The Singular-Bulk Problem of KKLT

Arthur Hebecker (Heidelberg)

based on recent paper with **Xin Gao and Daniel Junghans**

(includes comments on work with **Hamada/Shiu/Soler**)

Outline

- The importance of realizing de Sitter
  (or even just realistic quintessence) in string theory.
- Brief comment on progress in our 10d understanding of gaugino-condensation effects.
- **Main part:** The Singular-Bulk Problem of KKLT.

de Sitter in String Theory

- Metastable de Sitter or realistic quintessence are arguably the most important challenges in string phenomenology (and in the Swampland program)

- Leading candidates: KKLT, LVS

- Various objections/criticism have been raised .....

Woodard, Danielsson, Van Riet, Bena, Grana, Sethi, Dvali, ...

Danielsson/Van Riet, Ooguri/Palti/Shiu/Vafa, Garg/Krishnan '18

Moritz/Retolaza/Westphal

Gautason/Van Hemelryck/Van Riet/Venken '17...'19

Bena/Dudas/Grana/Lüst, Blumenhagen/Kläwer/Schlechter '18...'19

Carta/Moritz/Westphal, Gao/AH/Junghans '19...'20

- Also many new ideas for realizing dS space ....

e.g. Antoniadis/Chen/Leontaris '19
De Luca/Silverstein/Torroba '21

(1-slide reminder of) KKLT

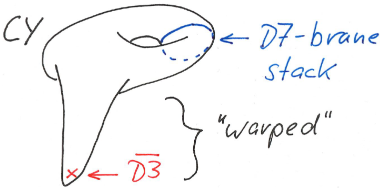
- CY with all complex-structure moduli fixed by fluxes;
  The only field left: Kahler modulus $T = \tau + ic$ with $\tau \sim \mathcal{V}^{2/3}$.

- $K = -3\ln(T + \overline{T})$; fluxes give $W = W_0 =$ const.,

  $\Rightarrow \quad V \equiv 0 \quad$ ('no scale') .

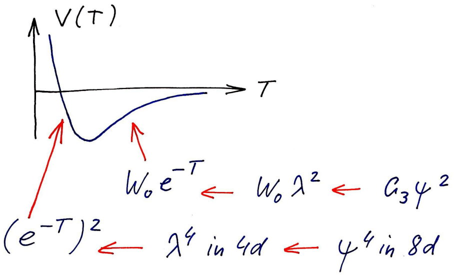
- Gaugino condensation on D7 brane stack: $W = W_0 + e^{-T}$.
  $\Rightarrow$ Stabilization in AdS.

- Small uplift by $\overline{\text{D3}}$-brane
  in a warped throat:
  $V \to V + c/\tau^2$.



The key role of gaugino condensation (or a corresponding instanton effect)

- A central ingredient in KKLT (and LVS) is an exponentially steep AdS minimum:



- Surprisingly, while the λ^4 4d gaugino term is standard (cf. WB or FVP), its 8d-origin ψ^4 remains unclear.

- As will be explained in detail in Gary Shiu's talk, a complete, regularized 10d + 8d action may be given:

$$
\begin{aligned}
& -\frac{1}{4}\int G\wedge *\overline{G} - \frac{1}{2}\left(\int_\Sigma \overline{G}_{MNz}\overline{\Psi}\,\Gamma^{MN}\,\Psi + \text{c.c.}\right) \\
& +\frac{1}{2}\int_\Sigma \delta^{(0)}_\Sigma \left(\overline{\Psi}^c\,\Gamma_{MN}\,\Psi^c\right)\left(\overline{\Psi}\,\Gamma^{MN}\Psi\right) \\
& +\frac{3\,i}{16}\int_\Sigma \left(\overline{\Psi}^c[\nabla_M,\nabla_N]\Gamma^{KL}\Gamma^{MN}\Psi^c\right)\left(\overline{\Psi}\,\Gamma_{KL}\Psi\right).
\end{aligned}
$$

Hamada/AH/Shiu/Soler '21

- While this strengthens the LVS and (one part of) the KKLT construction, the simplest version of KKLT may have an entirely different issue ....

This new concern starts with the

The Throat Glueing Problem

Carta/Moritz/Westphal '19
(cf. also Freivogel/Lippert '08)

- Recall basic parametrics of KKLT:

$$V_{AdS} \sim -e^{-4\pi \mathrm{Re}(T)} \qquad \text{vs.} \qquad V_{Uplift} \sim e^{-8\pi K/3g_s M}\,.$$

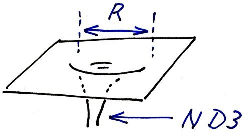
(Here K and M are the flux numbers of the two 3-cycles of the KS throat.)

