# I Review and Outlook



In this last chapter a review in the form of a short account of the historic development of electrodynamics is given. Additionally we will catch a glimpse of the general theory of relativity by showing that clocks run differently in the presence of gravitation.

## 30 Short History of Electrodynamics

I conclude with a short history of electrodynamics. For this purpose I have mainly used the following literature:
Sir Edmund Whittaker, A History of the Theories of Aether and Electricity
Emilio Segrè, Die großen Physiker und ihre Entdeckungen, Teil 1, Piper Band 1174
Wilhelm H. Westphal, Physik, Springer-Verlag
Wilhelm H. Westphal, Anhang I, Physikalisches Wörterbuch
Max Born, Emil Wolf, Principles of Optics, Historical Introduction
Edmund Hoppe, Geschichte der Physik
Encyclopedia Brittanica: Article 'Electromagnetic Waves' and 'Magnetism'
Woldemar Voigt, Theoretische Physik
J.D. Jackson and L.B. Okun, Historical roots of gauge invariance, Rev. Mod. Phys. 73 (2001) 663.

It is not easy to redraw a historical development. First of all, there is the question whether one has sufficiently complete sources. Secondly, it often happens that several persons are named for some discovery or explanation, sometimes even at separate times. One reason might be that one person did not know of the other's discovery. But it may also be that they observed or explained the phenomenon differently well. Sometimes they published the result particularly well so that their paper has become rather popular and they were considered alleged authors.

Who for example has explained how the rainbow comes about? Dietrich von Freiberg, Mahmud al Schirazi and Kamal al-Din who in the beginning of the 14th century found that sunlight is refracted twice and reflected once or twice inside the raindrop, or Descartes who found around 1625 that the total angle of reflection has an extremum so that a high intensity of light is reflected under a certain angle, or Fresnel and Airy who took around 1820 and 1836 the wave character of light into account? All of them contributed a piece to our knowledge.

Initially there were three different phenomena of electrodynamics observed by man without forseeing their connection. The most obvious one was light which offered him excellent orientation and which sometimes appeared to him both frightening and agreeable as in a flash of lightning and a rainbow.

Two other phenomena already known in antiquity were much rarer observed, the curious properties possessed by two minerals, amber ($\eta\lambda\epsilon\kappa\tau\rho o\nu$) and magnetic iron ore ($\eta\ \lambda\iota\theta o\zeta\ M\alpha\gamma\nu\eta\tau\iota\zeta$). The former, when rubbed, attracts light bodies; the latter has the power of attracting iron and has its name from Magnesia in Thessalia, where this stone is found. Thales of Milet (600 BC) is said to have known the properties of these minerals.

Accordingly, the investigation of these phenomena developed in parallel into a theory of light, of electrostatics, and of magnetostatics, before one realized that they are connected.

### 30.a Theory of Light to Fresnel

Hero of Alexandria argued that reflected light uses the shortest path and thus for light reflected at a mirror the angle of incidence equals the angle of reflexion. In antiquity and the middle ages one assumed that nature has final causes and thus asked *why*, not *how* does nature proceed?

Hero and Ptolemy held the opinion that men saw by means of rays of light issuing from the eye and reflecting from the objects seen. Alhazen held the correct view that light was issued from the sun or from some other luminous source and was reflected from the object seen into the eye. Alhazen made important discoveries in optics (1030): pinhole camera and parabolic mirror. Kepler learnt a lot from his work. Alhazen already knew that in refraction the incident, the reflected and the refracted beam lie in one plane.

Eye glasses were invented in the 13th century.

The explanation for the occurence of the rainbow by two refractions and one or two reflections of sunlight in the rain drop was given by Dietrich von Freiberg, Al-Shirazi, and Kamal al-Din at the beginning of the 14th century.

In 1621 Snell of Royen found experimentally the law of refraction. Descartes gave a theoretical derivation assuming that the velocity of the beams have given values in both media and that the component of the velocity vector parallel to the boundary is conserved. This derivation becomes correct if the vector of velocity is replaced by the wave-vector. However, Fermat introduced the principle of least time (1657) and derived from this the law of refraction (1661).

Hooke was probably the first who described in his *Micrographia* light as a wave, since he had observed diffraction. Considering theoretically the progression of the wave-front he derived the law of refraction. However, Huygens, developed in his *Traité de la lumière* (1678-1690) a wave theory of light. Important for the theory of diffraction but also refraction became his principle, which says: Each point of a wave-front may be regarded as the source of a secondary wave.' Newton is considered having put forward the theory of emanation, i.e. the idea that light is of corpuscular nature. This is not completely correct. Newton disliked to introduce imaginative hypotheses, which could not be proven experimentally. 'To avoid dispute, and make this hypothesis general, let every man here take his fancy; only whatever light be, I suppose it consists of rays differing from one another in contingent circumstances, as bigness, form or vigour.' Later however, he was in favour of the corpuscular nature of light.

