Name Index

Abraham, Max (1875-1922), 53
Aepinus, Franz Ulrich Theodor (1724-1802), 106
Airy, George B. (1801-1892), 103, 105
Alhazen (ibn al Haitham) (965-1038), 103
Ampère, André Marie (1775-1836), 107, 108
Arago, Dominique François Jean (1786-1853), 105, 107

Bernoulli, Daniel (1700-1782), 106
Biot, Jean Baptiste (1774-1862), 36, 105, 107
Bose, George Matthias, 105
Bradley, James (1692-1762), 104
Brewster, David (1781-1868), 64, 104
Browne, Thomas, 105

Cabeo, Niccolo (1585-1650), 108
Canton, John (1718-1772), 106
Cavendish, Henry (1731-1810), 106
Clausius, Rudolf Julius Emmanuel (1822-1888), 24, 109
Coulomb, Charles Augustin de (1736-1806), 5, 106, 107, 109

d'Alembert, Jean Baptiste le Rond (1717-1783), 70
Davy, Humphrey (1778-1829), 107
Desaguliers, Jean Théophile (1683-1744), 105
Descartes, René (1596-1650), 103–105
Dirac, Paul Adrien Maurice (1902-1984), ix, 110
Doppler, Christian (1803-1853), 87
Dufay, Charles (1698-1739), 105

Einstein, Albert (1879-1955), 79, 89, 110
Euler, Leonhard (1707-1783), 104

Faraday, Michael (1791-1867), 43, 107–109
Fermat, Pierre (1601-1665), 104
Fitzgerald, George Francis (1851-1901), 78, 109
Fizeau, Armand Hippolyte (1819-1896), 105, 108
Foucault, Léon (1819-1868), 108
Fourier, Jean Baptiste Joseph (1768-1830), 56, 57
Franklin, Benjamin (1706-1790), 106
Freiberg, Dietrich von (1250-1310), 103, 104
Fresnel, Augustin Jean (1788-1827), 105

Galilei, Galileo (1564-1642), 105
Gassendi, Pierre (1592-1655), 105
Gauss, Carl Friedrich (1777-1855), viii, 6
Gilbert, William (1540-1603), 105, 106
Giorgi, Giovanni (1871-1950), i, 6
Gounelle, E., 108
Gray, Stephen (1666-1736), 105
Green, George (1793-1841), 28, 106, 107
Guericke, Otto von (1602-1686), 105, 106

Hagen, Gotthilf (1797-1884), 64
Heaviside, Oliver (1850-1925), 109
Heisenberg, Werner (1901-1976), 110
Heitler, Walter (1904-1981), 110
Hero of Alexandria (2nd century BC or AD), 103
Hertz, Heinrich Rudolf (1857-1894), 109
Hooke, Robert (1635-1703), 104
Huygens, Christian (1629-1695), 104, 105

Jeans, James Hopwood (1877-1946), 110
Jordan, Pascual (1902-1980), 110

Kamal, al-Din al Farisi (-1320), 103, 104
Kelvin, William Thomson (1824-1907), 106–108
Kepler, Johannes (1571-1630), 103
Kirchhoff, Gustav Robert (1824-1887), 46, 107, 108
Kleist, Ewald Jürgen von (ca. 1700-1748), 105
Kohlrausch, Rudolf (1801-1858), 37, 108
Kopernikus, Nikolaus (1473-1543), 105
Kronecker, Leopold (1823-1891), iv

Lagrange, Joseph (1736-1813), 104
Lambert, Johann Heinrich (1728-1777), 107
Landau, Lev Davidovich (1908-1968), 53
Laplace, Pierre-Simon Marquise de (1749-1827), 104–107
Larmor, Sir Joseph (1857-1942), 100, 109, 110
Lenard, Philipp (1862-1947), 110
Lichtenberg, Georg Christoph (1744-1799), 106
Liénard, Alfred-Marie (1869-1958), 98, 109
Lifshitz, Eugenii Mikhailovich (1915-1985), 53
Lorentz, Hendrik Antoon (1853-1928), 5, 7, 70, 78, 109, 110
Lorenz, Ludvig Valentin (1829-1891), 70, 109

