taylor expanding at a point y_0 of the D7,

$$h(y_0 + \delta y) \approx h(y_0) + \partial_m h(y_0)\, \delta y^m\,,$$

we see that h runs negative near the D7: $|\widetilde{\delta y}| \lesssim 1/g_s M^2$.

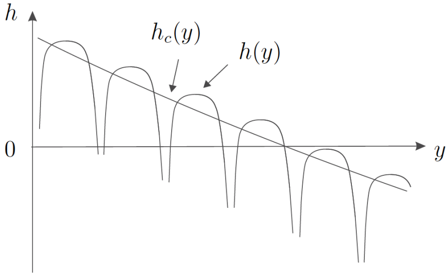
The singular-bulk problem (continued)

- A large singular region develops, eating up much of the classical geometry of the CY.



Singular-bulk problem with coarse-grained warp factor

- One may think in terms of a coarse-grained warp factor (cf. the coarse-grained electrostatic potential in a plasma).
- As it turns out, this quantity goes negative:



Escape routes and further problems

- There are, of course, possible escape routes (special O3/O7 arrangements; large $h^{1,1}$&N_c; stringy miracles).

Interesting recent suggestion:

- The dangerous, negative D3-tadpole-effects reside on O3 planes or on (curved) D7-stacks.

Carta/Moritz '21

- In the D7-case, it has been proposed that the corresponding singularities are resolved in an 'F-theoretic' way: by the a splitting into several branes of different type.
- The problem that remains are strongly curved regions and hence (at present) no control of the KKLT Kahler potential.

dS summary

- Establishing or disproving stringy de Sitter is a key goal.
- KKLT appears to survive the recent '10d line of attack'.
- It may fall victim to the bulk singularity problem.
- The LVS does not suffer from this issue.
- In parallel to (dis)proving KKLT/LVS in more and more detail, we should try to get stringy quintessence to work ...

An Aside on Quintessence:

- It is conceivable that all dS constructions will fail in the end.
- Quintessence is a natural way out, but this is also difficult..

see e.g. Cicoli/Pedro/Tasinato '12
(also: Cicoli/Burgess/Quevedo '11)

- In particular, one faces an *F*-Term Problem:

AH/Skrzypek/Wittner

- Namely, one needs an extremely large volume, where phenomenological SUSY-breaking implies:

$$e^K |D_x W|^2 \qquad \gg \qquad \left| e^K (|D_T W|^2 - 3|W|^2) \right|$$

⇒ completely new scalar-potential term needed!

Selection of recent work: Cicoli/DeAlwis/Maharana/Muia/Quevedo; Acharya/Maharana/Muia; Emelin/Tatar; Hardy/Parameswaran; Farakos

From Weak Gravity to Global Symmetries and Wormholes

- Another Swampland conjecture potentially relevant to phenomenology:

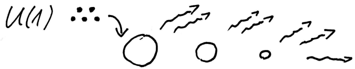
  No Global Symmetries

  see e.g. Banks/Dixon '88, Kamionkowksi/March-Russell, Holman et al. '92, Kallosh/Linde^2/Susskind '95, Banks/Seiberg '10

- But is this really a 'Swampland Conjecture'?

  – Indeed, consider an EFT with a global symmetry.

  – Standard BH evaporation physics will induce the expected violation.



  – the EFT is not constrained by this!

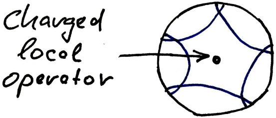
Introduction/Motivation (continued)

- Clearly, 'standard' BH evaporation physics is an overstatement. In, fact, at least in AdS (and with some assumptions), an independent argument for

  No (exact) global symmetries

  can be given.

Harlow/Ooguri '18



- However, our interest will be approximate global symmetries.
- Those are not forbidden and it is crucial to constrain their quality. In a sense, the low-energy effective field theorist has no other way to approach the issue.