Newton devoted considerable attention to the colours of thin plates. He supposed (*Opticks*) that 'every ray of light, in its passage through any refracting surface, is put into a certain transient state, which, in the progress of the ray, returns at equal intervals, and disposes the ray, at every return, to be easily transmitted through the next refracting surface, and, between the returns, to be easily reflected by it.'He found that the intervals between easy transmission vary with colour, being greatest for red and least for violet. If he had accepted the wave picture, he could have determined the wavelengths of visible light.

In 1717 the meanwhile known phenomenon of double-refraction was explained by Newton by light corpuscles of different shape which comes close the idea of a transversal polarization. Huygens' wave theory of light assigned elastic properties to the aether; However, he considered only longitudinal waves, and was forced to introduce two different kinds of these waves for double-refraction, one of which propagated isotropically, the other spheroidally. At that time Newton's explanation was generally accepted.

In this course we did not consider double-refraction and diffraction. They played an important role in the development of the theory of light. It should be remarked that double-refraction appears in anisotropic crystals where the dielectric constant is a tensor.

In 1675 Römer was able to determine the time light needs to transverse the distance from sun to earth by observing the eclipses of the moons of Jupiter. Until that time it was not clear whether light propagates instantly or at a finite velocity.

In 1728 James Bradley found the aberration of light, i.e. a change in the direction of the light from a star due to the perpendicular motion of the observer to the direction of the star. This was considered a proof of the corpuscular nature of light. In 1677 Römer already had conjectured such a phenomenon in a letter to Huygens.

In 1744 Maupertuis took up the old controversy between Descartes and Fermat. Convinced of the corpuscular nature of light but wishing to retain Fermat's method, he supposed that 'the path described is that by which the quantity of action is the least' and required that instead Fermat's $\int dt = \int ds/v$ the action $\int v ds$ should be extremal. In this way he introduced for the first time the principle of least action which was soon taken up by Euler and Lagrange, and which today is considered the principle governing all dynamics of nature.

In 1801 Thomas Young introduced the concept of the interference of two waves and brought Huygens' concept anew in play. He is able to explain Newton's rings with this concept. In 1808 Malus found that reflected light is normally partially polarized and found the angle of total polarization, now known as Brewster's angle (after eq. 18.22). The problem to explain the extraordinary beam in double-refracting crystals continued with explanations from both sides, in 1808 Laplace argued for corpuscles, in 1809 Young argued for waves, both agreeing that the medium has to be anisotropic. In 1815 when Brewster discovered crystals with two extraordinary beams (the case of three different eigenvalues of the dielectric tensor) the situation became even more complex.

In 1818 the French Academy announced a prize for the explanation of diffraction. The followers of the theory of emission (Laplace, Poisson, Biot) were confident of victory but Fresnel submitted a paper at the basis of the papers by Huygens and Young, in which he explained this phenomenon for several arrangements by means

of the wave theory. Poisson who studied the paper carefully found that in the centre of the shadow behind a circular disc there had to be a bright spot and asked for an experimental test. Arago found the bright spot and Fresnel received the prize. Since in 1818 Young was also able to explain aberration by means of the wave theory it became the leading theory.
In 1817 Young proposed for the first time, light might consist of transversal waves. This was supported by the observation that two light beams polarized perpendicular to each other do not show interferences. Fresnel picked up this idea and developed a successful theory of double-refraction, although Maxwell's equations were not yet available. Clever experiments by Airy (1831) showing that light irradiated under Brewster's angle suppresses Newton's rings, and that light propagated slower in water than air, proved the wave nature of light. (Wave-theory predicts in a medium with larger index of refraction a smaller velocity of light, in the corpuscular theory a larger one.)
Fresnel derived an expression for the change of velocity of light in moving matter which was confirmed experimentally by Fizeau (1851). However, there were several different theories on this subject among others by Stokes (1846). Different ideas competed on the question, to which extend matter would drag the aether.
It may be remarked that the aether as an elastic solid occupied many excellent scientists in the following time and made the theory of elasticity flourishing in the following years. Applied to the theory of light there remained the problem to suppress longitudinal waves..
There remained the puzzle whether the space above the earth is a plenum, which provides the necessary properties of elasticity for the propagation of light, or a vacuum, which allows the planets to move freely. This discussion existed already centuries before. Space was, in Descartes' view, a plenum (in contrast to a vacuum), being occupied by a medium which, though imperceptible to the senses, is capable of transmitting force, and exerting effects on material bodies immersed in it - the aether, as it was called. Gassendi, a follower of Copernicus and Galileo, re-introduced the doctrine of the ancient atomists that the universe is formed of material atoms, eternal and unchangeable, moving about in a space which except for them is empty, thus he re-introduced the vacuum. His doctrine was accepted not long afterwards by Newton and in fact became the departure point for all subsequent natural philosophy.