- For a metastable uplift to dS, the two potentials must match:

$$\Rightarrow \qquad \mathrm{Re}(T) \simeq K/g_s M\,.$$

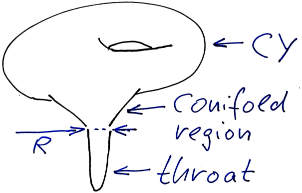
- At the same time, the throat carries $N = KM$ units of D3 charge, giving it a radius

$$R^4_{throat} \simeq g_s N\,.$$



Throat Glueing Problem (continued)

- The standard picture



suggests $g_s\,\mathrm{Re}(T) \sim R^4_{CY}$ and $R^4_{throat} < R^4_{CY}$.

- With the previous estimates, this leads to the problematic inequality

$$g_s N \lesssim K/M$$

or (using $K = N/M$)

$$\mathcal{O}(1) \lesssim 1/g_s M^2 .$$

Throat Glueing Problem (continued)

- The problem is that $g_s M \simeq R^2_{S^3} \gtrsim 1$

  KS, KPV, Klebanov/Herzog/Ouyang '01

  for supergravity control and $M \gtrsim 12$

  KPV (see also Bena/Dudas/Grana/Lüst, Blumenhagen/Kläwer/Schlechter)

  for metastability of the anti-D3-brane.

- Thus, the standard picture of a small throat glued into the large bulk of a CY can not be maintained.

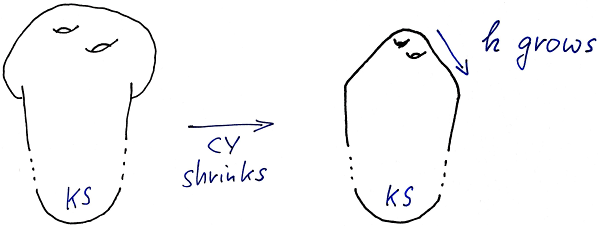
Is this deadly ?

- Not yet, since a priori the warp factor $h(y)$ of

$$ds_{10}^2 = h(y)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + h(y)^{1/2}\tilde{g}_{mn}dy^m dy^n$$

may come to rescue.

- The Kahler modulus corresponds to $h(y) \to h(y) + \text{const.}$ It is a flat direction 'at the level of GKP'. So we may simply make the bulk smaller than the throat!



The singular-bulk problem

Gao/AH/Junghans '21

- An actual problem is not that the geometry defies our standard intuition, it is that the CY may be forced into a singular regime, since $h < 0$.

- The danger of growing singularities as $h \to h - \text{const.}$ has already been discussed in the Appendix of Carta et a;., but without turning this into a quantitative problem for KKLT.

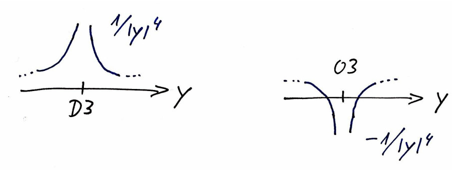
- The goal of the rest of the talk is exactly this:

  Demonstrate that, generically, the regime of KKLT is enforcing $\mathbf{h < 0}$ in a large portion of the CY geometry.

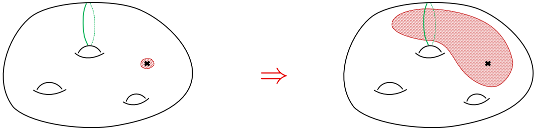
The singular-bulk problem (continued)

- Of course, small negative-h regions near O-planes are OK,

$h(y)$:



But our analysis reveals that a situation like this is generic:



The singular-bulk problem (continued)

- For quantifying the problem, a key insight is that the warped E3 size $\mathcal{V}_{E3}$ determines the exponential effect:

$$\mathrm{Re}(T) \sim N/g_s M^2 \qquad \Rightarrow \qquad \mathcal{V}_{E3} \sim N/M^2$$

with

$$\mathcal{V}_{E3} = \int_{E3} \sqrt{\tilde{g}}\, h(y) = \tilde{\mathcal{V}}_{E3} \, \langle\, h \,\rangle_{E3} \, .$$

- W.l.o.g., we use a CY-metric such that $\tilde{\mathcal{V}} = \int_{CY} \sqrt{\tilde{g}} = 1$. Hence $\tilde{\mathcal{V}}_{E3}$ is an $\mathcal{O}(1)$ number.

  $\Rightarrow$ We are constraining the warp factor on the E3 cycle:

$$\boxed{\langle\, h \,\rangle_{E3} \, \sim \, N/M^2}$$

The singular-bulk problem (continued)

- At the same time, h solves a Poisson-equation:

$$-\tilde{\nabla}^2 h = \tilde{\rho}_{D3} \qquad \text{with} \qquad \tilde{\rho}_{D3} \sim g_s N\,.$$

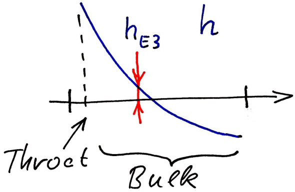
- So h is a compact-space Green's function for a charge distribution of

  $g_s N$ units of positive charge, localized at conifold

  $-g_s N$ units of negative charge, scattered in the CY.