Introduction/Motivation (continued)

- Again: Our interest is in quantitative conjectures against approximate global symmetries.
- One such approach is, of course, the Weak Gravity Conjecture:

$$\begin{aligned} \text{Gauge symmetry} &\rightarrow \text{global symmetry} \\ g &\rightarrow 0 \end{aligned}$$

(Ideal) claim of WGC: $g \gtrsim m/M_P$, where m is the mass of the lightest charge particle.

- Such a strong statement has not yet been proven.
- Rigorous progress has only been made in the context of the BH mass spectrum (i.e. masses of highly charged particles)

Cheung/Remmen .... Hamada/Noumi/Shiu .... '16..'20

Introduction/Motivation (continued)

- We want to consider a second route for approaching exact global symmetries:

AH/Daus/Leonhardt/March-Russell '20

gauge symm. → global symm. ← approx. global symm.

$g \rightarrow 0 \qquad\qquad 0 \leftarrow c$ (operator coeffs.)

- This second way of approaching a global symmetry is fundamentally different: no light vector is part of the EFT.

- Arguably, it is in fact the practically most useful way to think about a global symmetry

  (B-L, flavor symmetries, DM stability, flat axion potetials etc.)

What is the definition of an approximate global symmetry?

- Consider EFT with some (global) group action.
- Approximate Symmetry: All non-singlet operators are either irrelevant or have small coefficients ($c \ll 1$).
- Our goal: Quantify the smallness.

  see also Coleman/Lee, Rey $\sim$'90 .... Alonso/Urbano '17
  AH/Mikhail/Soler '18 ..... Alvey/Escudero '20
  (relies on wormholes – more details later...)

  see also Fichet/Saraswat '19

  (New conjecture inspired by BH evaporation:
  In a thermal plasma, the BH-induced violation effect should not exceed the effect of symmetry-violating local operators.)

- We want a derivation instead of a new conjecture (at least for a subclass of global symmetries).

Types of approximate global symmetries

- (1) Gauge derived

  Start with gauged U(1); ‘Higgs’ it using an axion

  ⇒ vector and axion become heavy

  ⇒ any light charged particle now sees an approximate global symmetry.

- (2) Accidental

  Spacetime and gauge symmetries forbid all relevant and marginal non-singlet operators.

- (3) Fine-tuned

  Coefficients of relevant and marginal non-singlet operators are small by landscape-type tuning.

Our focus will be on the gauge-derived case.

Minimal setting / basic idea

$$\mathcal{L} \supset \frac{1}{g^2}F^2 + |D\Phi|^2 + m^2|\Phi|^2 + f^2|\partial_\mu\varphi + A_\mu|^2$$

- If $m \ll gf$, the light field Φ sees a surviving global $U(1)$.
- φ is an axion, i.e. a scalar with gauged discrete shift symmetry $(\varphi \to \varphi + 2\pi n)$.
- As already discussed, instantons break its continuous shift symmetry with a strength bounded by $\boxed{S_{inst.} < M_P/f}$.
- Natural question: Can this be used to apply the Weak Gravity/Swampland logic to quantitatively constrain global symmetry violation? (of our lin.-realized global $U(1)$)

- For this, recall how a p-form is gauged by a $(p{+}1)$-form:

$$\frac{1}{g_p^2}|dA_p|^2 + \frac{1}{g_{p+1}^2}|dA_{p+1}|^2 \quad \to \quad \frac{1}{g_p^2}|dA_p + A_{p+1}|^2 + \frac{1}{g_{p+1}^2}|dA_{p+1}|^2$$

- Crucially, in the gauged/Higgsed version, the charged $(p-1)$-branes of the p-form theory cease to exist as independent objects:

  They would break gauge invariance

  $\delta A_{p+1} = d\chi_p$, $\delta A_p = -\chi_p$

- Instead, those brains can appear only as boundaries of the p-branes B_p charged under A_{p+1}:

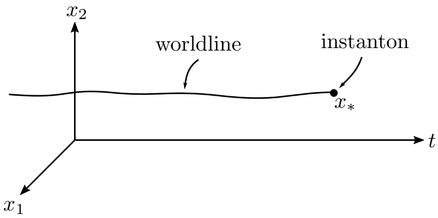
$$S \supset \int_{B_p} A_{p+1} + \int_{\partial B_p} A_p \,.$$

- In our case of **Gauge-derived global symmetries**, we gauge an axion (0-form) with a $U(1)$ vector (1-form).