## 30.b Electrostatics

Thales of Milet (600 BC) is said to have known that rubbed amber (Greek 'elektron') attracts light bodies. Around 1600 Gilbert discovered that many other materials assume the same property by rubbing. He coined the word 'electric' for this property. The word 'electricity' was introduced by Browne in 1646. Gilbert remarked essential differences between magnetic and electric forces. (Magnets are permanent in contrast to electrified bodies. Magnetic forces are not shielded by other substances. Magnets attract only magnetizable substances, electrified ones all substances.)
Otto of Guericke known for the preparation of the vacuum in the Magdebourgous spheres made pretty early a number of important electric discoveries - his *Experimanta nova magdeburgica* appeared in 1672 - For the first time he introduced the distinction between conductors and non-conductors, he observed electric attraction and repulsion, the phenomenon of influence. He designed the first reasonably working electrostatic generator. It seems that his discoveries did not receive general attention.
In 1708 Wall compared the spark which flashes over rubbed amber with thunder and flash, an indication that a flash is an electrostatic discharge.
In 1729 Gray found that electricity is transferred by certain substances which Desaguliers called non-electrics or conductors. Gray found that electricity is assembled at the surface of bodies. In 1734 DuFay observed that there are two kinds of electricity, vitreous and resinous electricity; similar ones repel each other, whereas dissimilar ones attract each other.
Improved electrostatic generators were designed between 1744 and 1746 by Johann Heinrich Winkler, George Matthias Bose and Benjamin Wilson.
The capacitor in form of a Leyden jar was invented in 1745 by Pieter van Musschenbroek, and independently probably a bit earlier by Ewald von Kleist, described by J. G. Krüger in 1746.
In 1746 William Watson concluded that 'in charging or discharging of a Leyden jar electricity is transferred, but it is not created or destroyed.' 'Under certain circumstances, it was possible to render the electricity in some bodies more rare than it naturally is, and, by communicating this to other bodies, to give them an additional quantity, and make their electricity more dense.' This was a first indication of the conservation of charge.

Similar experiments conducted by Benjamin Franklin after a talk by Dr. Spence who had come from Scotland to America, brought him in 1747 to the conclusion that 'the total amount of electricity in an insulated system is invariable.' Franklin became popular by the introduction of the lightning rod. He realized that lightning was an electric discharge.

The introduction of the signs for charges is ascribed to both Franklin and Lichtenberg (1777): 'I call that electricity positive, which, stimulated by blank glass, is transferred to conductiong bodies; the opposite one I call negative.'

Aepinus and Wilcke found that 'ordinary matter' (this is approximately what we nowadays call matter without outer electrons) repels itself, particles of the 'electric fluid' (nowadays called outer electrons) are repelling themselves, too, and ordinary matter and the electric fluid attract each other. Further, they realized that glass and even air is impermeable for the electric fluid despite the fact that the electric interaction acts over larger distances.

Aepinus explained in 1757 the phenomenon of influence (or electric induction), which had already been observed by Guericke, Canton, and Wilcke, by the electrostatic forces and the free mobility of the electric fluid. Wilcke described in 1762 many experiments in connection with influence and argues that dielectric media are polarized in an electric field.

Joseph Priestley communicates in his work *The History and present State of Electricity* ... which did not receive much attention an experiment conducted by Franklin and repeated by him that inside a metallic box there is no electric force and the interior sides do not carry any charges. He concludes that charges of equal sign repel each other with a force inversely proportional to the square of the distance. 'May we not infer from this experiment that the attraction of electricity is subject to the same laws with that of gravitation, ... since it is easily demonstrated that were the earth in the form of a shell, a body in the inside of it would not be attracted to one side more than another?'

