Completing the D7-brane local gaugino action

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based on recent work with Y. Hamada / G. Shiu / P. Soler

Outline

- dS in string theory – some brief comments (Including the recent issue of the 'singular-bulk problem'). Gao/AH/Junghans '20
- Towards a 10d understanding of gaugino condensation effects (as required for both KKLT and LVS).
- Main point: Explicit form of the 4-fermion piece in the type-IIB D7-brane action.

de Sitter in String Theory

- Existence of metastable de Sitter is arguably the most important question in string phenomenology (and in the Swampland program)

- Leading candidates: KKLT, LVS

- Various objections/criticism have been raised .....

Woodard, Danielsson, Van Riet, Bena, Grana, Sethi, Dvali, ...

Danielsson/Van Riet, Ooguri/Palti/Shiu/Vafa, Garg/Krishnan '18

Moritz/Retolaza/Westphal

Gautason/Van Hemelryck/Van Riet/Venken '17...'19

Bena/Dudas/Grana/Lüst, Blumenhagen/Kläwer/Schlechter '18...'19

Carta/Moritz/Westphal, Gao/AH/Junghans '19...'20

- Also many new ideas for realizing dS space ....

e.g. Antoniadis/Chen/Leontaris '19

(1-slide reminder of) KKLT

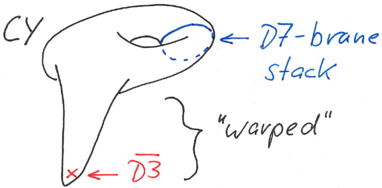
- CY with all complex-structure moduli fixed by fluxes;
  The only field left: Kahler modulus $T = \tau + ic$ with $\tau \sim \mathcal{V}^{2/3}$.

- $K = -3\ln(T + \overline{T})$; fluxes give $W = W_0 =$ const.,

  $\Rightarrow \quad V \equiv 0 \quad$ ('no scale') .

- Gaugino condensation on D7 brane stack: $W = W_0 + e^{-T}$.
  $\Rightarrow$ Stabilization in AdS.

- Small uplift by $\overline{\text{D3}}$-brane
  in a warped throat:
  $V \to V + c/\tau^2$.



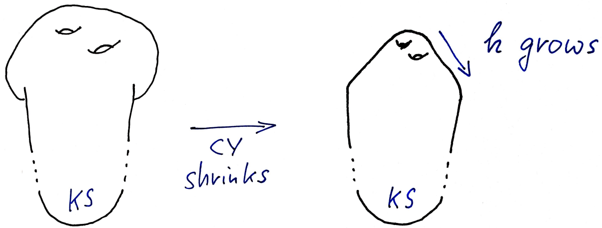
An important comment:

- There exists a parametric problem with fitting the throat (with a metastable $\overline{D3}$ and correct uplift energy) in the CY.

  [the 'Throat gluing problem'] Carta/Moritz/Westphal

- This can in principle be overcome at the price of significant warping in the bulk CY.

$$ds_{10}^2 = h(y)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + h(y)^{1/2}\tilde{g}_{mn}dy^m dy^n$$



... thus, this 'throat gluing problem' is in itself not deadly.
However, it entails the

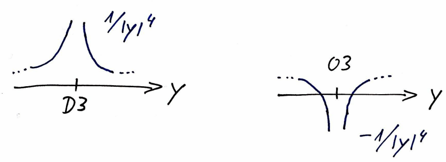
<u>Singular-Bulk Problem</u>

Gao/AH/Junghans '20

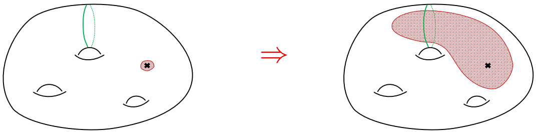
...which may destroy the whole framework.

- Indeed, while small negative-h regions near O-planes are OK,

$h(y)$:

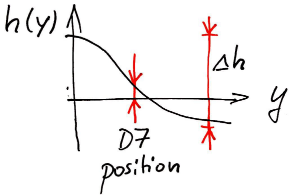


our analysis reveals that a situation like this is generic:



The singular-bulk problem explained:

- The warp factor h is a solution to a Poisson equation on the CY. It has typical variation $\Delta h \sim g_s N$.
- At the location of the D7-stack with gaugino condensate, it must have a value $\sim N/M^2 \ll g_s N$.
- Generically, this forces h to go negative in a large fraction of the bulk.



- A resolution through strongly curved regions in F-theory has been proposed, but how to derive the KKLT Kahler potential?