Malus, Étienne Luis (1775-1812), 104
Maupertuis, Pierre-Louis-Moreau (1698-1759), 104
Maxwell, James Clerk (1831-1879), 4, 34, 49, 108, 109
Mayer, Tobias (1723-1762), 107
Michell, John (1724-1793), 107
Michelson, Albert Abraham (1852-1931), 77, 78, 109
Minkowski, Hermann (1864-1909), 53, 78, 110
Mössbauer, Rudolf (1929-), 111
Morley, Edward Williams (1838-1923), 77, 78, 109
Mossotti, Ottaviano Fabrizio (1791-1863), 24, 106, 109
Musschenbroek, Pieter van (1692-1761), 105, 107

Neckam, Alexander (1157-1217), 106

Neumann, Franz Ernst (1798-1895), 108, 109
Newton, Isaac (1643-1727), 104, 105, 107

Ørsted, Hans Christian (1777-1851), 107
Ohm, Georg Simon (1787-1854), 45, 107

Pais, Abraham (1918-2000), 110
Pauli, Wolfgang (1900-1958), 110
Peregrinus, Petrus (Pierre de Maricourt) (13. Jhdt.), 106, 108
Planck, Max (1858-1947), 110
Poincaré, Henri (1854-1912), 109, 110
Poisson, Siméon-Denis (1787-1840), 10, 105–107
Poynting, John Henry (1852-1914), 51, 109
Priestley, Joseph (1733-1804), 106
Ptolemaios, Klaudius (2. Jhdt. n. Chr.), 103

Rayleigh, John William Strutt (1842-1919), 110
Riemann, Bernhard (1826-1866), 108, 109
Robison, John (1739-1805), 106
Römer, Olaf (1644-1710), 104
Rubens, Heinrich (1865-1922), 64

Sanuto, Livio (etwa 1530-1580), 106
Savart, Félix (1791-1841), 36, 107
al-Schirazi, Mahmud ibn (1236-1311), 103, 104
Schwarzschild, Karl (1873-1916), 109
Snell, Willibrord van Royen (1581-1626), 62, 104
Sommerfeld, Arnold Johannes Wilhelm (1868-1951), 53
Stokes, George Gabriel (1819-1903), vii, 105
Stoney, Johnstone (1826-1911), 108
Sturgeon, William (1783-1850), 107

Taylor, Brook (1685-1731), 107
Thales v. Milet (640-546), 103, 105
Thomson, Joseph John (1856-1940), 109
Thomson, William (Lord Kelvin) (1824-1907), 106–109

Voigt, Woldemar (1850-1919), 110
Volta, Alessandro Graf (1745-1827), 107

Wall, 105
Watson, William (1715-1787), 105
Weber, Wilhelm (1804-1894), 37, 108
Wheatstone (1802-1875), 108
Wheeler, William, 108
Wideröe, Rolf (1902-), 44
Wiechert, Emil (1861-1928), 98, 109
Wien, Wilhelm (1864-1928), 110
Wiener, Otto Heinrich (1867-1927), 109
Wilcke, Johann Carl (1732-1796), 106, 108, 109
Wilson, Benjamin (1721-1788), 105
Winkler, Johann Heinrich, 105

Young, Thomas (1773-1829), 103–105

Subject Index

aberration, 104, 105
action, 92, 104
addition theorem
  velocities, 82
amber, 103, 105
Ampere's law, 35
angular momentum conservation, 96

battery, 107
betatron, 43
Biot-Savart law, 36
Bohr's magneton, 39
boundary condition
  electric field, 23
  magnetic field, 41
Brewster's angle, 64, 104

capacities, 27
capacitor, 27
  plate, 28
  spherical, 27
charge
  influence, 28
charge conservation, 4, 49, 95, 105
charge density, 3
  freely moving, 22, 49
  polarization, 22, 50
charge density per area, 3
circuit
  electric, 45–46
Clausius-Mossotti relation, 25
coil
  long, 45
  magnetic, 36, 42
components
  contravariant, 78
  covariant, 78
conducting plane, 29
conducting sphere, 30
conductor, 105
  electric, 26
  force on, 26
conservation laws, 95–97
corkscrew rule, 35
Coulomb force, 5, 11
Coulomb law, 106
curl, v
current
  electric, 4
current density
  electric, 4
  free charges, 41
  freely moving, 49
  magnetization, 41, 50
  polarization, 50
current law, 46