In 1760 Daniel Bernoulli conjectured that there might be a $1/r^2$-law for the electrostatic interaction. In 1769 John Robison was presumably the first to measure a $1/r^n$-dependence with $n = 2 \pm 0.06$. In 1771 Cavendish declared that the interaction falls off with an inverse power less than 3. It took many years until Robison's and Cavendish's results were published. In 1775 Cavendish gave comparative results for the conductances of various materials. (iron, sea water, etc.)

In 1785 Coulomb verified by means of the torsion balance invented by Michell and independently by himself the $1/r^2$-law with high precision. This torsion balance served also for the determination of the gravitional constant (Cavendish).

In 1813 Poisson showed that the electrostatic potential obeys the equation, now called after him. In 1777 Laplace had shown that the operator, now called Laplacian, applied to the gravitational potential in matter free space yields zero. Poisson had included the regions filled with matter, and explicitely stated that one has an analog equation in electrostatics. Thus he has introduced the electrostatic potential and has stated that it is constant over the surface of a conductor. In 1828 George Green continued the calculations of Poisson. We know Green's theorem (B.67). Green's functions are named after him.

William Thomson (Lord Kelvin) (1845) and Mossotti (1847) formulated on the basis of Faraday's considerations the relation between electric field and polarisation, $\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P} = \epsilon\mathbf{E}$, $\rho_P = -4\pi \,\mathrm{div}\,\mathbf{P}$, which we showed in sect. 6.

## 30.c Magnetostatics

Magnets were already known in antiquity. Their name is derived from the city of Magnesia in Thessalia, where load stone (magnetite $Fe_3O_4$) occurs naturally, which has the property to attract other load stone and iron. Already about the year 1000 in China magnetic needles were known to have directive properties. The English encyclopedist Alexander Neckam reports on the compass.

In 1269 The crusader Petrus Peregrinus de Maricourt gave a precise description of magnetic stones in his *Epistola de magnete*. He laid an iron needle on a round magnetic stone and marked the directions which was assumed by the needle. He found that these lines formed circles like the meridians of the earth which passed through two points, which he called poles. He observed that a magnet broken into two pieces constitute again magnets with North and South poles; thus no magnetic monopoles exist.

In 1588 the idea of two magnetic poles of the earth was first noted by Livio Sanuto. In 1600 William Gilbert gave a comprehensive review in his work *De magnete*. He emphasizes that the earth is a large magnet.

Similarly to the force law between charges the force between poles of a magnet was investigated. Newton found a law close to $1/r^3$. In 1750 Michell found the $1/r^2$-law based on own measurements and on those of Brook Taylor and Musschenbroek, similarly in 1760 Tobias Mayer and in 1766 Lambert. This led soon to the idea of a 'magnetic fluid' in the sense of magnetic charges similarly to electric ones. Coulomb put forward the thesis that magnetism is captured in molecules and only inside molecules both magnetic fluids can be separated and yield magnetization. (Taylor series are named after Brook Taylor, although they were known before.)
In 1824 Poisson introduced a magnetic potential besides the electric one similarly to the one in subsection 11.b and introduced magnetization quantitatively. In 1828 this theory was extended by Green.
William Thomson (Lord Kelvin) introduced the equations $\operatorname{div}\mathbf{B} = 0$ and $\operatorname{curl}\mathbf{H} = \mathbf{0}$ for current-free space, introduced the relation $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$, obtained the expression for the magnetic energy density and concluded that in the relation from the expression $\mathbf{B} = \mu\mathbf{H}$, which had already been given in 1824 by Poisson with a tensor $\mu$ for anisotropic crystals the tensor $\mu$ has to be symmetric. He coined the notions susceptibility and permeability.

## 30.d Set out for Electrodynamics

For a long time electricity and magnetism were two separate phenomena. A first hint on a connection was the observation that lightnings made compass needles deflected. Occasionally it happened that during lightning the magnetization of magnets was reversed or that iron became magnetic: It is reported that in 1731 a flash hit a box filled with knives and forks, which melted. When they were taken up some nails which laid around were attracted. In 1681 a ship bound for Boston was hit by a flash. After this stroke the compasses showed into the opposite direction.
About 1800 the experimental situation improved when Volta invented what is called the Volta pile, a prototype of battery. Now it was possible to generate continuous electric currents with a power improved by a factor 1000 over the electrostatic one.
In 1820 Ørsted observed that a magnetic needle was deflected by a parallel flowing current. This discovery spread like wildfire in Europe. Biot and Savart determined in the same year quantitatively the force of a straight current on a magnet. On the basis of a calculation of Laplace for the straight wire and another experiment with a V-shaped wire, Biot abstracted in 1824 the force between a magnetic pole and a current element, which is basically what we now call the law of Biot and Savart.
In 1820 Ampère assumed a law of force of the form