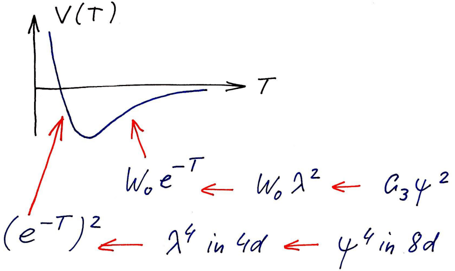
Carta/Moritz '21

... back to our main subject: D7-brane gauginos

- KKLT may survive through non-generic configurations or with better calculational techniques.

- The LVS is not affected (at least not obviously).

- A key ingredient in both is an exponentially steep AdS minimum:



- Surprisingly, while the λ^4 4d gaugino term is standard (cf. WB or FVP), its 8d-origin ψ^4 remains unclear.

Some recent history: 10d line of attack on dS

Moritz/Retolaza/Westphal '17
Gautason/Van Hemelryck/Van Riet '18

- The criticism was based on the established parts of the D7-gaugino–bulk-action:

$$\mathcal{L}_{10} \supset |G_3|^2 + G_3 \cdot \Omega_3 \langle \lambda\lambda \rangle \delta_{D7} \,.$$

Camara/Ibanez/Uranga '04, Koerber/Martucci '07
Baumann/Dymarsky/Klebanov/Maldacena/McAllister '06
Heidenreich/McAllister/Torroba '10

- It is clear what to expect:
  G_3 backreacts, becoming itself singular at the brane.

- Plugging this back into the action,
  one gets a divergent effect of type $(\delta_{D7})^2$.

- Now anything can happen....

Uplifted gaugino condensates rescued:

Hamada/AH/Shiu/Soler '18,'19; Kallosh '19; Carta/Moritz/Westphal '19
Bena/Grana/Kovensky/Retolaza, Kachru/Kim/McAllister/Zimet '19

- Singular gaugino effects have been observed before.

  Horava/Witten '96

- It has been shown that a highly singular $\langle\lambda\lambda\rangle^2$-term saves the day by 'completing the square'. Applied to our case:

$$\mathcal{L}_{10} \supset \left| G_3 + \Omega_3 \langle\lambda\lambda\rangle \, \delta_{D7} \right|^2 .$$

- Very roughly speaking, one now writes $G_3 = G_3^{flux} + \delta G_3$ and lets the second term cancel (most of) the δ-function.

  The result is (**very** roughly):

$$\mathcal{L}_{10} \supset \left| G_3^{flux} + \langle\lambda\lambda\rangle \right|^2 \quad \rightarrow \quad \left| D_T W_0 + \partial_T e^{-T} \right|^2 .$$

The non-locality issue

- While the above represents progress, it is not fully satisfactory.
- The reason is that the perfect square on the last slide was oversimplified. In fact, one needs

$$\mathcal{L}_{10} \supset \left| G_3 + P\left(\Omega_3 \langle \lambda\lambda \rangle \delta_{D7} \right) \right|^2 ,$$

where P stands for the projection on closed forms.

- But this is a non-local operation and is not suitable for the definition of a fundamental D7-brane gaugino term $\sim \psi^4$.

Also in the slightly different approach of Kachru et al., a ψ^4 term on the D7 has to be introduced which depends on the transverse volume (hence being non-local).

Getting the right 4d result without 10d-non-localities

- A key insight is that the (established part of the) 10d action can be rewritten as

$$-\left|G_+ - \sum_i \delta_i \lambda_i^2 \, \overline{\Omega}\right|^2 + |G_+|^2 - |G_-|^2 + \sum_{i,j} \lambda_i^2 \overline{\lambda}_j^2 \int \delta_i \delta_j \, \Omega \wedge * \overline{\Omega} \, .$$

- Here i runs over different D7 branes,

  δ_i are the corresponding δ-functions,

  and G_+ is the ISD part of the $G = H_3 - \tau F_3$.

- No projection is needed since G_+ **can** compensate the singular term inside the perfect square.

- Only the last term needs regularization and only for $i = j$.

- Let us start with a few simple manipulations with the (finite) contributions where $i \neq j$:

$$\int \delta_i^{(0)} \delta_j^{(0)}\, \Omega \wedge *\overline{\Omega} \sim \int \delta_i^{(0)} \delta_j^{(0)}\, J \wedge J \wedge J \sim \int \delta_i^{(2)} \wedge \delta_j^{(2)} \wedge J \equiv \mathcal{K}_{ij}$$

[Here we replaced the scalar δ-functions $\delta_i^{(0)}$ by 2-forms $\delta_i^{(2)}$ dual to the divisors Σ_i.]