d'Alembert operator, 70, 83
del operator, v
density of energy current, 94
density of force, 5
dielectric constant, 23
  Clausius, Mossotti, 24
dielectric displacement, 22
diffraction, 105
dipolar moment
  electric, 12–13, 20
  magnetic, 38
    current loop, 39
    Spin, 39
dipole
  electric
    angular momentum, 14
    energy, 14
    field, 12
    force, 14
    potential, 12
  magnetic
    field, 38
    force, 39
    torque, 40
dipole radiation
  electric, 74
  magnetic, 75
Dirac delta, ix, 3
displacement current, 109
distance, 81
  light-like, 81
  space-like, 81
  time-like, 81
divergence, v
Doppler effect, 87
Doppler shift, 88
double-refraction, 104

elasticity, theory of, 105
electro-magnet, 107
electromotive force, 45
electron, 108
electron radius
  classical, 11
electrostatics, 9–34

history, 105–106
energy
dipole
electric, 15
magnetic, 48
electrostatic, 11, 31
inductance, 46
quadrupole
electric, 15
energy balance, 51–52
energy current
plane wave, 56
energy current density, 51
energy density, 94
electromagnetic, 51
electrostatic, 11
plane wave, 56
energy momentum tensor, 94–97
energy, momentum conservation, 96
equation of continuity, 4, 83
equation of motion, 91
eye glasses, 104

field
charge distribution, 9
dipole
magnetic, 38
magnetic, divergency free, 108
of current distribution, 36
field lines
made visible, 108
field strength
electric, 9
magnetic, 41
field tensor, 85–86
dual, 86
fields
point charge, 87
flux
dielectric displacement, 22
electric, 10
magnetic, 35, 43
force
between circuits, 37, 47
between currents, 107
Coulomb, 5
Lorentz, 5
on electric dipole, 15
on magnetic dipole, 39
force density
dielectric matter, 31
electromagnetic, 51–54, 94
four-momentum, 90
four-potential, 83
four-scalar, 81
four-tensor
antisymmetric, 85
symmetric, 94
four-vector, 82–84
current density, 82
Fourier integrals, 56–57
Fourier series, 56–57

Galilei invariance, 77
gauge
Coulomb, 70, 109
Lorenz, 70, 109
gauge transformation, 69
Gauss' theorem, viii
gradient, v
Green's theorem, viii
group velocity, 67

Hafele-Keating experiment, 111
Hagen-Rubens law, 64
Hertz dipole, 74

index of refraction, 62
inductance, 45–48
induction
magnetic, 35–37
influence, 105, 106
influence charge, 28
integral theorems, vii

Kirchhoff's rules, 46
Kronecker delta, iv

Liénard-Wiechert potential, 98
Lagrange function, 92–93
Laplacian, vi, ix
law of induction, 43, 107
Legendre functions
associated, x
Levi-Civita symbol, iv
light velocity, 108
constance, 79
lightning, 103
line integrals, vii
loop of current, 38
Lorentz contraction, 80
Lorentz force, 5, 107, 109
Lorentz force density, 89
Lorentz invariance, 77
Lorentz transformation, 77–80
proper, 79

magnetite, 103
magnetization, 41
magnetostatics, 9, 35–42

history, 106–107
Maxwell's equations, 4, 49–50, 85
in matter, 50
mechanics
relativistic, 89–91
metallic reflection, 64
metric tensor, 78
Michelson-Morley experiment, 77
Minkowski force, 89
Minkowski space, 78
momentum
electromagnetic, 109
momentum balance, 52–54
momentum density, 94
electromagnetic, 52
monopole
electric, 20
magnetic, 38, 42, 106
movement of center of mass, 96
multipole
electric, 18–21
potential, 19

optics, history, 105

parabolic mirror, 103
permeability, 41
phase velocity, 67
pinhole camera, 103
plasma frequency, 61
plasma oscillations, 61
point charge, 3, 10
field, 98
radiation, 100
Poisson's equation, 10
polarization, 22
potential
advanced, 72
charge distribution, 9
charge, current distribution, 71–72
electric, 9
magnetic, 42, 107
retarded, 109
retarted, 72, 84
scalar, 69
potentials, 69–70
Poynting vector, 51, 56, 94, 109
proper time, 81