$$\mathbf{K} = I_1 I_2 \oint\oint \hat{\mathbf{r}}_{12}\,(f_1(r_{12})(\mathrm{d}\mathbf{r}_1 \cdot \mathrm{d}\mathbf{r}_2) + f_2(r_{12})(\hat{\mathbf{r}}_{12} \cdot \mathrm{d}\mathbf{r}_1)(\hat{\mathbf{r}}_{12} \cdot \mathrm{d}\mathbf{r}_2))\,,$$
$$\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2, \quad \hat{\mathbf{r}}_{12} = \frac{\mathbf{r}_{12}}{r_{12}}. \tag{30.1}$$

between two circuits with currents $I_1$ and $I_2$. In comparison with his measurements he obtained $f_1 = A/r_{12}^2$, $f_2 = B/r_{12}^2$. Each one of these contributions yields separately by an appropriate choice of $A$ and $B$, resp., the force between two closed circuits, compare (9.21). Ampère had already observed that the force on a line element of the conductor is perpendicular to it, which is fulfilled by $B = -3A/2$. Thus his force law includes already the Lorentz force, although he did not use the notion of a magnetic field.
After some preliminary work by Ampère and Arago, William Sturgeon constructed in 1825 an electro-magnet which could hold twenty times its own weight.
In 1821 Humphrey Davy found that the conductance ('conducting power') of a metal is proportional to its cross-section and inverse proportional to its length. Georg Wilhelm Ohm found in his *Die Galvanische Kette* (1826-1827) the linearity between the current through a conductor and the voltage applied to the conductor. In 1845 Kirchhoff formulated the current and the voltage laws (13.10, 13.11) named after him.
In 1812 Michael Faraday, a bookbinder journeyman interested in science applied for a position at the Royal Institution in London. Its director, Humphrey Davy accepted the application, hardly anticipating that he had accepted one of the greatest future experimentalists to his institute. (After Davy's death Faraday became director of the institute.) Shortly after Ørsted's discovery Faraday investigated the known experiments in electricity and magnetism, which he reviewed in his *Historical Sketch of Electro-Magnetism* (1821). Inspired by the influence i.e. the effect of a charge on charges on a conductor, he investigated, whether a current may excite a current on another circuit. He found that this happened when the current in the first circuit changed. This was the starting point for the law of induction. (1831)

When a politician asked Faraday, what his discoveries are worth, he answered 'Presently I do not know, but may be they can be taxed one day.' Well-known are also Faraday's investigations on electrolysis. Since he himself did not enjoy a classical education he asked William Wheeler, a philosopher and mathematician from Cambridge to help him choose appropriate termini. They introduced the names electrode, anode, cathode, ion, electrolysis which are still in use. Faraday discovered diamagnetism, too.

Faraday often used the concept of electric and magnetic field lines. He made them visible by plaster shavings and iron filings. These procedures were not new, but they were not popular with mathematical physicists in the succession of Newton who preferred the concept of long-distance action. Already Wilcke made electric field lines visible. Many experiments of Faraday on electrostatics were already performed by Wilcke. A survey of experiments of both physicists on the same topic is given in the *History of Physics* by Hoppe. The lines of magnetic force were already made visible by Niccolo Cabeo (1629) and by Petrus Peregrinus (1269). The reader should consider why electric and magnetic lines of forces can be made visible by prolate bodies of large dielectric constant and susceptibility, resp.

Faraday had a rather precise imagination of the magnetic field. He considered it as tubes of lines with the property that the product of magnitude and cross-section is constant, which corresponds to a divergency free field. He stated that the induced current is proportional to the number of field lines crossed by the circuit; we say today proportional to the change of the magnetic flux.

In 1890 the name electron was coined by Johnstone Stoney. Before also (today's) electrons were called ions.

## 30.e Electrodynamics and Waves

In 1845 Faraday observed that polarized light transversing glass changes its plain of polarization if a magnetic field is applied parallel to the ray. From this he conjectured that light is an electromagnetic phenomenon.

In order to obtain a unified theory of electromagnetism there were mainly two directions of effort. One started out from the law of induction and introduced the vector-potential $\mathbf{A}$, the other stayed mainly with the theory of action on distance following Ampère's investigations and introduced velocity dependent forces.

The vector-potential was introduced on the basis of various considerations. In 1845/48 Franz Neumann found in that the voltage of induction could be expressed as the time-derivative of the integral $\oint d\mathbf{r} \cdot \mathbf{A}(\mathbf{r})$. In 1846 the vector potential was also introduced by Wilhelm Weber and William Thomson (Lord Kelvin) on the basis of other considerations which today are no longer that convincing. In 1857 Kirchhoff used it.