- Thus, instantons become boundaries of worldlines.

- In other words: Instantons automatically destroy globally-charged particles

  (cf. many stringy examples: Ibanez/Marchesano/Rabadan '01 .... Antoniadis/Kiritsis/Rizos .... Uranga ... Blumenhagen/Cvetic/Kachru/Weigand .... Martucci '15)



By the WGC for axions, this particle-number violation is suppressed by $\exp(-S_{inst.}) \sim \exp(-M_P/f)$

- Moreover, according to the magnetic WGC for axions (for the dual B_2-theory with strings) the string tension is bounded by $T \lesssim M_P f$.

AH/Soler '17

- This implies a UV-cutoff for the EFT:

$$\Lambda \ \sim \ \sqrt{M_P f}$$

Hence, in total, the global-symmetry violation is bounded below by

$$\exp(-S_{inst.}) \ \sim \ \exp(-M_P^2/\Lambda^2)$$

- Very intriguingly, this is the same as the plasma-motivated bound of Fichet/Saraswat and as the bound expected from wormholes:

$$S_{WH} \ \sim \ M_P^2 \int \mathcal{R} \ \sim \ M_P^2/\Lambda^2\,.$$

An example with 'UV-complete' instantons:

$$\mathcal{L}_1 = -\frac{1}{e^2} F^2 + \overline{\psi} i \not{D} \psi \quad , \qquad \mathcal{L}_2 = -f^2 (\partial \varphi)^2 - \frac{1}{g^2} \mathrm{tr} G^2 + \frac{\varphi \, \mathrm{tr} G \tilde{G}}{8\pi^2}$$

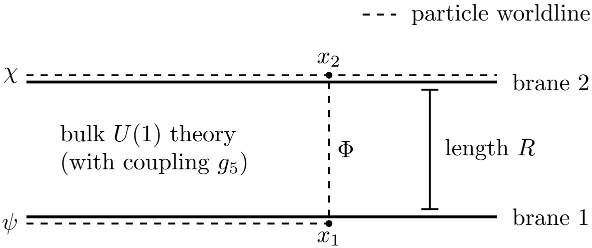
- Gauge: $\partial \varphi_\mu \;\to\; \partial_\mu \varphi + A_\mu$ and take ψ in the N of $SU(N)$.
- Now standard $SU(N)$ instantons induce a 't Hooft vertex

$$\mathcal{O} = e^{-S_I} \, \overline{\psi}_L \psi_R \, e^{i\varphi} + \mathrm{h.c.}$$

- After gauge-fixing to $\varphi = 0$, as appropriate in the IR, this is precisely the effect we claimed on general grounds.

For more general situations and stringy origins of such models
see e.g. Anastasopoulos/Bianchi/Dudas/Kiritsis '06

A simple 5d example on $S^1/\mathbb{Z}_2$:



- The 5d $U(1)$ is Higgsed on brane 2.
- The field ψ on brane 1 becomes globally charged.
- This global $U(1)$ is broken exponentially weakly (by the massive charged 5d particle Φ, required by the WGC)
- The resulting toy-model 'exotic instanton' has an action consistent with our general result.

A potential loophole

- If the light charged particle has $U(1)$-charge $n \gg 1$, the low-energy observers may be mislead:

  They see an n-instanton effect $(-\exp(-n\, M_P^2/\Lambda^2))$ and take it for a single-instanton effect $(-\exp(-M_P^2/\tilde{\Lambda}^2))$.