quadrupolar moment
electric, 12–14, 20
quadrupole
electric
energy, 14
potential, 12
quadrupole radiation
electric, 75

radiation field, 73
radiation resistance, 75
rainbow, 103, 104
reflection, 62–64
refraction, 62–64
relativity
general theory, 110
special theory, 77–97
relativity, theory of
history, 109–110
resistor, 107
ohmic, 45
rest energy, 90

scalar product, iv
scalar triple product, v
Snell's law, 62, 104
solid angle
element of, 16
sphere
dielectric, 24
spherical coordinates, 16
divergence, 17
gradient, 16
Laplacian, 17
volume element, 16
spherical harmonics, x, 17
addition theorem, 20
Stokes' theorem, vii
strength of pole, 42
stress tensor, 94, 109
electromagnetic, 52
electrostatic, 34
surface integrals, vii
susceptibility
electric, 23
magnetic, 41
systems of units, i–iii, 6–7

TE-mode, 66
TEM-mode, 66
time averages, 58–59
time dilatation, 80
gravitational, 111
TM-mode, 66
torque
on electric dipole, 15
on magnetic dipole, 40
total reflection, 64
transformation of fields, 86

units, i–iii, 6–7

vector algebra, iv
vector analysis, v
vector gradient, v
vector potential, 36–37, 69, 108
vector product, iv
velocity of light, 5, 104
    vacuum, 56
voltage law, 46
voltage source, 45
volume integrals, viii

wave
    at surface, 64
    circularly polarized, 58
    electromagnetic, 55–59
    in conductor, 60–61
    linearly polarized, 58
    longitudinal, 61
    plane, 55–56
    transversal, 105
wave equation, 55
wave guide, 65–68
wave packet, 67
world velocity, 82

Contents Electrodynamics

A Basic Equations **3**

1 Basic Equations of Electrodynamics **3**
1.a Charges and Currents . . . 3
1.a.α Charge Density . . . 3
1.a.β Current and Current Density . . . 4
1.a.γ Conservation of Charge and Equation of Continuity . . . 4
1.b Maxwell's Equations . . . 4
1.c Coulomb and Lorentz Force . . . 5

2 Dimensions and Units **6**
2.a Gaussian Units . . . 6
2.b Other Systems of Units . . . 6
2.c Motivation for Gaussian Units . . . 7

B Electrostatics **9**

3 Electric Field, Potential, Energy of the Field **9**
3.a Statics . . . 9
3.b Electric Field and Potential . . . 9
3.b.α Electric Potential . . . 9
3.b.β Electric Flux and Charge . . . 10
3.b.γ Potential of a Charge Distribution . . . 10
3.c Coulomb Force and Field Energy . . . 11

4 Electric Dipole and Quadrupole **12**
4.a The Field for $r > R$ . . . 12
4.b Transformation Properties . . . 13
4.c Dipole . . . 13
4.d Quadrupole . . . 14
4.d.α Symmetries . . . 14
4.d.β Symmetric Quadrupole . . . 14
4.e Energy, Force and Torque on a Multipole in an external Field . . . 14

5 Multipole Expansion in Spherical Coordinates **16**
5.a Poisson Equation in Spherical Coordinates . . . 16
5.a.α The Gradient . . . 16
5.a.β The Divergence . . . 17
5.a.γ The Laplacian . . . 17
5.b Spherical Harmonics . . . 17
5.c Radial Equation and Multipole Moments . . . 18
5.d Point Charge at $\mathbf{r}'$, Cylindric Charge Distribution . . . 20

6 Electric Field in Matter **22**
6.a Polarization and Dielectric Displacement . . . 22
6.b Boundaries between Dielectric Media . . . 23
6.c Dielectric Sphere in a Homogeneous Electric Field . . . 24
6.d Dielectric Constant according to Clausius and Mossotti . . . 24

7 Electricity on Conductors **26**
7.a Electric Conductors . . . 26
7.a.α Boundary Conditions at the Surface of the Conductor . . . 26
7.a.β Force acting on the Conductor (in Vacuo) . . . 26
7.b Capacities . . . 27
7.c Influence Charges . . . 28
7.c.α Space free of Conductors . . . 29
7.c.β Conducting Plane . . . 29
7.c.γ Conducting Sphere . . . 30