In 1848 Kirchhoff and in 1858 Riemann realized that the equations of forces for charges and currents differed by a factor which is the square of a velocity $c$. Two charges $q_1$ and $q_2$ at distance $r$ exert the Coulomb force $q_1 q_2/r^2$ on each other, two wires of length $l$ at distance $r$ ($r \ll l$) carrying currents $I_1$ and $I_2$ exert the force $kI_1I_2l/(c^2r)$ on each other with a number $k$, which may be determined by the reader. The determination of $c$ showed that this velocity agreed well with that of light. In 1834 first measurements of the propagation of electricity were performed by Wheatstone, in 1849 by Fizeau and Gounelle, and in 1850 by Foucault. They yielded velocities which were larger or smaller by factors of two or three-half from the velocity of light. (That some velocities were larger then light velocity was only possible because some arrangements were not linear).

In 1851 the construction of cables under water for the transmission of electric signals began (Dover-Calais). In 1854 William Thomson (Kelvin) found that at sufficiently high frequencies a damped wave propagates with approximately constant velocity. Kirchhoff showed by calculation that the velocity for a circular cross-section agrees with the velocity $c$, which appears in the ratio of the forces between charges and currents. This value had been measured shortly before by Weber and Kohlrausch to $3.1\times10^{10}$cm/sec.

Finally it was Maxwell who succeeded due to his imagination and analytic facilities to present the equations of electrodynamics in closed form. Studying Faraday's *Experimental Researches* he had learned a lot and still maintained the necessary abstraction. In 1857 he wrote to Forbes that he was 'by no means as yet a convert to the views which Faraday maintained', but in 1858 he wrote about Faraday as 'the nucleus of everything electric since 1830.'

Maxwell still worked a lot using mechanical analogies when he considered the fields **B** and **D** as velocities of an incompressible fluid. In 1861 he realized that in the equation $\operatorname{curl}\mathbf{H} = \frac{4\pi}{c}\mathbf{j}$ the displacement current $\dot{D}/(4\pi)$ had to be added to **j**, so that conservation of charge was guaranteed. From these equations he found that the velocity of light in vacuum was given by the factor $c$ appearing in the ratio between forces between charges and currents, which agreed very well with the measured ones. He concluded: 'We can scarcely avoid the interference that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena.' Maxwell's equations contained the potentials $\Phi$ and **A**, where he used the gauge we call Coulomb gauge. In 1864 he presented the complete set of electrodynamic equations in his paper *On a Dynamical Theory of the Electromagnetic Field*. In 1871 his monograph *Treatise on Electricity and Magnetism* was published.

In 1867 Ludvig Valentin Lorenz published his theory of electromagnetism, which contained the displacement current and the expressions (21.14) and (21.15) for the retarded potentials in which he had used the gauge named after him. The paper was based on the potential theory of Franz Neumann. In 1858 Riemann had found the retarded potentials, too. However, his paper was published only in 1867 together with that of Lorenz. Much of what Ludvig Lorenz found, was later attributed to the Dutch Hendrick Lorentz who wrote comprehensive papers on electrodynamics. This might be also due to their nearly equal names as well as Maxwell's inadequate criticism (1868) 'From the assumptions of both these papers we may draw the conclusions, first, that action and reaction are not always equal and opposite, and second, that apparatus may be constructed to generate any amount of work from its resources.' Ironically Maxwell did not realize that the fields contained energy and momentum. The Lorentz-Lorenz relation (1880) which is equivalent to the Clausius-Mossotti relation (6.34) when one replaces $\epsilon$ by the square $n^2$ of the index of refraction goes back to both of them.

In his *Treatise on Electricity and Magnetism* Maxwell derived the stress tensor of an electromagnetic field. The Poynting vector as the current density of electromagnetic energy was found by Poynting (1884) and by Heaviside (1885). In 1893 J.J. Thomson finally found that electromagnetic momentum can be expressed by the Poynting vector.

In 1889 Heaviside gave the expression (1.17) for the force on a charge moving in a magnetic field. J.J. Thomson who investigated cathode rays, had given it as half this amount in 1881. In 1895 Lorentz gives the correct result in his treatise. Today it is called Lorentz force. Already in 1864 Maxwell gave the contribution $\mathbf{v} \times \mathbf{B}$ to the electromotive force in a moving body.