- With this, the correct regularization is almost obvious:

Let $[\Sigma_i], [\Sigma_j] \in H^2(X, \mathbb{Z})$ be arbitrary smooth 2-forms dual to Σ_i, Σ_j and define:

$$\mathcal{K}_{ij} = \int [\Sigma_i] \wedge [\Sigma_j] \wedge J \qquad \text{for both} \qquad i \neq j \quad \text{and} \quad i = j\,.$$

- Using this well-defined $\mathcal{K}_{ij}$ and integrating out the dynamical part of G (with $G^{(0)}$ representing the flux or harmonic part), one has

$$-\left|G_+^{(0)} - \sum_i \frac{\lambda_i^2}{V_{i,\perp}}\,\overline{\Omega}\right|^2 + |G_+^{(0)}|^2 - |G_-^{(0)}|^2 + 3!\sum_{i,j}\lambda_i^2\,\overline{\lambda}_j^2\,\mathcal{K}_{ij}$$

[Here $V_{i,\perp} \equiv V/V_{\Sigma_i}$ is the brane-transverse volume.]

- Now, the first key observation is that this expression can be brought precisely into the form expected from 4d supergravity.

  [The proof uses manipulations familiar from the App. of GKP and from Grimm/Luis '04.]

- For example:

$$G^{(0)}_{(0,3)} = \int G^{(0)} \wedge *\Omega = -i\, W$$

$$G^{(0)}_{(3,0)} = \int G^{(0)} \wedge *\overline{\Omega} = -2\, e^{-\varphi}\, D_{\bar{\tau}} \overline{W}$$

- The result reads:

$$-e^{K} \left(\left| \frac{e^{-K/2}}{4} (\partial_{T_\alpha} f_i) \lambda_i^2 + D_{T_\alpha} W \right|^2 + |D_\tau W|^2 - 3|W|^2 \right)$$

[Here f_i is the D7-brane gauge-kinetic function.]

- From this, the gaugino condensate contribution to the scalar potential follows straightforwardly with (roughly) $\lambda^2 \to e^{-T}$.

8d covariant action

- The second key observation is that our regularized quartic gaugino expression $\mathcal{K}_{ii}$ has a local, covariant representation through 8d brane fermions.
- To see this, focus on a single brane Σ:

$$\mathcal{K}_{ii} \qquad \to \qquad \mathcal{K}_{\Sigma\Sigma} = \int [\Sigma] \wedge [\Sigma] \wedge J = \int_{\Sigma} [\Sigma] \wedge J$$

- Recall that the Chern class of the line bundle defining a divisor is identical to the Poincare dual 2-form of this divisor:

$$[\Sigma] = c_1(\mathcal{O}(\Sigma))$$

- Hence:

$$\mathcal{K}_{\Sigma\Sigma} = \int_{\Sigma} c_1(\mathcal{O}(\Sigma)) \wedge J = \int_{\Sigma} F(N) \wedge J\,.$$

Crucially, the field strenght $F(N)$ of the normal bundle can be expressed through the brane fermion!

- After a significant amount of Fierzing and other spinor manipulations one arrives at the complete, regularized action (displayed here for a single brane):

$$-\frac{1}{4}\int G\wedge *\overline{G}-\frac{1}{2}\left(\int_\Sigma \overline{G}_{MNz}\overline{\Psi}\,\Gamma^{MN}\,\Psi+\text{c.c.}\right)$$
$$+\frac{1}{2}\int_\Sigma \delta^{(0)}_\Sigma\left(\overline{\Psi}^c\,\Gamma_{MN}\,\Psi^c\right)\left(\overline{\Psi}\,\Gamma^{MN}\Psi\right)$$
$$+\frac{3\,i}{16}\int_\Sigma\left(\overline{\Psi}^c[\nabla_M,\nabla_N]\Gamma^{KL}\Gamma^{MN}\Psi^c\right)\left(\overline{\Psi}\,\Gamma_{KL}\Psi\right).$$

- The last term, evaluated for a brane with gaugino zero-mode, gives the desired $\lambda^2\overline{\lambda}^2 K_{\Sigma\Sigma}$.

- Direct confirmation through the analysis of 8d/10d supergravity would be very desirable!

For related recent work see Retolaza/Rogers/Tatar/Tonioni '21

Summary / Conclusions

- One should certainly not simply believe in metastable stringy de Sitter but try to establish it.
- For KKLT, I would argue that the 'singular-bulk problem' is the most serious challenge at the moment.
- The LVS appears to not to be threatened by this.
- Both the LVS and KKLT rely on the interplay of gaugino condensation and 10d flux and (surprisingly), the underlying 8d/10d lagrangian is not fully understood.
- We made progress by proposing a explicit form for the required 8d local 4-fermion-operator.