8 Energy, Forces and Stress in Dielectric Media **31**
8.a Electrostatic Energy . . . 31
8.b Force Density in Isotropic Dielectric Matter . . . 31
8.c MAXWELL's Stress Tensor . . . 34

C Magnetostatics **35**

9 Magnetic Induction and Vector Potential **35**
9.a AMPERE's Law . . . 35
9.b Magnetic Flux . . . 35
9.c Field of a Current Distribution . . . 36
9.d Vector Potential . . . 36
9.e Force Between Two Circuits . . . 37

10 Loops of Current as Magnetic Dipoles **38**
10.a Localized Current Distribution and Magnetic Dipole . . . 38
10.b Magnetic Dipolar Moment of a Current Loop . . . 39
10.c Force and Torque on a Dipole in an External Magnetic Field . . . 39
10.c.α Force . . . 39
10.c.β Torque . . . 40

11 Magnetism in Matter. Field of a Coil **41**
11.a Magnetism in Matter . . . 41
11.b Field of a coil . . . 42

D Law of Induction **43**

12 FARADAY's Law of Induction **43**

13 Inductances and Electric Circuits **45**
13.a Inductances . . . 45
13.b Elements of Circuits . . . 45
13.c KIRCHHOFF's Rules . . . 46
13.d Energy of Inductances . . . 46
13.d.α Constant Inductances . . . 47
13.d.β Moving Loops of Currents . . . 47
13.d.γ Constant Magnetic Fluxes . . . 47
13.d.δ Force between two Electric Circuits . . . 47
13.d.ϵ Energy of a Magnetic Dipole in an External Magnetic Induction . . . 48
13.d.ζ Permanent Magnetic Moments . . . 48

E MAXWELL's Equations **49**

14 Complete Set of MAXWELL's Equations **49**
14.a Consistency of MAXWELL's Equations . . . 49
14.b MAXWELL's Equations for Freely Moving Charges and Currents . . . 49

15 Energy and Momentum Balance **51**
15.a Energy . . . 51
15.b Momentum Balance . . . 52

F Electromagnetic Waves **55**

16 Electromagnetic Waves in Vacuum and in Homogeneous Isotropic Insulators **55**
16.a Wave Equation . . . 55
16.b Plane Waves . . . 55
16.c Superposition of Plane Periodic Waves . . . 56
16.c.α Insertion on Fourier Series and Integrals . . . 56
16.c.β Back to Maxwell's Equations . . . 57
16.c.γ Time averages and time integrals . . . 58

17 Electromagnetic Waves in Homogeneous Conductors **60**
17.a Transverse Oscillations at Low Frequencies . . . 60
17.b Transverse Oscillations at High Frequencies . . . 61
17.c Longitudinal = Plasma Oscilations . . . 61

18 Reflection and Refraction at a Planar Surface **62**
18.a Problem and Direction of Propagation . . . 62
18.b Boundary Conditions, Amplitudes . . . 62
18.c Discussion for $\mu_1 = \mu_2$ . . . 63
18.c.α Insulator, $|\sin\alpha_2| < 1$: Refraction . . . 63
18.c.β Insulator, $|sin\alpha_2| > 1$: Total Reflection . . . 64
18.c.γ Metallic Reflection, $\alpha = 0$ . . . 64
18.c.δ Surface Waves along a Conductor . . . 64

19 Wave Guides **65**
19.a Wave Guides . . . 65
19.b Solution for a Rectangular Cross Section . . . 67
19.c Wave Packets . . . 67

G Electrodynamic Potentials **69**

20 Electrodynamic Potentials, Gauge Transformations **69**
20.a Potentials . . . 69
20.b Gauge Transformations . . . 69

21 Electromagnetic Potentials of a general Charge and Current Distribution **71**
21.a Calculation of the Potentials . . . 71
21.b Gauge Condition . . . 72

22 Radiation from Harmonic Oscillations **73**
22.a Radiation Field . . . 73
22.a.α Near Zone (Static Zone) . . . 73
22.a.β Far Zone (Radiation Zone) . . . 73
22.b Electric Dipole Radiation (Hertz Dipole) . . . 74
22.c Magnetic Dipole radiation and Electric Quadrupole Radiation . . . 75
22.c.α Magnetic Dipole Radiation . . . 76
22.c.β Electric Quadrupole Radiation . . . 76