Already Wilcke (1758) and Faraday (1837) introduced the notion of polarization of an insulator. The idea that magnetization is related to atomic currents was already found in the work of Coulomb, Ampère and Thomson (Kelvin). This connection is not clearly stated in Maxwell's formulation. It is the merit of Lorentz that in 1895 he introduced in his *Elektronentheorie* the fields **E** and **B** as fundamental fields and clarified that **D** and **H** are due to polarization and magnetization. 'Seat of the electromagnetic field is the empty space. In this space there is only one electric and one magnetic field-vector. This field is generated by atomistic electronic charges, onto which the fields in turn act ponderomotorically. A connection of the electromagnetic field with the ponderable matter exists only since the electric elementary charges are rigidly tied to the atomistic building blocks of matter.' Lorentz was able to provide a clear cut between electrodynamics and the properties of condensed matter.

Alfred Liénard (1898) and Emil Wiechert (1900) determined the potentials of an arbitrarily moving point charge.

In 1873 Maxwell already realized that the magnetic field is invariant under a gauge transformation $\mathbf{A} \to \mathbf{A}+\nabla\chi$. However, he did not consider the consequences for the scalar potential. In 1904 Lorentz gave the general gauge transformation.

Besides to Lorentz we are indebted to Henri Poincaré, Oliver Heaviside and Heinrich Hertz for working out Maxwell's theory more clearly so that it found a broad distribution.

In 1900 and 1903, resp., Larmor and Schwarzschild introduced the principle of least action for the combined system of the electromagnetic field and charged particles.

Since 1878 Michelson and collaborators determined the velocity of light with high precision. Finally Heinrich Hertz succeeded in 1886 to produce electromagnetic waves (Hertzscher Dipol) and to detect them, initially in the range of meters, later also shorter ones. In 1890 Wiener proved the wave-nature of light by reflecting it on a mirror and obtaining a periodic blackening of the photographic emulsion by the standing waves.

## 30.f Theory of Relativity

In order to determine the velocity of the earth against the postulated ether Michelson and Morley performed their experiment initially in 1887 with the negative result: No motion against the ether was detected. In 1889 Fitzgerald postulated that all material objects are contracted in their direction of motion against the ether. Lorentz gave an expression for this contraction in 1892 up to order $v^2/c^2$ (Lorentz contraction, subsection 23.b.$\beta$). Essential was Lorentz's observation that the assumption of an aether carried along with matter was wrong.

In 1887 Voigt realized that the homogeneous equation $\Box\Phi = 0$ with the d'Alembert operator $\Box$ (20.13) is form

invariant under a class of linear transformations of $\mathbf{x}$ and $t$. LARMOR gives in his paper *Ether and Matter* written in 1898 and published in 1900 already the transformation (23.2). It is unknown whether this had an influence on LORENTZ. Already in 1898 POINCARÉ expressed doubts on the concept of simultaneity. In 1899 LORENTZ stated the transformation called after him, but with an undetermined scale factor, which corresponds to the factor $f$ after eq. (23.14).
In 1904 LORENTZ found that MAXWELL's equations without charges and currents are invariant under the transformations (23.2) provided that the fields are transformed in an appropriate way (see section 25). In 1905 POINCARÉ realized that the charge and current densities could be transformed so that the full set of MAXWELL's equations are invariant in form under LORENTZ transformations (compare sections 24 and 25).
In 1905 EINSTEIN without the knowledge of LORENTZ's paper and simultaneously with POINCARÉ's work mentioned above formulated the theory of special relativity in a general and complete way. He realized that the idea of a constant velocity of light in all systems of inertia constitutes a reality which governs all physics including mechanics and not only electrodynamics and which has to replace GALILEIan invariance. The reason that it took so long to develop the theory of (special) relativity and to convince scientists that it describes the reality, is the role of time in this theory.
It was (and still is for some persons) difficult to accept that the idea of absolute (that is independent of the system of inertia) simultaneity has to be abandoned. More on the history can be found in *A. Pais, "Subtle is the Lord ..." Albert Einstein, Oxford University Press*. Another problem is that now the ether as a system of reference disappeared.
An elegant formulation of the four-dimensional space was introduced by MINKOWSKI in 1908, which was considered by EINSTEIN initially as superfluous, but later as very useful. Starting from the special theory of relativity, which acts in a planar space, EINSTEIN developped the general theory of relativity assuming that gravitation yields a curved space.