  So they expect a cutoff $\tilde{\Lambda} = \Lambda/\sqrt{n}$ that is too low and suspect a violation of our bound.

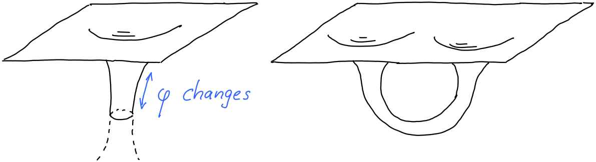
- In examples we studied, light high charges can only be constructed at the price of lowering the EFT-cutoff.
  $\Rightarrow$ probably no loophole (but more work needed).

- We already mentioned the parametrically similar wormhole-based arguments against global symmetries – let us develop this line of thought ....

Euclidean wormholes / gravitational instantons

- In Euclidean Einstein gravity, supplemented with an axionic scalar φ , instantonic solutions exist:

Giddings/Strominger '88
...



- The 'throat' is supported by the kinetic energy of $\varphi = \varphi(r)$, with r the radial coordinate of the throat/instanton.

- A wormhole-end looks like an instanton to the low-energy observer

  (recently revived in the Swampland/WGC context by

  Montero/Uranga/Valenzuela, Heidenreich/Reece/Rudelius '15
  AH/Mangat/Theisen/Witkowski '17, ...)

Euclidean wormholes (continued)

- The underlying lagrangian is simply

$$\mathcal{L} \sim M_P^2\,\mathcal{R} + f^2|d\varphi|^2\,, \quad \text{now with} \quad \varphi \equiv \varphi + 2\pi\,.$$

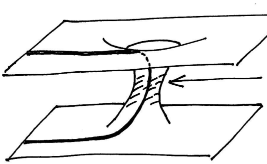
- This can be dualized $(dB_2 \equiv f^2 * d\varphi)$ to give

$$\mathcal{L} \sim M_P^2\,\mathcal{R} + \frac{1}{f^2}|dB_2|^2\,.$$

- The 'throat' exists due the compensation of these two terms:
  Placing one unit of flux (of $H_3 = dB_2$) on the transverse S^3 of radius R, we have

$$M_P^2\,R^{-2} \sim \frac{1}{f^2}R^{-6} \quad \Rightarrow \quad M_P\,R^2 \sim \frac{1}{f}\,.$$

- Thus, the instanton action is $S \sim M_P/f$
- This coincides parametrically with the lowest-action instanton of the WGC.
- The maximal WH-curvature scale is $\sqrt{f\, M_P}$, which should not exceed the UV cutoff:
$$f\, M_P < \Lambda^2 \quad \Rightarrow \quad S \sim M_P^2/\Lambda^2$$
- This agrees with our WGC-bound on global-symm.-violation
- Also technically (cf. our Appendix), one finds a new class of wormholes carrying our gauge-derived global charge:



H_3-flux associated with the axion Higgsing the U(1)

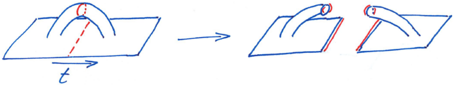
Euclidean wormholes - conceptual issues

- However, euclidean wormholes come at the price of deep conceptual issues.

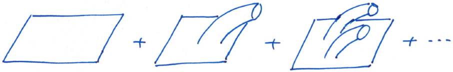
  Hawking '78..'88, Coleman '88, Preskill '89
  Giddings/Strominger/Lee/Klebanov/Susskind/Rubakov/Kaplunovsky/..
  Fischler/Susskind/...

  Recent review: AH, P. Soler, T. Mikhail '18

- First, with wormholes come baby universes:



- Second, with baby universes comes a 'baby universe state' (α vacuum) encoding information on top of our 4d geometry.