H Lorentz Invariance of Electrodynamics **77**

23 Lorentz Transformation **77**
23.a Galilei and Lorentz Transformation . . . 77
23.b Lorentz Transformation . . . 78
23.b.α Time Dilatation . . . 80
23.b.β Lorentz Contraction . . . 80

24 Four-Scalars and Four-Vectors **81**
24.a Distance and Proper Time as Four-Scalars . . . 81
24.a.α Space-like distance $s^2 < 0$ . . . 81
24.a.β Time-like distance $s^2 > 0$ . . . 81
24.a.γ Light-like distance $s^2 = 0$ . . . 81
24.b World Velocity as Four-Vector . . . 82
24.c Current Density Four-Vector . . . 82
24.d Four-Potential . . . 83

25 Electromagnetic Field Tensor **85**
25.a Field Tensor . . . 85
25.b Maxwell's Equations . . . 85
25.b.α The Inhomogeneous Equations . . . 85
25.b.β The Homogeneous Equations . . . 86
25.c Transformation of the Electric and Magnetic Fields . . . 86
25.d Fields of a Point Charge in Uniform Motion . . . 87
25.e Doppler Effect . . . 87

26 Relativistic Mechanics **89**
26.a Lorentz Force Density . . . 89
26.b Lorentz Force Acting on a Point Charge . . . 89
26.c Energy and Momentum of a Mass Point . . . 89
26.d Equation of Motion . . . 91

27 Lagrangian Formulation **92**
27.a Lagrangian of a Massive Charge in the Electromagnetic Field . . . 92
27.b Lagrangian Density of the Electromagnetic Field . . . 92

28 Energy Momentum Tensor and Conserved Quantities **94**
28.a The Tensor . . . 94
28.b Conservation Laws . . . 95
28.b.α Charge . . . 95
28.b.β Energy and Momentum . . . 96
28.b.γ Angular Momentum and Movement of Center of Mass . . . 96

29 Field of an arbitrarily Moving Point-Charge **98**
29.a Liénard-Wiechert Potential . . . 98
29.b The Fields . . . 98
29.c Uniform Motion . . . 99
29.d Accelerated Charge Momentarily at Rest . . . 99
29.e Emitted Radiation $\beta \neq 0$ . . . 100

I Review and Outlook **103**

30 Short History of Electrodynamics **103**
30.a Theory of Light to Fresnel . . . 103
30.b Electrostatics . . . 105
30.c Magnetostatics . . . 106
30.d Set out for Electrodynamics . . . 107
30.e Electrodynamics and Waves . . . 108

30.f Theory of Relativity . . . 109
30.g From Classical to Quantum Electrodynamics . . . 110

31 Gravitational Time Dilatation 111
31.a Light Quantum in the Gravitational Field . . . 111
31.b Principle of Equivalence . . . 111

Appendices i

A Connection between different Systems of Units i

B Formulae for Vector Calculus iv
B.a Vector Algebra . . . iv
B.a.α Summation Convention and Orthonormal Basis . . . iv
B.a.β Scalar Product . . . iv
B.a.γ Vector Product . . . iv
B.a.δ Multiple Products . . . v
B.b Vector Analysis . . . v
B.b.α Differentiation in Space, Del Operator . . . v
B.b.β Second Derivatives, Laplacian . . . vi
B.b.γ Derivatives of Products . . . vi
B.c Special Expressions . . . vi
B.d Integral Theorems . . . vii
B.d.α Line Integrals . . . vii
B.d.β Surface Integrals . . . vii
B.d.γ Volume Integrals . . . viii
B.d.δ Volume Integrals of Products . . . viii
B.e The Laplacian of $1/r$ and Related Expressions . . . ix
B.e.α The Laplacian of $1/r$ . . . ix
B.e.β Representation of a Vector Field as a Sum of an Irrotational and a Divergence-free Field ix

C Spherical Harmonics x
C.a Eigenvalue Problem and Separation of Variables . . . x
C.b Associated LEGENDRE Functions . . . x
C.c Orthogonality and Normalization . . . xii
C.d Remark on Completeness . . . xii

Name Index xv

Subject Index xvii

Contents xxi