  $\Rightarrow$ $\boxed{\Delta h \simeq g_s N}$

- Combined with $h_{E3} \simeq N/M^2 \ll \Delta h$, this leads to large negative regions.



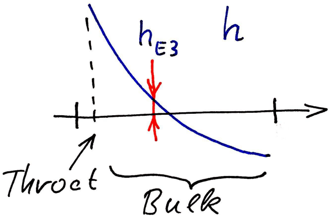
The singular-bulk problem (continued)

- In different words: $$\frac{|\tilde{\partial} h|}{h_{E3}} \gtrsim g_s M^2 \gtrsim M \gg 1$$

- By Taylor expanding at a point y_0 of the E3,

$$h(y_0 + \delta y) \approx h(y_0) + \partial_m h(y_0)\, \delta y^m \,,$$

we see that h runs negative near the E3: $|\tilde{\delta y}| \lesssim 1/g_s M^2$.



The singular-bulk problem (continued)

- Alternative view of the problem:

$$R_6 = h^{-5/2}|\tilde{\partial} h|^2 - \frac{3}{2}h^{-3/2}\tilde{\nabla}^2 h \qquad \Rightarrow \qquad R_6 \gtrsim g_s^2 M^5/\sqrt{N}$$

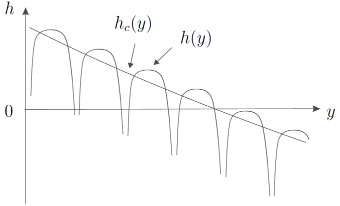
Imposing $g_s M \gtrsim 1$, $M \gtrsim 12$ and $R_6 \lesssim 1$ implies $N \gtrsim 3 \cdot 10^6$.

This exceeds the largest know tadpole of $7 \cdot 10^4$.

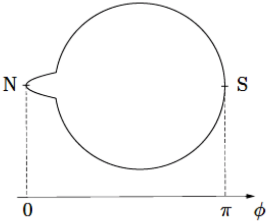
Taylor/Wang '15

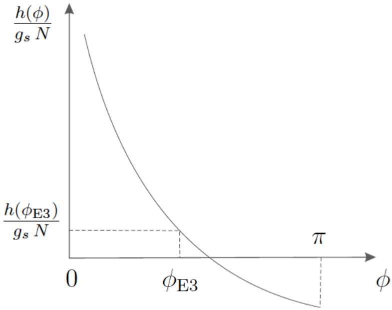
- Crucially:

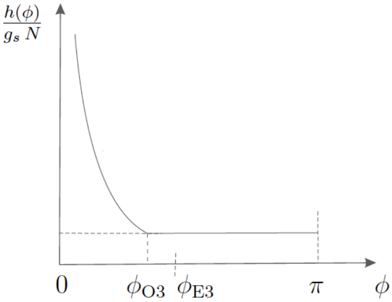
All of this applies also to the coarse grained warp factor h_c:



An S^3 toy model:







Escape routes

- One option, suggested by the toy model, is a very special arrangement of the O3s (or the curved O7/D7s).
  Very challenging to study this in proper CY geometries!

- Another option is to observe that the problematic 'small parameter' changes if the E3 is replaced by gaugino condensation:
  $$1/g_s M^2 \quad \rightarrow \quad N_c/g_s M^2 \,.$$

- However, $N_c \gg 1$ appears to always come with $h^{1,1} \gg 1$.
  Louis/Rummel/Valandro/Westphal '12, Carta/Moritz/Westphal '19

- But large $h^{1,1}$ is problematic due to the scaling
  $$\tau \sim (h^{1,1})^{3.2\cdots4.3}\,, \quad \mathcal{V} \sim (h^{1,1})^{6.2\cdots7.2} \qquad (h^{1,1} \gg 1)\,.$$
  Demirtas/Long/McAllister/Stillman '18

- One ends up with τ and hence the total tadpole too large ....

Escape routes (continued)

- A logical possibility is to just accept the singularities and ask how string theory resolves them.
- In a simple case (D7s wrapped on K3),
  this has been analysed using F-theory-inspired dualities.
  Carta/Moritz '21
- A non-singular but strongly curved geometry appears to arise.
- But: What happens to the classical Kahler potential in such a situation? How to repeat the KKLT analysis in this setting?

Summary / Conclusions

- One should not simply believe that metastable stringy de Sitter is possible/impossible but try to demonstrate it.

- Concerning the '10d-line-of-attack', things appear to be working out in favor of KKLT and LVS.

- However, KKLT may fall victim to the bulk singularity problem.

- The escape routes appear complicated and non-generic, but that does not make them hopeless.

  (Note: LVS appears not to suffer from this issue.)