## 30.g From Classical to Quantum Electrodynamics

In 1900 Max Planck derived an interpolation formula between the two limit cases of the energy distribution of a black body radiator as a function of the frequency radiated, namely the Rayleigh-Jeans law (1900-1905) for low frequencies and the Wien law (1896) for high frequencies, the Planck radiation law. It agreed excellently with the observations. A few months later he postulated that this can be explained by the fact that electromagnetic radiation of frequency $\nu = \omega/(2\pi)$ cannot have arbitary energies but only integer multiples of $h\nu$, where $h$ is a new fundamental constant now called Planck constant. This quantization of energy was soon confirmed by the photoelectric effect: The kinetic energy of electrons emitted from the surface of a metal by means of light is independent of the intensity of light but depends on its frequency (Lenard 1902).
It took a quarter of a century from this observation to the quantum theory of electrodynamics. First the quantum theory for the particles which hitherto had been considered point masses had to be developed until it was possible to quantize the electromagnetic field (P.A.M. Dirac 1927, P. Jordan and W. Pauli, 1928; W. Heisenberg and W. Pauli, 1929; see e.g. W. Heitler, The Quantum Theory of Radiation).

# 31 Gravitational Time Dilatation

## 31.a Light Quantum in the Gravitational Field

Finally we will consider an effect of the general theory of relativity, which can be derived in an elementary way, namely, the different behaviour of clocks in a gravitational potential. The statement is that clocks at different distances from a massive body run differently fast, those at further distance faster, the closer ones slower. This is an effect which had been observed in the Hafele-Keating experiment. In this experiment cesium atomic beam clocks were carried in an airplane around the earth (J.C. Hafele and R. E. Keating, Science 177, 166 (1972)). In this experiment one can observe time dilatation due to different velocities of the airplanes with respect to the center of the earth; but the effect that clocks run differently in different gravitational potentials is of the same order of magnitude. We will now explain this second effect.
We give two explanations: The first one uses the conservation of energy. (Actually, the theorem of conservation of energy does not hold in generally in the general theory of relativity. If however, the space becomes sufficiently plane at large distances then it is still valid. Therefore we need not consider this objection.) If a body of mass $m$ falls the height $h$ in a field of gravitation of acceleration $g$, then it gains $\delta E = mgh$ of kinetic energy. This holds at least for masses of velocity $v \ll c$.
As a consequence, light quanta will gain energy in falling in the field of gravitation and they loose energy while climbing against the field. If this were not true, then one could construct a perpetuum mobile by letting particles and anti-particles falling in the gravitational field, and having them irradiated into light quanta. These could now move up and recombine to a particle anti-particle pair, where one could extract the gained potential energy from the system. Since the energies of all masses are changed by $\delta E = mgh = \frac{gh}{c^2}E$, the same has to hold for light quanta, that is we find for light quanta of energy $E = \hbar\omega$

$$\delta\omega = \frac{\delta E}{\hbar} = \frac{gh}{c^2}\frac{E}{\hbar} = \frac{gh}{c^2}\omega. \tag{31.1}$$

This loss of frequency while leaving a gravitational field is known as the red shift in a gravitational field. It can be measured by means of the Mössbauer effect. A loss of frequency at a height of about 20 m is already sufficient. Thus if we compare the course of two atomic clocks down and up at a difference of height $h$, then one observes that the frequency of the lower clock is smaller by $\delta\omega$. The upper clock is thus faster by a factor of

$$1 + \frac{\delta\omega}{\omega} = 1 + \frac{gh}{c^2}. \tag{31.2}$$

## 31.b Principle of Equivalence

The general theory of relativity does not make use of quantum theory, i.e. it does not use the relation $E = \hbar\omega$. Instead it uses the principle of equivalence. This principle says that a system of reference which moves freely in the gravitational field behaves like a system of inertia. Let us assume we consider a system which moves like a freely falling elevator. Let us assume the lower clock is at a certain time relative to the elevator at rest and radiates upwards with frequency $\omega$. It takes the time $t = h/c$ until the light has arrived the upper clock. During that time the earth and the upper clock have gained the velocity $v = gt$ upwards as seen from the elevator. Thus an observer at the upper clock will observe a Doppler shift by the frequency $\delta\omega = \omega v/c$ (for the weak gravitational field we consider here it is sufficient to consider in subsection 25.e only the contribution linear in $\beta$). Thus we obtain the Doppler shift

$$\delta\omega = \frac{gh}{c^2}\omega, \tag{31.3}$$

which agrees with the result obtained above.
Now you may ask, how can one apply the principle of equivalence, if the gravitational field does not point everywhere in the same direction and is of the same strength. Then, indeed, the description becomes more complicated. Then the description can no longer be founded on a flat space, and one has to dig seriously into the general theory of relativity.