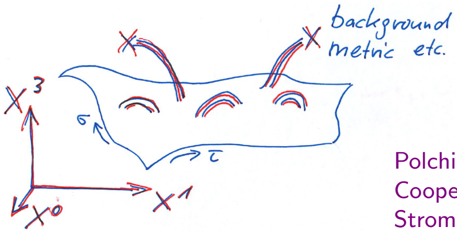


Conceptual issues (continued)

- In our concrete (single-axion) case, an α parameter now governs the naively calculable e^{-S_I}-effects.
- In the simplest approach, 4d measurements collapse α parameters to random constants.
- However, one should really include the full quantum dynamics of α parameters ...

Conceptual issues (continued)

- In 1+1 dimensions this corresponds to the target-space-dynamics of string theory.



Polchinski, Banks/Lykken/O'Loughlin, Cooper/Susskind/Thorlacius, Strominger '89…'92

- What is the analogue in 3+1 dimensions?

- Another key problem is a possible clash with locality on the CFT-side of AdS/CFT (factorization problem)

  Maldacena/Maoz '04, Arkani-Hamed/Orgera/Polchinski '07, ..., 'SYK'

Conceptual issues (continued)

- With all these problems in mind, maybe one should dismiss wormholes altogether?
- One option is to forbid topology change, but certainly (?) not in $d = 2$.
- Is there a reason to forbid topology change just in $d > 2$?
- A different argument is that these wormhole solutions have negative modes and should hence be dismissed.

  Rubakov/Shvedov '96, Maldacena/Maoz '04,
  see however Alonso/Urbano '17, ...
  Hertog/Trigiante/Truijen/Van Riet '04 ... '17/'18
- But, while this is even technically still an open issue, it does not appear to be a strong enough objection ....

Conceptual issues (continued)

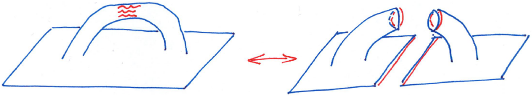
- Indeed, once a non-zero amplitude

  $$\text{universe} \rightarrow \text{universe} + \text{baby-universe}$$

  is accepted, the reverse process is hard to forbid.

- As a result, one gets all the wormhole effects.

- The negative mode issue may be saying:
  'Giddings-Strominger' does not approximate the amplitude well.



- ..hard to see how it would dispose of the problem altogether..

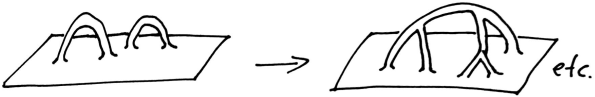
Recent developments related to Wormholes

- Recently, a concrete proposal for calculating the entropy of an evaporating BH has emerged (method of 'Islands')

Penington, Almheiri/Engelhardt/Marolf/Maxfield,
Almheiri/Mahajan/Maldacena/Zhao, ... '19/20

- The concrete mechanism by which entropy leaves the BH in this approach is related to euclidean WHs

- Motivated by this, a new 2d toy model developing Coleman's baby universe calculation has been suggested



Marolf/Maxfield '20

(For a different model see Ambjorn/Sato/Watabiki '21)

Recent developments related to wormholes (continued)

- In particular, Marolf/Maxfield proposed to mod out the naive BU Hilbert space by a certain equivalence (related to 1 BU → 2 BU transitions, etc.)

- It has then been proposed that, in $d \geq 4$, this equivalence should be so strong that the BU Hilbert space is 1-dimensional

  McNamara/Vafa '20

- This would not remove the effect of BUs completely, but it would get rid of the arbitrariness of α parameters

- But can we do a proper calculation in $d \geq 4$?

‘Global Symmetries’ – Summary/Conclusions

- The WGC for axions demands certain minimal-action instantons.
- This leads to a universal bound on the quality of gauge-derived global symmetries: $\gtrsim \exp(-M_P^2/\Lambda^2)$.

  (In agreement with other effects, such as wormholes.)
- But the latter come at the price of α vacua (and other disasters).
- Keep struggling with these fundamental unresolved issues!