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Strongly Correlated Bosons

Christopher Popp

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christopherpopp@gmx.de

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Model

Bosonic Hubbard Model

$$H = \sum_{x,y} t_{x,y} b_x^\dagger b_y + \sum_x U_x b_x^\dagger b_x^\dagger b_x b_x \tag{1}$$

$$[b_x^\dagger, b_y] = \delta_{x,y} \tag{2}$$

Physical effects

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- Pair formation
- Relation to certain spin systems in strong magnetic field
- In this talk: Systems with special lattice structure
    - Lattice can be modeled as line graph
    - Flat band in single particle spectrum
    - Elegant formulation of ground states via graph theory

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Graph theory

Blackboard

Example 1 Chequerboard lattice

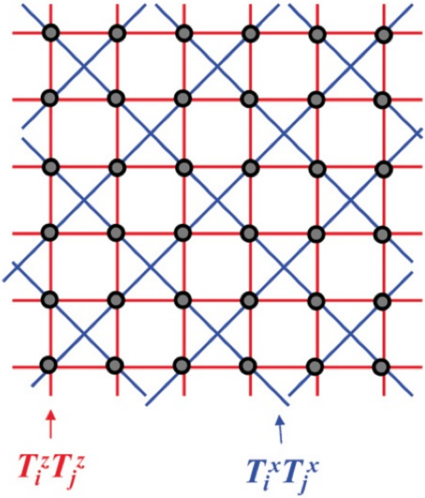

Figure: Chequerboard lattice

Example 2 Kagomé lattice

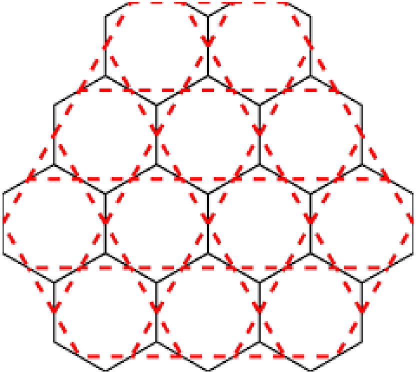

Figure: Kagomé lattice

Hubbard model for line graphs

$$H = \sum_{e,e'} t_{e,e'} b_e^\dagger b_{e'} + \sum_e U_e b_e^\dagger b_e^\dagger b_e b_e \tag{3}$$

$$t_{e,e'} = t \sum_{x \in V(G)} b_{xe} b_{xe'} \tag{4}$$

$$e, e' \in E(G) \tag{5}$$

$$U_e,\ \ t > 0 \tag{6}$$

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- Orientation of faces: Boundary of face orientated clockwise
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- Define

$$S = (s)_{f \in F(G), e \in E(G)} = \tag{7}$$

$$\begin{cases} 1 : & e \in C(f), e, f \text{ same orientation,} \\ -1 : & e \in C(f), e, f \text{ different orientation,} \\ 0 : & \text{else.} \end{cases} \tag{8}$$

Cycles as single particle states

- $$BS^T = 0 \tag{9}$$
  $$dim(ker(B)) = |F(G)| \tag{10}$$
  $$\Rightarrow \text{Columns of } S^T \text{ form basis of } ker(B) \tag{11}$$
  $$b_f^\dagger := \sum_{e \in E(G)} s_{fe} b_e^\dagger \tag{12}$$
  $$\Rightarrow b_f^\dagger |0\rangle \text{ is single particle ground state with energy 0} \tag{13}$$

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- $b_f^\dagger |0\rangle$ localized only on the enclosing cycle of f

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- Cycle set C: $\{c_i, i = 1, \ldots, N\}$, c_i: edge disjoint cycles
- Contraction C' of C: $F(c_i') \subset F(c_i) \forall i$ and $\cup_i F(c_i') \subset \cup_i F(c_i)$

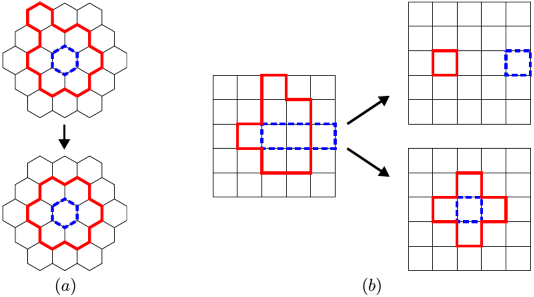

Figure: Contraction of cycle sets

Cycle sets as multi particle states

- Define multi particle states via cycle sets:

$$|\Phi(C)\rangle = O^\dagger(C)\,|0\rangle \tag{14}$$

$$O^\dagger(C) := \prod_i \sum_{f \in F(c_i)} b_f^\dagger \tag{15}$$

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- Can we find a basis?

Theorem

The states $|\Phi(C^{(u)})\rangle$ belonging to uncontractible cycle sets form a basis of the Fock space F_0 of the kernel of H.

Sketch of the proof

1. Recursive construction of states by successively adding particles
   - on a cycle of a disconnected face
   - on a cycle around a set of close packed cycles
   - $\rightarrow$ states correspond to uncontractible cycle sets
   - all states $\left|\Phi(C^{(u)})\right\rangle$ can be constructed like that, until a critical density

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2. Linear independence by unique combination of creation operators
3. Spanning property
    - a) Any ground state can be expressed as linear combination of $|\Phi(C)\rangle$
    - b) Each $|\Phi(C)\rangle$ can be decomposed into sum of uncontractable cycles

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Exceeding the critical density

- Construction only possible until lattice is close packed with cycles
- Above this critical density, multi particle states behave differently
- Hard to get exact results. $\rightarrow$ variational methods and numeric calculation.
- Simplifications
  - Weak interaction: Construction of Wannier Basis, Mean-field treatment
  - Strong interaction: Hardcore limit $U \rightarrow \infty$

Example: Chequerboard chain

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- Single particle state localized on cycle
- Multi particle states below critical density by putting particles on different cyles until all cycles are occupied
- Add one more particle. Three possibilities:
  1. Put particle in existent state. → minimal hopping, but strong interaction
  2. Spread particle on all sites. → lowers interaction, increases hopping
  3. Put particle on one of the cycles, s.th. the two particles on it avoid each other. Pair formation. → no interaction, but hopping

Pair formation

- Analytic result for hopping between cycles small enough and hardcore interaction: Option 3 (Localized pairs) [Mielke, 2017]
- Numeric results: Pair formation of hardcore bosons also occurs for general $t' \leq t$
- Use special properties of system for more exact results. Here: Reflexion symmetry

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Experimental realization

- Ultracold atoms in optical lattices (e.g. Kagomé lattice)
  - Standing waves of laser light trapping particles used to construct certain geometries.
- Nearly polarized spin systems. Single unpolarized spins "magnons" are well described by model

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- Drescher, M., Mielke, A.: Hard-core bosons in flat band systems above the critical density. Eur. Phy B90, 217 (2017)
- Motruk, J., Mielke, A.: Bose-Hubbard model on two-dimensional line graphs. J. Phys. A 45(22), 225,206 (2012)
- Pudleiner, P., Mielke, A.: Interacting bosons in two-dimensional flat band systems. Eur. Phys. J. B 88, 207 (2015)
- S.D. Huber, E. Altman, Phys. Rev. B 82, 184502 (2010)
- Mielke, A.: Pair Formation of Hard Core Bosons in Flat Band Systems. J. Stat. Phys. 171, 679-695 (2018)